# Electron-Ion Coulomb Shearing as a Dominant Growth Mechanism in Eddington-Limited Regimes

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## Abstract

Observations of supermassive black holes, from nearby systems to luminous objects in the early universe, continue to challenge classical growth models based on Eddington-limited accretion. When the Eddington limit is applied as a radiation force balance on electrons and combined with the standard assumption of perfect electron–ion coupling, the resulting growth rates fail to account for observed masses within available cosmic time. This work demonstrates that the inconsistency arises not from a breakdown of Eddington physics, but from the failure of the coupling assumption itself under radiation-dominated conditions.

We present Coulomb Shearing, a physically grounded mechanism in which finite, time-dependent Coulomb coupling allows electrons and ions to respond differently to anisotropic forcing near rotating black holes. Radiation pressure acting on electrons saturates at the Eddington limit along preferentially polar or near-polar trajectories, while ions, no longer rigidly bound to the electron response, continue inward. The result is a sustained coexistence of electron-dominated outflow and ion-dominated inflow, emerging naturally from species-resolved dynamics rather than from imposed geometry or jet assumptions.

The paper develops this mechanism quantitatively by computing differential accelerations, coupling and residence timescales, and species-dependent shearing windows, and by mapping these into observable consequences and growth behavior. The framework yields concrete, testable predictions, including stratified outflow composition, polarization and Faraday signatures tied to viewing geometry, and enhanced effective growth relative to classical Eddington-limited expectations.

The full calculation pipeline is executed on real astrophysical systems, including M87* and Sgr A*, using fixed inputs and a pre-committed evaluation protocol that admits only pass, fail, or data-insufficient outcomes. By combining first-principles plasma physics with observationally anchored testing, this work reframes black hole growth as a species-resolved transport problem and establishes Coulomb Shearing as a decisive mechanism that either succeeds quantitatively in the relevant regimes or fails transparently when confronted with data.

## Correspondence and Reproducibility

Correspondence regarding technical errors, reproducibility concerns, or conflicting observational evidence is welcome and may be directed to the author at the contact address listed on the title page. Where possible, claims in this work are structured to permit independent verification using publicly available data. A best-effort attempt has been made to eliminate technical and typographical errors; however, errors may remain. Corrections and substantive technical critique are explicitly invited, and will be incorporated in subsequent versioned releases where warranted.

## Note on the Use of AI Tools

AI-assisted tools were used in a bounded, support role: (i) language editing for clarity and conformity with common scientific style, (ii) LaTeX formatting and document tooling, (iii) assistance in locating publicly available inputs used in worked examples, and (iv) supplemental mathematical checks in limited sub-derivations. All model claims, physical assumptions, and final technical judgments are the author's responsibility. Readers who identify errors, ambiguities, or conflicts with observation are encouraged to contact the author so the issue can be evaluated and, if necessary, corrected or treated as disconfirming.

## Versioning and Errata Policy

A best-effort attempt has been made to eliminate technical and typographical errors; however, errors may remain. Corrections and substantive technical critique are explicitly invited, and will be incorporated in subsequent versioned releases where warranted.

## Executive Summary

This paper addresses the long-standing tension between observed black hole masses and classical growth limits without discarding established radiation physics. The Eddington limit is treated throughout as physically correct when applied as a radiation force balance on electrons. The central problem examined here is not the limit itself, but the additional assumption commonly imposed in growth models: that electrons and ions remain perfectly and instantaneously coupled under all conditions relevant to accretion. When that coupling assumption is enforced, growth timescales become incompatible with observations across a wide range of systems, including both nearby and early-universe black holes.

The core claim of this work is that, in radiation-dominated regimes around rotating black holes, electron–ion coupling fails in a controlled and quantifiable way. The paper develops a species-resolved framework in which matter is preferentially transported onto polar or near-polar trajectories, where forcing becomes strongly anisotropic. Radiation pressure acts directly on electrons and saturates at the Eddington limit, while ions respond differently because Coulomb coupling is finite and time-dependent. Under these conditions, joint motion is not enforced over the relevant residence times, leading to sustained electron-dominated outflow concurrent with ion-dominated inflow. This process is referred to as Coulomb Shearing and is treated as the dominant driver of mass growth in the regimes examined.

The manuscript proceeds by building this mechanism step by step. It first establishes the geometric and kinematic conditions under which polar or near-polar transport arises in rotating black hole spacetimes. It then computes the differential forcing experienced by electrons and ions along those trajectories and evaluates when Coulomb coupling fails to maintain collective behavior. Species-dependent outcomes are derived, producing well-defined shearing windows and separation behavior that depend on physical conditions rather than on imposed assumptions.

These species-resolved dynamics are then connected to observable consequences. The framework predicts stratified outflow structure, specific polarization and Faraday signatures under defined viewing geometries, and a clear separation between axial electron-dominated regions and surrounding ion-bearing material. The same dynamics also yield a modified growth channel in which ion inflow is no longer bound by electron saturation, leading to quantitatively enhanced effective growth relative to classical Eddington-limited expectations.

To ensure that these claims are testable rather than interpretive, the paper adopts a pre-committed evaluation protocol. Inputs are fixed, uncertainties are handled explicitly, and each system evaluated is assigned one of three outcomes: pass, fail, or data-insufficient. There is no parameter tuning to improve agreement and no reclassification to avoid failure. The full calculation pipeline is executed on real astrophysical systems, including M87* and Sgr A*, with results presented as computed distributions rather than illustrative examples.

Overall, the paper reframes black hole growth as a species-resolved transport and coupling problem rather than a purely radiative one. By combining plasma-scale physics, relativistic geometry, and observation-facing tests within a single quantitative framework, it provides a decisive assessment of Coulomb Shearing as a mechanism that either succeeds in explaining growth in radiation-dominated

regimes or fails transparently when confronted with data.

# Contents

## Roadmap and dependency order

This manuscript is intentionally modular, but several definitions are used across multiple sections. To reduce circular cross-references, the dependency order is fixed as follows:

1. **Problem statement and scope.** Section 1.1 defines the growth paradox and clarifies that Coulomb Shearing is a stratification and growth-channel mechanism, not a jet-launching model.
2. **Transport to polar-adjacent trajectories.** Section 2 defines the polar delivery fraction $f_{\text{polar}}$ and the residence-time control variable $t_{\text{res}}$.
3. **Differential forcing and the closure test.** Section 3 defines the projected differential acceleration $\Delta a_{\parallel}$, the coupling time $t_c$, the screening scale $\lambda_D$, and the admissibility functional that determines whether closure can fail locally.
4. **Species-dependent windows.** Section 4 maps the closure test into $(r_{\min}(Z), r_{\max}(Z))$ windows and the associated shearing probability $P_s(Z)$.
5. **Observable consequences.** Section 5 defines the scorable transverse observables (P1–P4) and their required viewing-geometry conditions.
6. **Growth-channel consequence.** Section 6 defines the two-channel growth mapping and the inverse-age consequence.
7. **Evaluation protocol.** Section 8 locks inputs, uncertainty handling, and PASS/FAIL/DATA-INSUFFICIENT scoring, including the falsification leverage requirement.
8. **Executed examples and outputs.** Section 9 demonstrates the calculation pipeline on named anchors and provides computation-backed figures.

Supporting material is partitioned as follows: Appendix A is the symbol registry, Appendix B contains derivations, Appendix C fixes source admissibility and provenance rules, and Appendix D fixes the worked-example input packet rules. When a key assumption is required for interpretation, it is stated in the main text and linked to the relevant appendix for traceability, rather than being discoverable only by appendix reading.

**How source classes map onto this regime-first structure.** The manuscript does not treat "quasar," "quasi-star," or "long-jet system" as mechanism labels. Instead, each source class is decomposed into the physical regime declarations that the pipeline actually evaluates (transport geometry, forcing channel specialization, plasma composition, and admissibility gates). In this mapping, "quasar" denotes a high-luminosity SMBH regime that often motivates electromagnetic specializations; "black hole star" denotes a conditional external environment hypothesis that is only invoked when its required inputs are available; and "extremely long jets" denotes an observational consequence class whose interpretation is tied to the effective-geometry and residence-time controls rather than to a separate mechanism.

# 1 The Growth Paradox

## 1.1 Observational problem

A persistent and quantitatively defined growth paradox exists. Supermassive black holes with masses $\gtrsim 10^9\, M_\odot$ are observed at redshifts $z \gtrsim 6$, corresponding to cosmic ages $\lesssim 1\,\mathrm{Gyr}$ after the Big Bang. Under classical Eddington-limited growth with perfect electron–ion coupling, such masses cannot be assembled within the available time, even under continuous accretion at the Eddington rate.

A canonical observational anchor is the quasar ULAS J1120+0641 at redshift $z = 7.085$, corresponding to a cosmic age of approximately $0.77\,\mathrm{Gyr}$. Its central black hole mass is measured to be $(2.0 \pm 0.7) \times 10^9\, M_\odot$, inferred from virial broad-line methods calibrated against local active galactic nuclei. The observed bolometric luminosity is close to the Eddington luminosity for this mass, with an inferred Eddington ratio of order unity. These values are reported by Mortlock et al. (2011), who emphasize that sustained near-Eddington accretion is already assumed in their interpretation.

A second anchor is the quasar SDSS J010013.02+280225.8 at redshift $z = 6.30$, corresponding to a cosmic age of approximately $0.88\,\mathrm{Gyr}$. Its black hole mass is estimated at $(1.2 \pm 0.2) \times 10^{10}\, M_\odot$, with a bolometric luminosity again consistent with accretion near the Eddington limit. These measurements are reported by Wu et al. (2015), who note that the inferred mass already strains standard growth scenarios.

Under classical Eddington-limited growth, the black hole mass evolves as

$$M(t) = M_0 \exp\left(\frac{t}{t_{\mathrm{Sal}}}\right), \tag{1}$$

where $M_0$ is the initial mass and $t_{\mathrm{Sal}}$ is the Salpeter timescale,

$$t_{\mathrm{Sal}} = \frac{\epsilon}{1-\epsilon}\frac{c\,\sigma_T}{4\pi G m_p} \approx 45\,\mathrm{Myr} \tag{2}$$

for a radiative efficiency $\epsilon = 0.1$. This expression follows directly from equating the accretion luminosity to the Eddington luminosity and assuming perfect coupling between electrons and ions.

Starting from an initial mass of $M_0 = 10^2\, M_\odot$, representative of a stellar-remnant progenitor, growth over $0.77\,\mathrm{Gyr}$ allows for approximately 17 e-foldings, yielding a maximum mass of $\sim 2 \times 10^9\, M_\odot$ only if accretion proceeds continuously at the Eddington limit without interruption. This bound is already saturated by ULAS J1120+0641 and is exceeded by SDSS J010013.02+280225.8 by a factor of approximately six. Any reduction in duty cycle, increase in radiative efficiency, or delay in the onset of accretion worsens the discrepancy.

Alternatively, one may compute the minimum required super-Eddington factor. To reach $1.2 \times 10^{10}\, M_\odot$ from $10^2\, M_\odot$ within $0.88\,\mathrm{Gyr}$ requires an effective e-folding time of $\sim 35\,\mathrm{Myr}$, corresponding to a sustained luminosity exceeding the Eddington limit by a factor of $\gtrsim 1.3$ under perfect coupling. Larger seed masses reduce but do not eliminate this requirement unless seed masses exceed $\sim 10^6\, M_\odot$, which shifts rather than resolves the time-budget problem.

These quantitative failures do not imply incorrect mass measurements, erroneous redshifts, or invalid luminosity estimates. They also do not imply that the Eddington limit itself is incorrect. They demonstrate that, under the classical assumption of perfect electron–ion coupling, the Eddington-limited growth law is insufficient to account for the observed masses within the available cosmic time.

### 1.2 The Eddington limit as a correct and foundational result

The Eddington limit is a physically correct and foundational result. It expresses the balance between the outward force exerted by radiation on electrons and the inward gravitational force acting on the mass associated with those electrons in an ionized plasma.

Radiative momentum transfer couples directly to electrons through Thomson scattering, characterized by the Thomson cross section $\sigma_T = 6.65 \times 10^{-29}\,\mathrm{m}^2$. Gravity acts on mass, which in a baryonic plasma is dominated by ions, with a characteristic mass per electron of approximately the proton mass $m_p = 1.67 \times 10^{-27}$ kg. For a spherically symmetric radiation field of luminosity $L$ produced by a central mass $M$, the outward radiative force per electron is

$$F_{\mathrm{rad}} = \frac{\sigma_T L}{4\pi r^2 c}, \tag{3}$$

while the inward gravitational force on the associated mass column is

$$F_{\mathrm{grav}} = \frac{G M m_p}{r^2}, \tag{4}$$

where $G$ is the gravitational constant and $c$ is the speed of light.

Equating these forces yields the classical Eddington luminosity,

$$L_{\mathrm{Edd}} = \frac{4\pi G M m_p c}{\sigma_T}, \tag{5}$$

which represents the luminosity at which outward radiative acceleration of electrons exactly balances inward gravitational acceleration of the electron-associated mass column. Every symbol in this expression has a direct physical meaning and standard units: $G$ in $\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$, $M$ in kilograms, $m_p$ in kilograms, $c$ in $\mathrm{m\,s}^{-1}$, and $\sigma_T$ in $\mathrm{m}^2$.

The physical content of this balance is unambiguous. Radiative momentum transfer is mediated by electron-scale interaction cross-sections, while gravitational acceleration is set by the total mass associated with each electron, which in a baryonic plasma is dominated by ions. The translation of this electron-level force balance into a bound on bulk mass inflow requires an additional assumption: that electrons and ions move together as a single effective fluid. This assumption is not part of the force balance itself but a closure used to relate electron dynamics to ion dynamics.

The Eddington limit is not violated, invalidated, or replaced in this work. It remains the correct expression of radiative force balance at the electron level and the appropriate starting point for any discussion of accretion regulated by radiation pressure.

### 1.3 The closure assumption suppressed by angular averaging

The conversion of the Eddington electron force balance into a bound on ion inflow relies on a specific closure assumption: that electrons and ions remain sufficiently and instantaneously coupled that an electron-level force balance can be promoted to a constraint on bulk mass inflow. In standard accretion treatments, this closure is not imposed pointwise along individual trajectories, but is instead enforced after angular and temporal averaging of the radiation field and plasma response.

Under this assumption, the outward radiative force experienced by electrons is immediately transmitted to ions through Coulomb interactions, and the Eddington limit on electron acceleration becomes a limit on the inward motion of the mass-dominant ionic component. In this framework, ions experience radiation force only indirectly, mediated entirely by their coupling to electrons.

Finite electron–ion coupling, two-temperature plasmas, and differential response to external forcing are well-established features of plasma physics and are extensively documented in both laboratory and astrophysical contexts [1, 2]. The existence of finite Coulomb relaxation timescales and localized electron–ion disequilibrium is therefore not in question.

In classical accretion theory, the Eddington limit is derived and applied using angle-averaged radiation fields and effective single-fluid descriptions, so that electron-level force balance is promoted to a bound on bulk mass inflow only after angular and temporal integration [3, 4]. This averaging suppresses trajectory-resolved differential acceleration by construction and constrains ion inflow only in an integrated sense, not along individual plasma trajectories.

In standard accretion treatments, however, the Eddington limit is applied after angular and temporal averaging, implicitly assuming that electron–ion coupling remains effective on the scales relevant for mass transport, so that electron-level force balance can be promoted to a bound on bulk inflow [3].

What is not standard in classical accretion closures is the use of this finite coupling to reopen the logical step that converts an electron-level radiative force balance into a global ion inflow bound under strongly anisotropic, trajectory-resolved conditions. In particular, the Eddington limit is typically applied after angular and temporal averaging, implicitly assuming that any differential electron response is rapidly erased before it can influence mass transport.

Coulomb coupling, however, has finite strength and finite response time. The joint motion of electrons and ions is therefore not guaranteed locally and temporarily when differential forcing is large or rapidly varying. This statement does not introduce a mechanism or outcome; it identifies a physical property of the coupling itself.

The circumstances under which this distinction matters are restricted. They require anisotropic forcing rather than isotropic averaging, trajectory channels that are polar or near-polar rather than equatorial, and an environment shaped by black hole rotation, which introduces spin-linked anisotropy in both radiation fields and plasma dynamics.

Finite electron–ion coupling permits sustained differential acceleration along polar trajectory channels in such anisotropic environments. The averaged closure that elevates the electron Eddington balance into a bound on ion inflow therefore does not apply locally, and ion inflow is not constrained in the same manner.

**Why angular averaging does not constrain a polar channel.** The step that promotes the single-particle electron force balance into a bound on *bulk* inflow is not the Eddington derivation itself; it is an additional closure that replaces trajectory-resolved forcing with an averaged field. Let $F(r, \theta, \phi, t)$ denote the local radiative flux. Standard Eddington-style inflow bounds implicitly appeal to an effective flux

$$\langle F \rangle_{\Omega,t}(r) \equiv \frac{1}{4\pi\,\Delta t} \int_{t_0}^{t_0+\Delta t} \int F(r, \theta, \phi, t)\, d\Omega\, dt, \tag{6}$$

together with an assumption that the plasma response rapidly enforces co-motion so that this averaged forcing is representative of the momentum budget of the inflow.

A polar trajectory, however, does not sample $\langle F \rangle_{\Omega,t}$. It samples a *restricted* forcing history $F[r(t), \theta(t), \phi(t), t]$ along a narrow tube around $\theta \simeq 0$, in a geometry where (i) the radiation field is generically anisotropic because the emitting disk is not spherically symmetric, and (ii) the plasma dynamics are anisotropic because magnetic stresses and frame-dragging guide transport into a funnel-like region. In that situation, the quantity controlling whether electron–ion locking is maintained is not an angular average, but the *trajectory-resolved differential forcing integrated over the residence time.*

Accordingly, the claim made here is conditional and testable: if the forcing along the polar tube has coherence over times comparable to the coupling time (Differential Forcing and the Onset of Coulomb Shearing), then angular averaging is not the appropriate closure and the bulk inference can fail *locally* even when the global Eddington derivation remains correct. If instead the polar forcing decorrelates on times $\ll t_c$ or the medium is sufficiently collisional that $t_c \ll t_{\rm res}$ everywhere, then the standard closure is recovered and the polar exception disappears.

### 1.4 JWST compact high-redshift objects: Little Red Dots and black-hole-star hypotheses

JWST surveys have revealed a populous class of extremely compact sources with blue rest-UV emission but a red rest-optical continuum, commonly referred to as *Little Red Dots* (LRDs). In multiple samples, LRDs span approximately $z \sim 2$–11 and appear at number densities that can exceed those of UV-selected or X-ray-selected AGN at similar epochs, with several objects showing broad recombination lines and other AGN-consistent signatures in NIRSpec spectroscopy [5–8].

These sources matter to the growth paradox for two independent reasons. First, if even a subset of LRDs host accreting black holes at $z \gtrsim 4$, their abundance shifts the problem from a small number of rare, luminous quasars to a potentially common growth channel in the early Universe. Second, their extreme compactness tightens geometric and timescale constraints for any mechanism that couples radiative forcing, ion transport, and species-dependent separation.

The observational label *LRD* is descriptive and does not uniquely fix a physical interpretation. In addition to obscured-AGN interpretations, the literature also uses the term *black hole star* as a synonym for *quasi-star* scenarios in which a black hole accretes inside a massive envelope [9]. The evaluation logic in this manuscript therefore treats JWST LRDs and related compact high-$z$ objects as an explicit target class, while keeping the interpretation conditional on auditable inputs rather than taxonomy.

**Audit boundary and non-exclusivity.** This manuscript does *not* claim that the LRD phenomenology (red rest-optical continua, extreme compactness, line properties, or inferred number densities) is uniquely explained by COULOMB SHEARING within the present release. LRDs are included here because they are a high-impact *target class* for the same falsifiable closure-failure predictions defined elsewhere in this work, not because the existing JWST data are asserted to already demonstrate COULOMB SHEARING. Obscured-AGN and extreme-starburst interpretations remain viable in the literature and are not ruled out by any argument in Section 1.4. The discriminant proposed by COULOMB SHEARING is narrower: if an LRD is confirmed to host an accreting black hole and if the near-funnel environment permits closure failure, then COULOMB SHEARING predicts window-locked, species-linked transverse observables (P1–P4) under matched beam, whereas alternative explanations do not generically imply those same inequalities. Accordingly, the LRD connection in this section should be read as a *test commitment* and a scope declaration, not as a completed evaluation.

## 2 Transport of Matter to Polar Trajectories

**Horizon radius (definition used throughout).** The exterior-domain boundary is the outer Kerr event-horizon radius,

$$r_{\mathrm{H}} \equiv r_{+} = r_{\mathrm{g}}\left(1+\sqrt{1-a_*^2}\right). \tag{7}$$

### Geometric and Coordinate Framework

All calculations are performed in the Kerr spacetime describing a rotating black hole of mass $M$ and dimensionless spin parameter $a_*$. The geometry is expressed in Boyer–Lindquist coordinates $(r, \theta, \phi)$ via the line element in equation (8) and the auxiliary functions in equation (9), with $\theta = 0$ aligned with the spin axis. The term *polar* denotes trajectories with $\theta \ll 1$, and *equatorial* denotes motion confined to $\theta \simeq \pi/2$.

The Kerr geometry explicitly breaks spherical symmetry through rotation. As a result, both the structure of allowed orbits and the dynamical response of infalling matter depend on inclination relative to the spin axis.

### 2.1 Accretion Geometry in Rotating Black Hole Systems

The spacetime exterior to a rotating black hole is described by the Kerr metric. In Boyer–Lindquist coordinates, the line element is

$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right) dt^2 - \frac{4Mar\sin^2\theta}{\Sigma}\, dt\, d\phi + \frac{\Sigma}{\Delta} dr^2 + \Sigma\, d\theta^2 + \left(r^2 + a^2 + \frac{2Ma^2 r\sin^2\theta}{\Sigma}\right)\sin^2\theta\, d\phi^2, \tag{8}$$

with

$$\Sigma = r^2 + a^2\cos^2\theta, \qquad \Delta = r^2 - 2Mr + a^2, \tag{9}$$

and $a \equiv J/M$ (setting $G = c = 1$), with $a_* \equiv a/M \in [0, 1)$.

The off-diagonal $dt\, d\phi$ term encodes frame dragging and couples azimuthal motion to time. Because this term scales as $\sin^2\theta$, rotational effects vary with polar angle. As a result, trajectories at different inclinations can experience distinct dynamical environments even at the same radius.

The equations of motion for test particles in Kerr spacetime further reflect this anisotropy through the separation of variables in the Hamilton–Jacobi formalism, where the effective potentials governing $r$ and $\theta$ motion differ in form and constraint. Polar and equatorial trajectories are therefore dynamically inequivalent at the level of the fundamental equations, not merely in their boundary conditions.

### 2.2 Why Matter Preferentially Accesses Polar Inflow Channels

#### 2.2.1 Frame-Dragging and Angular Momentum Reorientation

A defining feature of Kerr spacetime is frame-dragging: the forced precession of orbital planes induced by spacetime rotation. The characteristic Lense–Thirring precession frequency for inclined motion at radius $r$ is given, to leading order, by

$$\Omega_{\mathrm{LT}}(r) = \frac{2GaM}{c^2 r^3}, \tag{10}$$

where $a$ is the dimensional spin parameter. This frequency increases rapidly toward smaller radii.

The angular momentum evolution induced by frame-dragging may be written schematically as

$$\frac{d\vec{L}}{dt} = \vec{\Omega}_{\mathrm{LT}} \times \vec{L}, \tag{11}$$

which represents precessional reorientation rather than radial or polar forcing. Frame-dragging therefore does not select a preferred inclination, but prevents long-term confinement to a single orbital plane in the absence of external enforcing torques.

For matter entering the gravitational influence of the black hole with angular momentum misaligned from the spin axis, frame-dragging acts as a deterministic *precessional* torque: it changes the *node* (orientation) of the orbital plane about the spin axis. This does not, by itself, drive mass toward the poles or increase inclination; it does not provide a polar transport force. Accordingly, frame-dragging is used here only to justify that misaligned inflow does not remain locked to a single, permanently equatorial orbital plane in the absence of other aligning torques. Any sustained access to polar-adjacent inflow channels must be supplied by the separate vertical transport mechanisms discussed below (pressure, magnetic, and geometric effects), and the delivered fraction is quantified by $f_{\text{polar}}$ rather than assumed.

### 2.2.2 Disk–Polar Coupling and Vertical Transport

Accretion disks in rotating systems are not dynamically isolated from their polar surroundings. Rotation necessarily introduces vertical stresses and forces that couple disk material to regions above and below the equatorial plane. In magnetized accretion flows, the vertical component of the momentum equation takes the schematic form

$$\rho \frac{dv_\theta}{dt} = -\frac{1}{r}\frac{\partial P}{\partial \theta} - \frac{1}{8\pi r}\frac{\partial}{\partial \theta}\left(B^2\right) + \text{(geometric terms)}, \tag{12}$$

where $P$ is the fluid pressure and $B$ the magnetic field strength.

The associated polar-adjacent mass flux may be expressed schematically as

$$\dot{M}_\theta \sim \rho v_\theta r^2 \Delta\Omega, \tag{13}$$

where $\Delta\Omega \ll 4\pi$ denotes a limited solid-angle contribution. No assumption is made that $\dot{M}_\theta$ dominates the accretion budget; only that it is generically non-zero and continuously replenished.

The vertical pressure and magnetic gradients implied by disk rotation act continuously, not intermittently. As a result, matter is transported into polar-adjacent regions through deterministic vertical motion. These regions are therefore dynamically fed by the disk rather than remaining empty or isolated, even when the associated mass flux constitutes only a small fraction of the total accretion flow.

### 2.2.3 Reduced Centrifugal Barriers Along the Spin Axis

Centrifugal support against radial infall depends on the projection of angular momentum perpendicular to the radial direction. In spherical coordinates, the centrifugal term appearing in the effective potential scales as

$$V_{\text{cent}} \propto \frac{L^2}{r^2}\sin^2\theta, \tag{14}$$

where $L$ is the specific angular momentum.

This geometric reduction does not generate polar inflow by itself; polar trajectories are accessed through the transport mechanisms described here.

Near the equatorial plane, $\sin^2\theta \approx 1$, and substantial angular momentum must be removed for matter to reach small radii. In contrast, along polar or near-polar trajectories with $\theta \ll 1$, the centrifugal barrier is intrinsically suppressed. Matter approaching the spin axis therefore accesses smaller radii without requiring the same degree of angular momentum dissipation demanded by equatorial inflow.

#### 2.2.4 Quantitative flow fraction estimate

The preceding arguments establish that polar-adjacent access channels are generically populated, but they do not by themselves specify what fraction of the total inflow budget occupies those channels. Since the growth enhancement implied later in the manuscript depends on the near-horizon delivery fraction $f_{\rm polar}$, this subsection defines a minimal, auditable estimator for the polar mass-flow fraction on a control surface at radius $r$.

**Continuity-based estimator on a fixed-radius control surface.** Consider a spherical control surface at radius $r$ partitioned into (i) a polar-cap region of half-opening angle $\theta_{\rm cap}$ about each pole and (ii) the remaining equatorial-dominated region. Define the total polar solid angle (two caps) as

$$\Omega_{\rm polar}(\theta_{\rm cap}) \equiv 4\pi \left(1 - \cos\theta_{\rm cap}\right) \;\simeq\; 2\pi\,\theta_{\rm cap}^2 \quad (\theta_{\rm cap} \ll 1). \tag{15}$$

A minimal estimator for the polar delivery fraction on that surface is then

$$f_{\rm polar}(r) \;\equiv\; \frac{\dot{M}_{\rm polar}(r)}{\dot{M}_{\rm in}(r)} \;\approx\; \left(\frac{\Omega_{\rm polar}}{4\pi}\right)\left(\frac{\rho_{\rm polar}}{\rho_{\rm disk}}\right)\left(\frac{|v_{r,\rm polar}|}{|v_{r,\rm disk}|}\right), \tag{16}$$

where $\rho_{\rm polar}$ and $|v_{r,\rm polar}|$ denote characteristic density and inward radial speed in the polar-cap region, and $\rho_{\rm disk}$ and $|v_{r,\rm disk}|$ the corresponding characteristic values in the disk-dominated region on the same control surface.

**Interpretation and conservative bounds.** Equation (16) separates three physically distinct controls:

- **Geometric access:** $\Omega_{\rm polar}/4\pi$ depends only on the adopted polar-cap angle $\theta_{\rm cap}$.
- **Mass loading:** $\rho_{\rm polar}/\rho_{\rm disk}$ captures whether the polar-adjacent region is a low-density funnel, a mass-loaded sheath, or a thicker inflow boundary layer.
- **Inward delivery:** $|v_{r,\rm polar}|/|v_{r,\rm disk}|$ captures the reduced centrifugal barrier near the spin axis and any corresponding increase in inward radial speed relative to the disk body.

**How the ratios are to be instantiated (not asserted).** The estimator in Eq. (16) is intentionally written so that $\rho_{\rm polar}/\rho_{\rm disk}$ and $|v_{r,\rm polar}|/|v_{r,\rm disk}|$ are *not* free narrative choices. In any worked example where growth enhancement is evaluated, these ratios must be supplied by one of the following auditable paths: (i) direct extraction from a declared GRMHD snapshot on the same control surface $r$ using the same cap definition $\theta_{\rm cap}$; (ii) observational constraints on polar-cap density (e.g., Faraday-active sheath bounds) combined with an explicit disk-density proxy on the same scale; or (iii) a deliberately conservative bound set that biases *against* large $f_{\rm polar}$ and is reported as such.

This matters because GRMHD commonly yields disk-dominated accretion with comparatively low polar density, and in MAD-like states the polar funnel can be strongly evacuated by magnetic pressure.

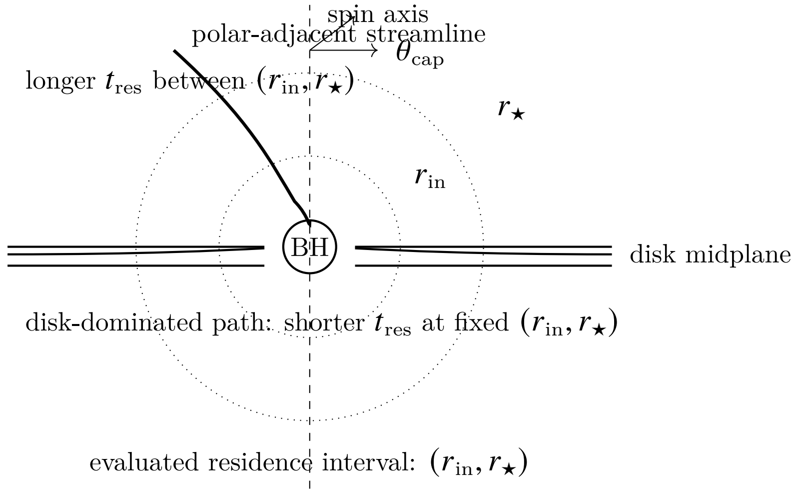


Figure 1: Schematic geometry for disk-dominated versus polar-adjacent trajectories and the residence interval used to define $p(t_{\rm res})$ between $(r_{\rm in}, r_\star)$. The figure is conceptual and does not assert a specific metric solution or jet-launching mechanism.

In those regimes $f_{\rm polar}$ may be $\ll 1$ even if polar-adjacent trajectories exist, and Eq. (16) should be treated as a *bottleneck* rather than an assumption of substantial polar mass loading. Therefore, unless a worked example explicitly reports the above ratios (with provenance), the manuscript does not claim a large polar delivery fraction for that system.

The estimator is intentionally agnostic about the microphysics setting these ratios, and it does not assume that $f_{\rm polar}$ is large. Its role is to make the bottleneck explicit and externally auditable.

**Bottleneck criterion.** Later growth-rate implications scale with externally constrained polar delivery. Therefore, if independent constraints imply

$$f_{\rm polar}(r_\star) \ll 10^{-2} \tag{17}$$

on the near-horizon evaluation surface $r_\star$ used in a worked example, the corresponding global growth enhancement is necessarily small even if Coulomb closure failure occurs locally. In that regime the mechanism remains a viable stratification channel but is not a dominant growth pathway for that system under the declared inputs. This bottleneck is treated as a fail-capable external constraint wherever growth enhancement is evaluated.

### 2.3 Residence Time Along Polar Trajectories

Polar access does not imply instantaneous passage or free-fall. Matter on polar or near-polar trajectories remains subject to competing forces, non-radial motion, and curvature-induced deflection. A natural measure of persistence is the residence time,

$$t_{\rm res} = \int_C \frac{ds}{v(s)}, \tag{18}$$

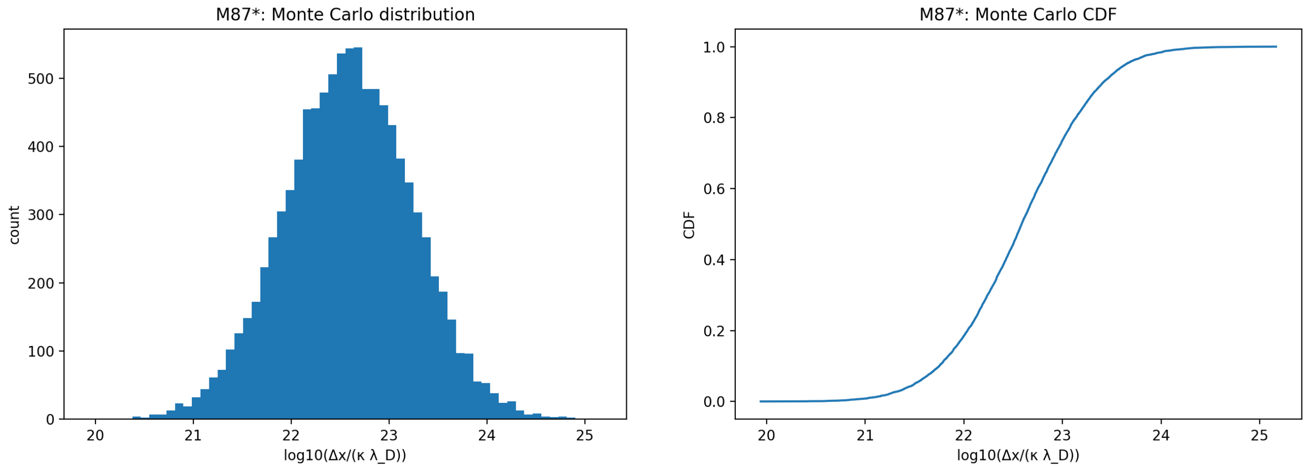


Figure 2: Executed Monte Carlo output from the worked-example pipeline (M87* anchor): histogram and CDF of the computed shearing-margin ratio used in the probability evaluation. Unlike Fig. 1, this figure is produced by the executed calculation described in Section 9 and is included here to ensure that the first-page figure set contains at least one computation-backed diagnostic rather than schematic geometry alone.

where $ds$ is the differential path length along a trajectory $C$ and $v(s)$ the local speed.

Purely radial free-fall would imply a characteristic velocity

$$v_{\rm ff} = \sqrt{\frac{2GM}{r}}, \tag{19}$$

but polar and near-polar trajectories experience non-radial curvature and stresses that reduce the net inward radial speed. A conservative lower bound on the residence time is therefore

$$t_{\rm res} \gtrsim \int_{r_1}^{r_2} \frac{dr}{\eta\, v_{\rm ff}}, \qquad 0 < \eta \le 1, \tag{20}$$

**Meaning of the slowdown factor $\eta$.** The parameter $\eta$ is a dimensionless transport efficiency comparing the trajectory-averaged inward radial speed to the local free-fall speed:

$$\eta \equiv \left\langle \frac{|v_r|}{v_{\rm ff}} \right\rangle_C, \qquad 0 < \eta \le 1, \tag{21}$$

where the average is taken along the trajectory segment $C$ contributing to $t_{\rm res}$. Values $\eta \simeq 1$ correspond to near-ballistic inflow, while $\eta < 1$ represents curvature, stresses, and path elongation that reduce the net inward radial speed.

**A priori choice of $\eta$ in reported calculations.** Although $\eta$ is a physical transport parameter and is not predicted by this paper, the calculations reported below do not permit post hoc selection of $\eta$. Therefore, all worked examples and all PASS/FAIL evaluations reported in this paper fix the conservative free-fall bound

$$\eta = 1. \tag{22}$$

This choice minimizes $t_{\rm res}$ for fixed $(r_1, r_2)$ and therefore makes Coulomb-closure failure hardest to satisfy. Any analysis that relaxes this lock must be presented as a separate, explicitly labeled extension and must propagate results across a declared interval $\eta \in [\eta_{\min}, \eta_{\max}]$ without selecting a single value after outcomes are known.

**Clarification: longer $t_{\rm res}$ does not weaken coupling.** The conservativeness of the $\eta = 1$ lock is strictly about *exposure time*: reducing $t_{\rm res}$ reduces the time available for the drag-limited displacement $\Delta x(t_{\rm res})$ to accumulate. It is *not* an assumption that fewer collisions imply weaker coupling. The coupling microphysics is carried by the local momentum-relaxation time $t_c$ (defined and evaluated later), and once $t_{\rm res} \gg t_c$ the dynamics are already in the strongly collisional, drag-limited regime. Increasing $t_{\rm res}$ in that regime increases the time over which the *already-coupled* slip response can accumulate; it does not reduce the strength of coupling, nor does it change $t_c$ by itself. Therefore, adopting $\eta < 1$ would typically *increase* $\Delta x(t_{\rm res})$ through the longer exposure interval while leaving the collision-dominated suppression encoded by $t_c$ intact. Any claim that smaller $\eta$ causes the coupling comparison to "fail" must be demonstrated by an explicit, consistent re-evaluation of $\Delta x(t_{\rm res})$ with the same $t_c$ model and declared state inputs, rather than inferred from collision counts alone.

**GRMHD context for plausible $\eta$ values (not used in reported calculations).** Global GRMHD simulations spanning SANE and MAD-like states consistently show that horizon-adjacent inflow is not globally ballistic outside the immediate plunging region: angular-momentum support, magnetic stresses, and pressure gradients reduce the radial component of the velocity below free-fall in the disk body and in funnel-adjacent boundary layers (e.g. [10, 11]). Simple viscous scalings also suggest $|v_r| \sim \alpha (H/r)^2 v_K$ for geometrically thick flows [2], motivating order-of-magnitude expectations such as $\eta \sim 10^{-2}$–$10^{-1}$ (up to order-unity factors), depending on state and radius.

This contextual range is provided only to indicate that $\eta < 1$ is physically generic in realistic flows. It is not used to set or tune any reported result: all worked examples and all PASS/FAIL evaluations reported in this paper fix the conservative free-fall bound $\eta = 1$ (Eq. (22)). Any future extension that adopts $\eta < 1$ must treat it as a declared external input and must propagate results across a stated interval $\eta \in [\eta_{\min}, \eta_{\max}]$ without selecting a single value after outcomes are known.

**Clarification: why $\eta < 1$ does not "restore coupling" by collision count alone.** The role of $\eta$ is to set the exposure time $t_{\rm res}$ available for the drag-limited displacement kernel to accumulate. Decreasing $\eta$ increases $t_{\rm res}$; it does not, by itself, change the microphysical coupling timescale $t_c$ computed from the declared local plasma state. In the collisional regime $t_{\rm res} \gg t_c$, the dynamics are already drag-limited (memoryless in $\Delta v$) and the cumulative displacement scales as $\Delta x(t_{\rm res}) \propto \Delta a\, t_c\, t_{\rm res}$. Therefore, it is not valid to infer that smaller $\eta$ makes coupling "stronger" based only on a larger number of collisions; the collision count is exactly what produces the drag-limited suppression through $t_c$, and the net effect of longer exposure must be evaluated by recomputing $\Delta x(t_{\rm res})$ with the same $t_c$ model and declared inputs.

### 2.3.1 Residence time sensitivity analysis (distribution form and required bracket)

The residence time $t_{\rm res}$ is an *exposure-time* control parameter for the drag-limited displacement $\Delta x(t_{\rm res})$ (Eq. (68)) and therefore for the admissibility functional $\mathcal{S} \equiv \Delta x/(\kappa\lambda_D)$ used to define $\chi_s$ and the species windows in later sections. A single-value estimate is therefore not the most faithful object-level description: realistic turbulent and magnetized flows admit a distribution of trajectory exposure times (and corresponding path geometries) even at fixed radius.

**Distribution form (general).** Let $p(t_{\rm res}\,|\,r,\theta)$ denote the residence-time distribution for the trajectory family actually sampled by the polar-adjacent evaluation segment at radius $r$ and polar angle $\theta$. Then the distribution-propagated shearing probability for a given species proxy (and any other fixed state inputs) is

$$P_s^{(\rm res)}(Z) \;\equiv\; \int_0^{\infty} P_s(Z\,|\,t_{\rm res})\; p(t_{\rm res}\,|\,r,\theta)\; dt_{\rm res}, \tag{23}$$

where $P_s(Z\,|\,t_{\rm res})$ is computed by the same kernel pipeline already declared in Species-Dependent Shearing Ranges and Probabilities, with the sole change that $t_{\rm res}$ is treated as an argument rather than a single locked value.

**Connection to GRMHD particle tracking (how $p(t_{\rm res})$ is obtained).** The most direct way to estimate $p(t_{\rm res}\,|\,r,\theta)$ is Lagrangian tracer tracking in GRMHD: inject tracers in the evaluated polar-adjacent region, measure the distribution of time spent between declared radii $(r_{\rm in}, r_\star)$ on the relevant streamline family, and report the empirical distribution (or a fitted parametric form) as a declared external input. Here $r_\star$ denotes the reference evaluation radius used later for step-by-step worked examples and control runs. When a worked example declares a specific value, it is written explicitly (e.g. $r_\star = 5\,r_g$). This manuscript does not require a particular simulation state or fit family in order to define the shearing criterion; it requires only that any adopted $p(t_{\rm res})$ be *declared* and then propagated without post hoc selection.

**Required bracket in the absence of an adopted $p(t_{\rm res})$.** When an empirical $p(t_{\rm res})$ is not declared, robustness must still be demonstrated by a fixed sensitivity bracket referenced to the free-fall exposure time on the same radial interval. Define the free-fall exposure time between $(r_{\rm in}, r_\star)$ as

$$t_{\rm ff}(r_\star) \;\equiv\; \int_{r_{\rm in}}^{r_\star} \frac{dr}{v_{\rm ff}(r)}, \qquad v_{\rm ff}(r) = \sqrt{\frac{2GM}{r}}, \tag{24}$$

and introduce the dimensionless residence factor

$$\tau \equiv \frac{t_{\rm res}}{t_{\rm ff}}. \tag{25}$$

A mandatory sensitivity report is then defined by evaluating the full pipeline outputs $(\chi_s, r_{\min}, r_{\max}, P_s)$ at

$$\tau \in \{0.1,\ 1,\ 10\}, \tag{26}$$

and reporting the resulting changes without selecting a preferred $\tau$ after outcomes are known.

**Minimum residence time required for closure failure (mechanical definition).** For any fixed state inputs and forcing model, the minimum residence time required to satisfy the admissibility inequality is defined implicitly by

$$\Delta x\left(t_{\rm res,min}\right) = \kappa\, \lambda_D, \tag{27}$$

where $\Delta x(t)$ is computed from Eq. (68) (or Eq. (69) when the constant-$\Delta a$ approximation is declared valid on the interval). Reporting $t_{\rm res,min}$ (or equivalently $\tau_{\min} \equiv t_{\rm res,min}/t_{\rm ff}$) is therefore a required diagnostic: it makes explicit how far below or above the threshold a given worked example lies.

**Interpretation discipline.** This sensitivity analysis does not relax the pre-committed evaluation logic. It adds an explicit robustness layer demanded by the fact that $t_{\rm res}$ is a transport-time variable that can vary across realizations. Where the conservative bracket $\tau = 0.1$ yields $P_s \ll 1$ while $\tau = 1$ yields $P_s \simeq 1$, the classification must report that the mechanism is transport-sensitive for that object and that empirical residence constraints are needed to resolve PASS versus FAIL under the declared comparator logic.

With this definition, the residence-time lower bound evaluates to an explicit scaling:

$$t_{\rm res} \gtrsim \frac{1}{\eta}\int_{r_1}^{r_2} \frac{dr}{\sqrt{2GM/r}} = \frac{2}{3\eta\sqrt{2GM}}\left(r_2^{3/2} - r_1^{3/2}\right). \tag{28}$$

If $r_2 \gg r_1$, this simplifies to

$$t_{\rm res} \gtrsim \frac{2}{3\eta}\sqrt{\frac{r_2^3}{2GM}}. \tag{29}$$

These expressions make the sensitivity explicit: $t_{\rm res} \propto \eta^{-1}$. Larger $t_{\rm res}$ increases the time available for differential drift, but it also increases the number of momentum-exchange times available when $t_c \ll t_{\rm res}$. Accordingly, admissibility is governed by the ratio $t_{\rm res}/t_c$ rather than by $t_{\rm res}$ alone. This bound establishes finite, non-negligible residence times without invoking model-dependent plasma dynamics.

To illustrate the magnitude implied by this bound, consider a representative supermassive black hole with mass

$$M = 10^9\, M_\odot, \tag{30}$$

and evaluate the residence time at a characteristic radius

$$r = 10\, r_g, \qquad r_g \equiv \frac{GM}{c^2}. \tag{31}$$

Substituting into the lower-bound estimate yields

$$t_{\mathrm{res}} \gtrsim \frac{1}{\eta}\sqrt{\frac{(10\, r_g)^3}{2GM}} = \frac{10^{3/2}}{\eta}\frac{GM}{c^3}. \tag{32}$$

For $M = 10^9\, M_\odot$, this corresponds to

$$t_{\mathrm{res}} \gtrsim \frac{10^{3/2}}{\eta} \times 5 \times 10^3\ \mathrm{s} \ \sim \ \frac{1.6 \times 10^5}{\eta}\ \mathrm{s}, \tag{33}$$

i.e., residence times on the order of days for $\eta \lesssim 1$. This estimate is conservative, as it neglects additional non-radial forces and curvature effects that further prolong residence.

The purpose of this estimate is solely to establish that polar-adjacent residence times are macroscopic rather than instantaneous; no inference regarding specific interaction mechanisms is made here.

Frame-dragging-induced precession, vertical forces, and reduced but nonzero angular momentum combine to slow purely radial motion. These effects prolong the time matter spends within polar-adjacent regions, yielding finite residence times that exceed instantaneous ballistic free-fall. Polar inflow therefore constitutes a sustained dynamical channel rather than a transient geometric curiosity.

## Conclusion

Rotating black hole systems can generate and continuously replenish non-equatorial, polar-adjacent inflow trajectories under a broad range of conditions. These trajectories arise from direction-dependent dynamics in Kerr spacetime, can be dynamically accessible in generic accretion flows, may experience reduced centrifugal barriers, and can persist for finite, non-negligible residence times. Crucially, however, their existence does not imply that the polar region carries a large fraction of the net mass inflow: standard GRMHD solutions commonly show disk-dominated accretion, and in magnetically arrested disk (MAD) states strong vertical fields can evacuate the polar funnel and suppress polar mass loading. The transport claim used in this manuscript is therefore limited to *access*, not *dominance*: Coulomb Shearing requires only that some material samples polar-adjacent segments for a finite interval, with the actual delivered fraction captured explicitly by $f_{\mathrm{polar}}$ and treated as a measurable bottleneck rather than an asserted large quantity.

The residence times derived here are sufficient for the drag-limited coupling dynamics (cf. Differential Forcing and the Onset of Coulomb Shearing) to approach the shearing threshold under forcing conditions characteristic of near-Eddington luminosity systems.

# 3 Differential Forcing and the Onset of Coulomb Shearing

## 3.1 Forces acting on electrons vs ions

Along the polar and near-polar trajectories established by the polar transport geometry and residence-time constraints, the plasma experiences external forcing whose effective acceleration differs by species because the relevant coupling channels scale differently with charge and mass. The distinction is not a matter of environmental coincidence. It follows directly from (i) the electron-specific radiative momentum channel, (ii) the Lorentz force scaling with charge-to-mass ratio, and (iii) the fact that gravity acts on mass.

**Radiation pressure.** In the parameter regime considered here, radiative momentum transfer couples predominantly through electron (Thomson) scattering, so the opacity per unit mass is effectively electron-dominated. As a result, ions experience negligible *direct* radiative acceleration along the same trajectory segment, aside from the acceleration communicated through electrostatic coupling to the electron population. This produces an intrinsic asymmetry in the force budget: any nonzero radiative forcing contributes primarily to the electron equation of motion. If other channels dominate the momentum exchange (e.g. bound–bound or bound–free opacity, or pair-regulated scattering), the partition of radiative forcing among species can differ; the analysis that follows assumes Thomson-dominated coupling, consistent with near-Eddington, highly ionized conditions.

**Magnetic forcing.** Magnetic forces act through the Lorentz interaction and therefore scale with charge-to-mass ratio. Since electrons and ions carry comparable charge magnitude but vastly different mass, the same electromagnetic environment generically implies much larger electron acceleration. Along polar or near-polar trajectories, where ordered magnetic structure and field curvature can be dynamically relevant, the electron response to electromagnetic forcing is correspondingly more rapid.

**Gravity.** Gravity accelerates matter proportional to mass and is therefore comparable per unit mass for both species when treated as test particles. However, within a baryonic plasma the inertia is dominated by the ions. In any closure that enforces joint motion, the ion inertia sets the effective mass that must be accelerated if the pair is to remain locked.

Taken together, these considerations imply that the net acceleration experienced by electrons and ions along the same polar trajectory is generically unequal. This follows directly from the force scalings: radiative momentum exchange is electron-dominated, electromagnetic forcing scales as $q/m$, and gravitational forcing scales with mass. These contributions can be combined into a single species-resolved dynamical quantity, introduced next.

## 3.2 Derivation of differential acceleration

Let the motion be evaluated along a specific polar or near-polar trajectory segment, parameterized by time $t$, and define $\hat{\mathbf{s}}$ as the unit vector tangent to that trajectory. We adopt the sign convention that the positive direction is *outward* along $\hat{\mathbf{s}}$ (away from the compact object), and the negative direction is *inward* along $-\hat{\mathbf{s}}$ (toward the compact object). This convention is used throughout the manuscript wherever $\Delta a$ appears.

Define the electron and ion accelerations along the trajectory as the tangential projections of the net force per particle,

$$a_e(t) \equiv \frac{1}{m_e}\,\mathbf{F}_e(t)\cdot\hat{\mathbf{s}}, \qquad a_i(t) \equiv \frac{1}{m_i}\,\mathbf{F}_i(t)\cdot\hat{\mathbf{s}}, \tag{34}$$

with $m_e$ the electron mass and $m_i$ the ion mass for the representative ion species under consideration.

The net force on each species is decomposed into physically distinct contributions:

$$\mathbf{F}_e(t) = \mathbf{F}_{\mathrm{rad},e}(t) + \mathbf{F}_{\mathrm{EM},e}(t) + \mathbf{F}_{g,e}(t), \tag{35}$$

$$\mathbf{F}_i(t) = \mathbf{F}_{\mathrm{rad},i}(t) + \mathbf{F}_{\mathrm{EM},i}(t) + \mathbf{F}_{g,i}(t). \tag{36}$$

Here $\mathbf{F}_{\mathrm{rad},e}$ denotes radiative momentum transfer acting directly on electrons; $\mathbf{F}_{\mathrm{rad},i}$ denotes any direct radiative force on ions (which is typically negligible compared to the electron channel in the regimes this mechanism targets); $\mathbf{F}_{\mathrm{EM}}$ denotes electromagnetic forces (Lorentz and any electric field contribution consistent with the local plasma state); and $\mathbf{F}_g$ denotes gravitational force.

With the sign convention fixed, the single dynamical quantity that drives separation is the *differential acceleration* along the shared trajectory,

$$\Delta a_{\parallel} \equiv a_e - a_i \tag{37}$$

where $a_e$ and $a_i$ are defined by Eqs. (34)–(36). A nonzero $\Delta a_{\parallel}$ is generically expected because at least one forcing channel (radiative and often electromagnetic) couples far more efficiently to electrons than to ions. In addition, $\Delta a_{\parallel}(t)$ may vary in time as the trajectory traverses spatial gradients in radiation intensity, electromagnetic structure, and gravitational potential.

Throughout this section, $\Delta a_{\parallel}$ denotes the along-trajectory (tangent-projected) differential acceleration used in the closure and displacement calculations; $\Delta a$ without a subscript is reserved for non-projected usage elsewhere.

**Auditable specialization: Thomson-dominated forcing near the Eddington scale.** For later numerical work it is useful to state an explicit, minimal force model that can be evaluated from system-scale observables. Consider a fully ionized plasma in which the dominant radiative coupling is Thomson scattering on electrons and direct radiative forcing on ions is negligible. Approximating the radiative momentum flux as locally radial with magnitude $L/(4\pi r^2 c)$, the direct radiative force per electron is

$$\mathbf{F}_{\mathrm{rad},e} \simeq \frac{\sigma_T L}{4\pi r^2 c}\,\hat{\mathbf{r}}, \qquad \mathbf{F}_{\mathrm{rad},i} \simeq \mathbf{0}, \tag{38}$$

and the gravitational forces are

$$\mathbf{F}_{g,e} = -\frac{GMm_e}{r^2}\,\hat{\mathbf{r}}, \qquad \mathbf{F}_{g,i} = -\frac{GMm_i}{r^2}\,\hat{\mathbf{r}}. \tag{39}$$

Here the force expressions are written in a minimal Newtonian form to provide an auditable scaling; in GR treatments the same relations apply with $L/(4\pi r^2)$ interpreted as the local radiative flux in the chosen plasma frame, related to the luminosity measured at infinity by the standard gravitational redshift factors.

**Scope and validity of the Thomson specialization.** Equation (38) assumes (i) electron-scattering dominates the radiative momentum transfer, (ii) photon energies satisfy $h\nu \ll m_e c^2$ so the scattering cross section is well-approximated by $\sigma_T$, and (iii) the ion radiative force is negligible because ions are fully stripped and line/bound-free opacity is suppressed. When these conditions are not met, the specialization can be generalized without altering the logic of the criterion by replacing $\sigma_T$ with an energy- and angle-averaged effective cross section $\bar{\sigma}_{\rm eff}$ and by allowing a nonzero ion radiative force term:

$$\mathbf{F}_{\mathrm{rad},e} \longrightarrow \frac{\bar{\sigma}_{\rm eff}\,L}{4\pi r^2 c}\,\hat{\mathbf{r}}, \qquad \mathbf{F}_{\mathrm{rad},i} \neq \mathbf{0} \;\; \big(\text{if bound-free/line coupling is present}\big). \tag{40}$$

In the Klein–Nishina regime, $\bar{\sigma}_{\rm eff} = \langle \sigma_{\rm KN}(\nu)\rangle < \sigma_T$, which reduces the radiative term in $\Delta a$. Any such correction is treated as a declared force-model choice under the evaluation protocol (Section 8).

Because gravitational acceleration is mass-independent, the gravitational contributions cancel in the differential acceleration $\Delta a$ when both species share the same trajectory direction (they differ only in the radiative and electromagnetic channels). If the trajectory tangent makes angle $\theta_s$ with the local radial direction, define $\mu_s \equiv \hat{\mathbf{r}} \cdot \hat{\mathbf{s}} = \cos\theta_s$.

**Electromagnetic forcing under the along-trajectory projection.** For each species $s \in \{e, i\}$, the electromagnetic force is the Lorentz force

$$\mathbf{F}_{\mathrm{EM},s} = q_s\left(\mathbf{E} + \mathbf{v}_s \times \mathbf{B}\right), \tag{41}$$

with $q_e = -e$ and $q_i = +Ze$. The criterion uses the along-trajectory (tangent) acceleration component, $a_s \equiv \mathbf{a}_s \cdot \hat{\mathbf{s}}$. To leading order, $\hat{\mathbf{s}}$ is parallel to the instantaneous velocity, so the magnetic part of the Lorentz force is orthogonal to $\hat{\mathbf{s}}$ and does not contribute to $a_s$:

$$(\mathbf{v}_s \times \mathbf{B}) \cdot \hat{\mathbf{s}} \simeq 0. \tag{42}$$

Therefore, the only electromagnetic contribution to the along-trajectory acceleration is the parallel electric field,

$$a_{\mathrm{EM},s} \equiv \frac{1}{m_s}\mathbf{F}_{\mathrm{EM},s} \cdot \hat{\mathbf{s}} \simeq \frac{q_s}{m_s}\,E_\parallel, \qquad E_\parallel \equiv \mathbf{E} \cdot \hat{\mathbf{s}}. \tag{43}$$

In ideal MHD, $E_\parallel \to 0$ in the comoving frame, so electromagnetic forcing does not introduce an additional free along-trajectory acceleration channel in that limit. If a non-ideal $E_\parallel$ model is included, it must be declared explicitly as part of the pre-committed evaluation protocol.

### 3.2.1 Regime-dependent $E_\parallel$ prescription (collisional vs collisionless)

If electromagnetic forcing is included through a nonzero $E_\parallel$ term, the prescription must respect the local collisionality regime. A resistive-MHD Ohmic closure is valid only when the electron mean free path is small compared to the system scale. In horizon-adjacent, magnetically dominated funnels the plasma can instead be weakly collisional or collisionless, in which case reconnection-mediated fields set the characteristic $E_\parallel$ scale.

**Collisionality parameter.** Define the electron mean free path using the electron thermal speed and the electron-ion collision frequency,

$$\lambda_{\rm mfp} \equiv \frac{v_{{\rm th},e}}{\nu_c}, \qquad v_{{\rm th},e} \equiv \sqrt{\frac{2k_BT_e}{m_e}}, \qquad \nu_c \equiv t_c^{-1}, \tag{44}$$

where $t_c$ is the electron-ion momentum relaxation time introduced below. Define the system-scale collisionality parameter

$$\chi \equiv \frac{\lambda_{\rm mfp}}{L_{\rm sys}}, \qquad L_{\rm sys} \equiv r, \tag{45}$$

so $\chi \ll 1$ indicates a collisional regime and $\chi \gg 1$ indicates a collisionless regime.

**Regime-dependent prescription (declared and auditable).** The evaluation uses the following declared $E_\parallel$ model:

$$E_\parallel(r) = \begin{cases} \eta_{\rm Sp}(r)\, j_\parallel(r), & \chi(r) < 0.1 \quad \text{(collisional)} \\ \epsilon_{\rm rec}\, B(r)\, v_A(r), & \chi(r) > 10 \quad \text{(collisionless reconnection)} \\ \min\left[\eta_{\rm Sp}(r)\, j_\parallel(r),\ \epsilon_{\rm rec}\, B(r)\, v_A(r)\right], & 0.1 \le \chi(r) \le 10 \quad \text{(transitional, conservative)} \end{cases} \tag{46}$$

**Interpretation and non-ad-hoc status of the prescription.** Equation (46) is not introduced as a new derivation of plasma microphysics. It is a *bounded, protocol-gated parametrization* of the along-trajectory electric field scale that is used *only* when an electromagnetic specialization with $E_\parallel \neq 0$ is explicitly invoked. The primary radiative-only track remains $E_\parallel = 0$ by construction. When $E_\parallel \neq 0$ is invoked, the intent is to (i) enforce a branch choice from declared state inputs through $\chi(r)$, (ii) prevent post hoc selection of an "optimistic" field estimate, and (iii) keep the resulting field scale auditable through the declared brackets on $j_\parallel$ and $\ell_j$.

**No claim of "collisionless" behavior at high density.** The collisionless reconnection branch in Eq. (46) is included to cover magnetically dominated, low-density funnel-adjacent domains *if* the declared plasma state implies $\chi \gg 1$. It is not a claim that the high-density states sometimes discussed for disk-body conditions (e.g., $n_e \sim 10^{13}$–$10^{15}\,\mathrm{cm}^{-3}$) are collisionless. For such densities, the evaluated mean free path is expected to yield $\chi \ll 1$ on horizon-adjacent scales, which forces selection of the collisional (Ohmic) branch and suppresses any use of the collisionless expression. In other words, whether a worked example is collisional or weakly collisional is decided by the computed $\chi(r)$ from declared inputs, not by narrative labeling.

**Auditable current-density scale for the collisional (Ohmic) branch.** The collisional branch uses $E_\parallel = \eta_{\rm Sp} j_\parallel$. To keep $j_\parallel$ auditable without imposing a detailed circuit model, adopt an Ampère-law scale estimate

$$j_\parallel \sim \frac{B}{\mu_0\,\ell_j}, \tag{47}$$

where $\ell_j$ is the characteristic transverse length scale over which the relevant field changes and therefore bounds the current density implied by the macroscopic field. For horizon-adjacent funnels, a physically grounded bracket is

$$\delta_e \;\le\; \ell_j \;\le\; r, \tag{48}$$

where $\delta_e \equiv c/\omega_{pe}$ is the collisionless electron skin depth computed from the declared $n_e$ and $\omega_{pe}$ is the electron plasma frequency. This bracket is not a fit: $\ell_j = r$ is a conservative large-scale closure estimate, while $\ell_j = \delta_e$ is a kinetic lower bound on sheet thickness that produces a conservative upper bound on $j_\parallel$ at fixed $B$. Any narrower choice of $\ell_j$ must be declared explicitly for a worked example and propagated uniformly. In Eq. (46), $\eta_{\rm Sp}$ is the Spitzer resistivity (appropriate only in the collisional regime), $j_\parallel$ is evaluated using the auditable Ampère-law scale estimate $j_\parallel \sim B/(\mu_0 \ell_j)$ with the physically grounded bracket $\delta_e \le \ell_j \le r$ (where $\delta_e \equiv c/\omega_{pe}$ is the electron skin depth computed from the declared $n_e$), and the collisionless reconnection form uses an efficiency parameter $\epsilon_{\rm rec}$ that is treated as a declared input when this branch is invoked. The Alfvén speed is

$$v_A \equiv \frac{B}{\sqrt{\mu_0 \rho}}, \tag{49}$$

with $\rho$ the mass density. The collisionless branch is intended to represent the observed order-of-magnitude reconnection field scale in fast reconnection, while remaining agnostic about detailed microphysics beyond the adopted efficiency $\epsilon_{\rm rec}$.

**Protocol constraint (no post hoc branch selection).** When any worked example includes $E_\parallel \neq 0$, the branch selection implied by $\chi(r)$ must be computed from the declared $(n_e, T_e)$ (through $t_c$) at the evaluated radius and applied uniformly. It is not permitted to select the collisional versus collisionless branch after outcomes are known. The radiative-only specialization remains the fixed case $E_\parallel = 0$.

**Transitional regime handling.** In the transitional band $0.1 \leq \chi \leq 10$, the conservative prescription in Eq. (46) uses the smaller of the collisional and collisionless estimates, so electromagnetic forcing cannot be increased by interpolation. Any alternative interpolation choice must be declared as an explicit variant and propagated across the evaluation set rather than introduced for a single object.

#### 3.2.2 Verification of magnetic decoupling (frozen-in check)

Several steps in the electromagnetic specialization implicitly assume that electrons can develop a sustained relative drift along the evaluated segment without remaining perfectly frozen into the magnetic field. In high-conductivity plasma the ideal frozen-in condition applies,

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} \;\simeq\; \mathbf{0}, \tag{50}$$

and non-ideal field-line slippage requires finite magnetic diffusivity. Therefore, when an $E_{\parallel} \neq 0$ channel is invoked, the evaluation must explicitly verify that magnetic diffusion on the local separation scale is not negligible.

**Magnetic Reynolds number (local).** Define the parallel conductivity using the same collision frequency already introduced for coupling,

$$\sigma_{\parallel} \;\simeq\; \frac{n_e e^2}{m_e \nu_{ei}}, \qquad \nu_{ei} \equiv \nu_c \equiv t_c^{-1}, \tag{51}$$

and define the magnetic diffusivity (SI units)

$$\eta_m \;\equiv\; \frac{1}{\mu_0 \sigma_{\parallel}}. \tag{52}$$

The local magnetic Reynolds number is then

$$R_{m,\text{local}} \;\equiv\; \frac{L\,v}{\eta_m} \;=\; \mu_0 \sigma_{\parallel} L v, \tag{53}$$

where $L$ is the characteristic separation scale and $v$ is the characteristic relative drift speed on that scale.

**Operational choice of $L$ and $v$ (consistent with the closure kernel).** To bind this check to the same quantities already computed in the closure criterion, take

$$L \equiv \Delta x(r_\star), \qquad v \equiv \Delta v(r_\star) \simeq \int_0^{t_{\text{res}}} \Delta a(t)\,dt, \tag{54}$$

and in the constant-$\Delta a$ specialization use $\Delta v \simeq \Delta a\, t_{\text{res}}$.

**Decoupling admissibility thresholds.** The evaluation adopts the following auditable thresholds:

$$R_{m,\mathrm{local}} < 10 \implies \text{magnetic slippage is plausible on the separation scale (decoupling admissible),} \tag{55}$$

$$R_{m,\mathrm{local}} > 100 \implies \text{frozen-in is strong on the separation scale (decoupling not admissible).} \tag{56}$$

Values in $10 \le R_{m,\mathrm{local}} \le 100$ are treated as transitional and must be reported explicitly.

**If frozen-in holds: required modification to the force balance.** If a worked example falls in the frozen-in regime ($R_{m,\mathrm{local}} \gtrsim 100$), then treating electron displacement as independent of the field is not valid. In that case, relative drift that attempts to separate charges drags field structure with it, generating a magnetic pressure response that opposes further separation. A minimal along-trajectory correction is introduced by adding a magnetic-pressure-gradient term to the effective differential acceleration:

$$\Delta a_{\mathrm{eff}} \;\equiv\; \Delta a - \frac{1}{\rho}\frac{\partial}{\partial s}\left(\frac{B^2}{2\mu_0}\right) L_B^{-1}, \tag{57}$$

where $L_B$ is the declared scale length for field-strength variation along the displacement direction (or along the trajectory projection used for the criterion) and $\rho$ is the local mass density. This correction is not invoked unless the frozen-in check fails; when it is invoked, $L_B$ must be declared and propagated without post hoc selection.

**Scope discipline.** This frozen-in verification is a gate on the electromagnetic specialization. The radiative-only specialization ($E_{\parallel} = 0$) does not rely on magnetic slippage and is unaffected by this check. When $E_{\parallel} \neq 0$ is invoked, $R_{m,\mathrm{local}}$ must be computed and reported for that worked example at the evaluated radius.

**Clarification: frozen-in does not imply identical species acceleration histories.** This manuscript does *not* assume electron-only detachment from magnetic field lines while ions remain frozen-in. In the magnetized regime, both species can remain magnetized and still experience different *external* force projections by species (e.g., radiative forcing acting directly on electrons but not ions, or differing inertia under the same guiding-center geometry). The purpose of the frozen-in check above is therefore restrictive: it determines whether an electromagnetic specialization that presumes magnetic slippage on the separation scale is admissible. If frozen-in is strong on the evaluated scale, then electron displacement cannot be treated as independent of the field and the electromagnetic slippage path is disallowed under the declared protocol.

Accordingly, the Coulomb Shearing mechanism is evaluated as a *drag-limited* species-relative slip along a declared polar segment, not as macroscopic charge separation or as a claim that electrons "escape" magnetic attachment. Where magnetic stresses enforce strict co-motion on the relevant scale, the admissibility test fails and the segment is classified as inactive ($\chi_s = 0$).

**Note on transverse electromagnetic forces.** The specialization adopted here concerns only the along-trajectory differential acceleration used to compute $\Delta x(t_{\rm res})$ from $\Delta a_{\parallel}(t)$ in the closure criterion. Electromagnetic forces can strongly influence the trajectory geometry through transverse (normal) deflections, but such deflections are not an additional independent driver of *along-trajectory* charge separation unless they generate a sustained $E_{\parallel} \neq 0$ or alter the transport time and path length (captured separately through $t_{\rm res}$ in Section 02). A fully vectorial generalization of the criterion (tracking the separation vector relative to the local screening geometry) is possible. In the present framework, electromagnetic forcing contributes to $\Delta a_{\parallel}$ only through an explicitly declared $E_{\parallel}(r)$ model (Eq. (43)); the radiative-only specialization is recovered by setting $E_{\parallel} = 0$.

Then, separating radiative forcing from the only electromagnetic contribution that survives the along-trajectory projection (the parallel electric field; Eq. (43)), the differential acceleration can be written as

$$\Delta a \equiv a_e - a_i \simeq \mu_s \, \frac{\sigma_T L}{4\pi r^2 c \, m_e} + \left(\frac{q_e}{m_e} - \frac{q_i}{m_i}\right) E_{\parallel}, \tag{58}$$

Setting $E_{\parallel} = 0$ yields the Thomson specialization

$$\Delta a \equiv a_e - a_i \simeq \mu_s \, \frac{\sigma_T L}{4\pi r^2 c \, m_e}. \tag{59}$$

Evaluated at the Eddington luminosity $L_{\rm Edd} = 4\pi G M m_p c/\sigma_T$, this becomes

$$\Delta a(L_{\rm Edd}) \simeq \mu_s \, \frac{m_p}{m_e} \, \frac{GM}{r^2}, \tag{60}$$

so the electron–ion differential acceleration is enhanced over the local gravitational scale $GM/r^2$ by the factor $m_p/m_e \approx 1836$.

Physically, the time integral of $\Delta a$ corresponds to the accumulation of relative velocity between the species, and the second time integral corresponds to their relative displacement. This will be made explicit only after the Coulomb restoring model is specified, because the relevant question is not whether $\Delta a$ exists (it generally does), but whether Coulomb coupling can dynamically suppress the resulting drift on the timescales available along the polar trajectory.

### 3.3 Coulomb restoring force and coupling timescale

The mechanism that resists electron–ion separation is Coulomb coupling: electrostatic interaction enforces quasi-neutrality and transfers momentum between species via collisions and collective response. The standard “instantaneous locking” closure is therefore a model assumption about the efficacy and speed of this coupling, not a physical identity.

**Scope of the coupling-break claim (explicit regime constraint).** The species-resolved forcing argument in this manuscript does *not* assert that electrons and ions remain separated over macroscopic times (e.g., hours to days) in dense, fully collisional disk plasma. Instead, the framework evaluates whether *any* non-negligible species-relative drift can accumulate *locally* along a polar trajectory segment before coupling suppresses it. In practice, this is a regime test: if the electron–ion momentum-relaxation time is far shorter than the residence time available for differential forcing, then frequent momentum exchange enforces co-motion and the mechanism is *inactive* on that segment (formalized below by setting the shearing inadmissibility flag to $\chi_s = 0$).

This constraint directly excludes the high-density collisional limit often associated with disk body conditions (e.g., $n_e \sim 10^{13}$–$10^{15}\,\mathrm{cm}^{-3}$ for plausible temperatures), where Spitzer-type estimates typically give $t_c$ that is orders of magnitude shorter than dynamical residence times. The mechanism is therefore evaluated only where the local plasma state and trajectory geometry allow $t_c$ to be comparable to, or longer than, the effective forcing interval, which is expected (if at all) in the lower-density, funnel-adjacent polar domain rather than in the dense disk interior.

To define an explicit and auditable coupling timescale, we adopt the following plasma assumptions for the analysis in this part: (i) the medium is sufficiently ionized that electrons and ions constitute well-defined species, (ii) quasi-neutrality holds on scales larger than the Debye length,

$$n_e \simeq Z\,n_i, \tag{61}$$

with $n_e$ the electron number density, $n_i$ the ion number density, and $Z$ the ion charge state, and (iii) the coupling is in the weakly coupled, collisional regime where a Spitzer-type momentum relaxation time is meaningful (equivalently, the Coulomb logarithm $\ln\Lambda$ is large and binary small-angle scattering dominates the momentum exchange rate).

Under these conditions, a concrete and widely used choice for the electron–ion momentum relaxation (coupling) timescale is the Spitzer electron–ion collision time, which we denote $t_c \equiv \tau_{ei}$. For a single ion species with charge state $Z$,

$$t_c \equiv \tau_{ei} = \frac{3\,(2\pi)^{3/2}\,\varepsilon_0^2\,m_e^{1/2}\,(k_B T_e)^{3/2}}{n_i\,Z^2\,e^4\,\ln\Lambda}, \tag{62}$$

where $\varepsilon_0$ is the vacuum permittivity, $k_B$ is Boltzmann's constant, $T_e$ is the electron temperature, $e$ is the elementary charge, and $\ln\Lambda$ is the Coulomb logarithm. This timescale represents the characteristic time for Coulomb interactions to transfer momentum between electrons and ions of charge $Z$ and thereby relax species-relative drift.

**Order-of-magnitude scaling and relevance to polar jet-base plasma.** Equation (62) implies the familiar scaling $\tau_{ei} \propto T_e^{3/2}/(n_e Z^2)$ (up to $\ln\Lambda$). For later numerical checks, it is useful to record a convenient normalization (Spitzer form) [1]:

$$t_c \approx 3\times 10^5\,\mathrm{s}\,\left(\frac{T_e}{10^8\,\mathrm{K}}\right)^{3/2}\left(\frac{10^8\,\mathrm{cm}^{-3}}{n_e}\right)\left(\frac{20}{\ln\Lambda}\right)\frac{1}{Z^2}, \tag{63}$$

where $n_e$ is the electron density and $\ln \Lambda$ is the Coulomb logarithm.

**Plasma-regime caveat and effective** $\ln \Lambda$**.** Equations (62)–(63) adopt a Spitzer-type momentum relaxation time, which is appropriate for weakly coupled, approximately Maxwellian, collisional plasmas. In polar funnels the distribution can be anisotropic and non-thermal, and strong magnetization and collective processes (wave-particle scattering, micro-instabilities) can modify the effective momentum-exchange rate relative to the binary-collision estimate. In this manuscript we therefore treat $\ln \Lambda$ as an explicit, reportable *input* and interpret it operationally as an effective parameter that absorbs these kinetic corrections rather than as a fixed constant. Sensitivity to this choice is transparent via

$$t_c(\ln \Lambda) = t_c(\ln \Lambda_0)\,\frac{\ln \Lambda_0}{\ln \Lambda}, \qquad \ln \Lambda_0 \equiv 20, \tag{64}$$

and the evaluation protocol requires any worked example or regime map to declare the $\ln \Lambda$ value or bound used (Pre-Committed Evaluation Protocol (Anti-Post-Hoc)).

The admissibility question is therefore parameter-local and falsifiable: if $t_c \ll t_{\text{res}}$ then momentum exchange rapidly enforces co-motion and suppresses drift; if instead $t_c \gtrsim t_{\text{res}}$, then sustained differential forcing can generate resolvable displacement within the available residence time.

Equation (62) is not invoked to claim that coupling is weak or strong a priori. Its role is definitional: it provides a finite, parameter-dependent coupling speed that can be compared directly to the time over which differential forcing acts along a polar trajectory. The closure of perfect co-motion corresponds to the limiting assumption $t_c \to 0$, which is not guaranteed by plasma physics and therefore must be tested against trajectory-resolved forcing.

**Relation to electron–ion equilibration, collisionless heating, and two-temperature accretion literature.** A large literature treats electron–ion equilibration and energy partition in weakly collisional accretion plasmas, including relativistic Coulomb exchange rate fits and their use in two-temperature flow modeling [e.g. 12–18]. Related work also emphasizes that collective processes and micro-instabilities can mediate momentum and energy exchange more rapidly than binary Coulomb collisions, and therefore can modify the effective coupling experienced by electrons and ions [e.g. 19]. The present mechanism is not a claim that two-temperature plasmas are newly discovered, nor a replacement for that literature. The specific novelty asserted here is a *trajectory-resolved closure test*: rather than solving for a bulk electron temperature given a heating fraction, we ask whether enforced electron–ion co-motion remains admissible along a polar (funnel-adjacent) trajectory segment under sustained, species-asymmetric forcing, with failure defined by a displacement criterion that is mapped into scorable, window-locked observables in Observable Consequences: Jets and Stratification and scored under Pre-Committed Evaluation Protocol (Anti-Post-Hoc). Much of the two-temperature accretion-flow literature does not surface this mechanism because it typically (i) evolves a single bulk velocity field for the plasma (electrons and ions share the same fluid motion), (ii) treats electron thermodynamics and ion thermodynamics separately while maintaining quasi-neutral flow kinematics, and (iii) focuses on energy partition and emission modeling rather than on spatial charge-separation failure as a trajectory-local admissibility question. Within this manuscript, those collisionless and quasilinear uncertainties are not ignored; they are absorbed transparently into declared coupling parameters (e.g., effective $\ln \Lambda$ and the regime branching rules above), so that stronger-than-Coulomb microphysical coupling moves the mechanism toward explicit inadmissibility rather than being silently assumed away.

**Regime exclusion: dense collisional plasma enforces the closure.** If the local plasma state satisfies $t_c \ll t_{\rm res}$, then the drag closure defined below drives $\Delta v$ to its quasi-steady value on a timescale negligible compared to the available residence time, preventing cumulative drift from building over the trajectory segment. In this limit, enforced electron–ion co-motion remains admissible under the present closure, and the shearing inadmissibility flag should be taken as $\chi_s = 0$ (Eq. (78)) along those segments. This work therefore does not claim shearing in dense, fully collisional disk plasma conditions; those conditions constitute an explicit failure regime for the mechanism as posed here.

For later use in the explicit failure criterion, we also define the Debye length that bounds quasi-neutral response,

$$\lambda_D \equiv \left( \frac{\varepsilon_0 \, k_B T_e}{n_e \, e^2} \right)^{1/2}, \tag{65}$$

which sets the characteristic scale over which significant charge separation can develop before electrostatic fields and collective plasma response become dominant.

### 3.4 Shearing admissibility condition

Coulomb Shearing is framed here in terms of the admissibility of enforced electron–ion co-motion under sustained differential forcing along a polar or near-polar trajectory. The operative question is binary: does Coulomb coupling maintain joint motion over the available residence time, or does relative displacement grow to a scale where quasi-neutral locking cannot be sustained along that trajectory segment?

However, Coulomb coupling does act continuously. In the collisional Coulomb-coupling regime defined above, a minimal trajectory-resolved model for the suppression of relative drift is a linear momentum-exchange (drag) closure on the species-relative velocity,

$$\frac{d\,\Delta v}{dt} + \frac{1}{t_c}\,\Delta v = \Delta a_{\parallel}(t), \tag{66}$$

where $t_c$ is the electron–ion momentum relaxation time defined in Eq. (62). Equation (66) is not an outcome claim; it is the simplest auditable way to encode the statement “collisions act continuously to reduce species-relative drift”.

For an arbitrary forcing history $\Delta a_{\parallel}(t)$, Eq. (66) implies the trajectory-resolved *drag-limited* relative velocity and displacement

$$\Delta v(t) = \int_0^t e^{-(t-t')/t_c}\,\Delta a_\parallel(t')\,dt', \tag{67}$$

$$\Delta x(t) = \int_0^t \left[t_c\left(1 - e^{-(t-t')/t_c}\right)\right]\Delta a_\parallel(t')\,dt'. \tag{68}$$

This explicitly shows the continuous suppression: forcing contributions earlier than a few $t_c$ are exponentially "forgotten" in $\Delta v$, and the effective displacement kernel saturates at $t_c$ for $t \gg t_c$.

In the special case where $\Delta a_\parallel(t)$ is approximately constant over the residence segment, Eq. (68) reduces to the closed form

$$\Delta x(t) \simeq \Delta a_\parallel \left[t_c\,t - t_c^2\left(1 - e^{-t/t_c}\right)\right]. \tag{69}$$

The limiting behaviors make the key timescale dependence explicit:

$$t \ll t_c: \quad \Delta x(t) \simeq \tfrac{1}{2}\Delta a_\parallel\,t^2 \qquad \text{(free-drift limit)}, \tag{70}$$

$$t \gg t_c: \quad \Delta x(t) \simeq \Delta a_\parallel\,t_c\,t \qquad \text{(drag-limited accumulation)}. \tag{71}$$

**Why short $t_c$ does not get "ignored" (and when it kills the effect).** A very short coupling time does not eliminate differential forcing; it limits the *net* species-relative drift to the drag-limited form in Eq. (71). In the strongly collisional limit $t_c \ll t_{\rm res}$, the number of Coulomb momentum-relaxation events is $\sim t_{\rm res}/t_c \gg 1$, but that large count is not itself the deciding quantity - it is precisely what produces the exponential memory loss in $\Delta v$ (Eq. (67)) and the linear-in-$t_c$ suppression of displacement in Eq. (71). Concretely, even if $t_{\rm res}$ were days, a microsecond–millisecond $t_c$ drives $\Delta x(t_{\rm res}) \propto \Delta a\,t_c\,t_{\rm res}$ to be tiny unless $\Delta a$ is extraordinarily large. The mechanism is therefore *self-disabling* in the high-density collisional regime: small $t_c$ directly suppresses cumulative drift, and the admissibility test below will fail (yielding $\chi_s = 0$) unless the declared parameters imply $\Delta x(t_{\rm res})$ can reach the screening threshold.

Therefore the relevant question is not whether $\Delta a \neq 0$ (it generally is), but whether the *drag-limited* displacement implied by Eqs. (68)–(71) exceeds the admissible scale over the available residence time $t_{\rm res}$.

Coulomb coupling suppresses species-relative drift by transferring momentum between electrons and ions on the timescale $t_c$ (Eq. (62)). Separately, quasi-neutrality implies that charge separation is screened over the Debye length $\lambda_D$ (Eq. (65)). Accordingly, relative displacements approaching or exceeding $\lambda_D$ require electrostatic fields strong enough to restructure the charge distribution on that scale and are taken here to indicate the breakdown of the "locked co-motion" closure over the trajectory segment. The Debye length is therefore used solely as an explicit, auditable *screening / quasi-neutral locking* scale, rather than as a criterion for the cessation of collisional coupling.

Accordingly, the admissibility condition is posed in terms of the *drag-limited* displacement $\Delta x(t_{\rm res})$ implied by Eqs. (68)–(69). Introduce a dimensionless constant $\kappa \sim \mathcal{O}(1)$ to make the threshold explicit while keeping it falsifiable (its allowed bracket is declared explicitly and enforced by the locked protocol in Pre-Committed Evaluation Protocol (Anti-Post-Hoc), rather than tuned per object).

**Interpretation of the Debye-length threshold.** The Debye length $\lambda_D$ marks the physical scale at which collective electrostatic response dominates over collisional momentum exchange. Displacement reaching this scale indicates that the enforced single-fluid closure has broken down along the resolved trajectory segment. Reaching $\Delta x \sim \lambda_D$ indicates that collective electrostatic response and field-mediated dynamics must be treated explicitly, and that the assumption of enforced electron–ion co-motion can no longer be promoted without additional microphysical modeling. No claim of sustained charge separation or large-scale quasi-neutrality violation is implied.

Accordingly, the admissibility condition defined below governs the closure test. For systems operating near the Eddington luminosity and for polar residence times of the magnitude derived in section 2, explicit evaluation (section 9) shows that low-$Z$ species satisfy this condition within the polar channel.

$$\Delta x(t_{\rm res}) \geq \kappa \, \lambda_D, \tag{72}$$

**How $\Delta x(t_{\rm res})$ is obtained.** $\Delta x(t_{\rm res})$ is computed with continuous Coulomb drag, i.e. from Eq. (68) (or Eq. (69) when $\Delta a$ is approximately constant on $[0, t_{\rm res}]$).

**Why $\lambda_D$ is used and what it does *not* claim.** The choice of $\lambda_D$ does not assert that collective behavior switches discontinuously at a single length. It provides a conservative and auditable scale separating regimes where quasi-neutral shielding can plausibly maintain electron–ion locking (displacements well below $\lambda_D$) from regimes where sustained charge separation necessarily sources restoring fields and collective response (displacements comparable to or exceeding $\lambda_D$). The factor $\kappa \sim \mathcal{O}(1)$ is retained explicitly to acknowledge that the transition is not sharp and to allow the worked examples (Worked Examples and Source-Grounded Outputs) and window maps (Species-Dependent Shearing Ranges and Probabilities) to test robustness to the precise cutoff. Any alternative threshold would need the same properties: it must be definable from declared plasma inputs and must correspond to a clear failure of the enforced co-motion closure.

**Collective electric-field response and bounded separation.** No part of this manuscript requires, or asserts, macroscopic charge separation. If a species-relative displacement begins to approach the screening scale, the self-consistent collective electric field grows rapidly to restore quasi-neutrality on that scale. This response is not a secondary consideration - it is the physical mechanism that *limits* any drift once $\Delta x$ becomes comparable to the screening length. Accordingly, the admissibility threshold should be interpreted as a *bounded* closure-break indicator: it marks the point at which collective electrostatic response and field-mediated dynamics must be treated explicitly, rather than as a claim that $\Delta x \gg \lambda_D$ is realized or sustained.

Operationally, Coulomb Shearing is treated as a rate-limited leakage process up to a bounded scale $\Delta x = \mathcal{O}(\lambda_D)$, not as long-lived separation over many Debye lengths. If the drag-limited displacement fails to reach this bound within the available forcing interval, the segment is inactive ($\chi_s = 0$). If it does reach the bound, the interpretation is *only* that the enforced co-motion closure is no longer defensible without explicit collective-field modeling.

**Modeling choice and quantitative consequence (explicit).** The use of $\kappa\,\lambda_D$ in Eq. (72) is an operational boundary condition for the *closure test*, not a claim that plasma response changes discontinuously at $\Delta x = \lambda_D$. In particular, the transition from collisional drag-dominated suppression (encoded by $t_c$) to collective field-mediated response is not a sharp cutoff and can depend on magnetization, anisotropy, and micro-instability-driven scattering. The manuscript therefore treats $\kappa$ as an explicitly declared, order-unity uncertainty parameter rather than a hidden degree of freedom.

This criterion is not interpreted as centimeter-scale, long-lived charge separation: it is a bookkeeping threshold indicating that the enforced co-motion closure has become non-defensible without explicit collective-field modeling. Throughout this work, $\kappa$ is restricted to a declared bracket $\kappa \in [1, 10]$ and is never tuned per object; results that would require $\kappa \gg 10$ are classified as inactive under the protocol.

Quantitatively, replacing the screening scale $\lambda_D$ with any alternative microphysical displacement scale $s$ (e.g. an inertial length or magnetization-controlled effective scale in strongly ordered fields) corresponds to the substitution $\lambda_D \longrightarrow s$ in Eq. (72) and therefore shifts the inferred species-resolved boundary radii $(r_{\min}, r_{\max})$ in Section 4 in a controlled way. This is why $\kappa$ is retained explicitly and why the worked-example evaluations (Worked Examples and Source-Grounded Outputs) and regime maps (Species-Dependent Shearing Ranges and Probabilities) must declare the adopted $\kappa$ bound alongside the plasma inputs.

**How $\Delta x(t_{\text{res}})$ is obtained.** $\Delta x(t_{\text{res}})$ is computed with continuous Coulomb drag, i.e. from Eq. (68) (or Eq. (69) when $\Delta a$ is approximately constant on $[0, t_{\text{res}}]$).

A velocity-based alternative (e.g. requiring $\Delta v(t_{\text{res}})$ to exceed a thermal scale) can be formulated, but it corresponds to a different notion of failure: it tests whether the drift becomes dynamically large relative to microscopic speeds, rather than whether quasi-neutral locking can be maintained over the trajectory segment. The analysis here therefore adopts a displacement-to-screening threshold, since it directly targets the closure of interest: enforced joint motion under quasi-neutral coupling along a resolved segment of the trajectory.

For later numerical evaluation it is useful to state explicitly the two required comparisons that are implicit in the drag-limited form:

1. **Forcing-to-threshold comparison:** the integrated differential forcing must be large enough over the available residence time to produce $\Delta x(t_{\text{res}})/\lambda_D \gtrsim 1$ under the drag-limited dynamics.

2. **Residence-to-coupling comparison:** the residence segment must be long enough that continuous Coulomb drag does not eliminate the drift immediately, i.e. $t_{\text{res}}/t_c \nll 1$.

In the constant-$\Delta a$ approximation, these combine into the explicit dimensionless condition

$$\frac{\Delta x(t_{\text{res}})}{\lambda_D} \simeq \frac{\Delta a_{\parallel}}{\lambda_D}\left[t_c\, t_{\text{res}} - t_c^2\left(1 - e^{-t_{\text{res}}/t_c}\right)\right] \geq \kappa, \tag{73}$$

which reduces to $\frac{1}{2}\Delta a_{\parallel}\, t_{\rm res}^2/\lambda_D \geq \kappa$ when $t_{\rm res} \ll t_c$ and to $\Delta a_{\parallel}\, t_c\, t_{\rm res}/\lambda_D \geq \kappa$ when $t_{\rm res} \gg t_c$.

This admissibility test is binary by construction:

- If $\Delta x(t_{\rm res}) < \kappa\,\lambda_D$, enforced electron–ion co-motion remains admissible under the assumed dynamical closure.
- If $\Delta x(t_{\rm res}) \geq \kappa\,\lambda_D$, enforced electron–ion co-motion is no longer admissible and requires explicit dynamical evaluation.

**Regime specificity.** The admissibility inequality in Eq. (73) is not satisfied under typical dense, thermalized accretion-disk plasma conditions, where short coupling timescales and modest differential forcing preserve enforced electron–ion co-motion. The mechanism operates specifically in polar channels of rapidly accreting systems where residence times are extended and radiative forcing approaches Eddington intensity. In these regimes, $\chi_s = 1$ is the expected outcome for low-Z species.

Only in restricted regimes with sufficiently anisotropic forcing, extended residence time, and plasma conditions departing from the dense collisional limit can $\chi_s = 1$ arise, and those regimes must be identified by explicit evaluation rather than assumed.

The inequality is failure-capable because each quantity is explicitly definable and, where required, can be evaluated numerically: $\Delta a_{\parallel}(t)$ is species-resolved and trajectory-projected (Eq. (37)), $t_{\rm res}$ is set by the polar transport dynamics (section 2), and $\lambda_D$ and $t_c$ are explicit functions of $(n_e, T_e, Z, \ln\Lambda)$. No outcome language is required. The statement is solely about the validity of the enforced electron–ion co-motion closure under time-resolved differential forcing.

### 3.5 Relativistic correction factors

The specializations in this section are written in an auditable Newtonian form to keep the force and timescale bookkeeping explicit and executable from declared inputs. For near-horizon evaluation radii (e.g. $r \sim 5\,r_g$ in the M87* worked example), general-relativistic effects can introduce order-unity corrections. This manuscript therefore defines a minimal set of multiplicative correction factors that can be applied without changing any closure definitions or inequality structure.

**Consistency convention for relativistic correction factors.** All relativistic correction factors introduced in this subsection apply *uniformly and multiplicatively* to the same physical quantities whenever those quantities are evaluated, and are not optional or selectively invoked. In particular: (i) force terms must be evaluated in the same declared local frame before forming species differences; (ii) any time-dilation or redshift mapping used to relate local accelerations or timescales to a distant-observer frame must be applied consistently to *both* the differential acceleration $\Delta a$ and the residence-time kernel when those quantities are compared; and (iii) no worked example may include a relativistic correction for a quantity in one step while omitting it for the same quantity in another step.

Operationally, each worked example must declare whether the evaluation is performed entirely in the local orthonormal frame (in which case no additional mapping factors are introduced), or mapped to a distant-observer frame (in which case the same mapping is applied everywhere). Any calculation that violates this uniformity requirement is invalid under the declared evaluation protocol.

**Redshift and notation.** Let $g \equiv -g_{tt}$ denote the gravitational redshift factor evaluated on the relevant trajectory at radius $r$ in the exterior domain. Quantities in this section are defined as follows:

- The Newtonian-force specialization in Eq. (38) is treated as a baseline *coordinate-frame* expression.
- Proper-time quantities (denoted by "proper") are related to coordinate-time quantities (denoted by "coord") by the redshift factor $g^{1/2}$ at the evaluation radius.

### 3.5.1 Radiative forcing correction

A minimal correction for radiative momentum transfer in the near-horizon exterior region is written as a multiplicative factor on Eq. (38):

$$F_{\mathrm{rad},r}^{(\mathrm{corr})} = \left(\frac{\sigma_T L}{4\pi r^2 c}\right) g^{1/2} \left(1 + \frac{v_r}{c}\right), \tag{74}$$

where $v_r$ is the local radial velocity component along the trajectory segment (sign convention as in Section 2). The corrected electron radiative acceleration is then

$$a_{\mathrm{rad},e}^{(\mathrm{corr})} = \frac{F_{\mathrm{rad},r}^{(\mathrm{corr})}}{m_e}, \tag{75}$$

with the ion direct radiative term remaining negligible under the Thomson specialization unless explicitly declared otherwise.

### 3.5.2 Proper time vs coordinate time

Timescale comparisons in Sections 2 and 3.3 can be expressed either in coordinate time or proper time, provided the same convention is used consistently. A minimal mapping at the evaluation radius is

$$t_{\mathrm{res,proper}} = t_{\mathrm{res,coord}}\, g^{1/2}, \tag{76}$$

and similarly for any coupling or relaxation time expressed in the local frame. Under this mapping, any ratio $t_{\mathrm{res}}/t_c$ is invariant only if both numerator and denominator are expressed in the same time convention.

### 3.5.3 Frame-dragging correction to drift

If the evaluation track includes an electromagnetic drift contribution, a minimal frame-dragging correction can be applied to the standard $\mathbf{E} \times \mathbf{B}$ drift magnitude:

$$v_{\mathrm{drift}}^{(\mathrm{corr})} = c\, \frac{|\mathbf{E} \times \mathbf{B}|}{B^2} \left[1 + \left(\frac{\omega r}{c}\right)\right], \tag{77}$$

where $\omega \equiv -g_{t\phi}/g_{\phi\phi}$ is the local frame-dragging rate in the exterior domain and all quantities are evaluated on the declared trajectory segment.

#### 3.5.4 Conservative revision gate for worked examples

The primary admissibility metric in this section is the dimensionless displacement ratio $\Delta x(t_{\mathrm{res}})/\lambda_D$ (Eq. (73)). Relativistic corrections enter this ratio only through (i) the corrected differential forcing that sets $\Delta a$, and (ii) the time convention used for $t_{\mathrm{res}}$ and $t_c$. A conservative check is therefore imposed:

> If applying Eqs. (74)–(77) and the proper-time mapping in Eq. (76) changes $\Delta x(t_{\mathrm{res}})/\lambda_D$ by a factor greater than 3 for a declared worked example, that worked example must be re-issued under a "relativistic-corrected" control run in Appendix E before any PASS/FAIL outcome is used downstream.

This gate does not alter any mechanism claim. It is a bounded robustness requirement on the numerical instantiation of declared inputs at near-horizon evaluation radii.

$$\chi_s = \begin{cases} 1, & \text{enforced electron–ion co-motion inadmissible} \\ 0, & \text{enforced electron–ion co-motion admissible} \end{cases} \tag{78}$$

**Exterior-only domain and near-horizon limit.** The admissibility variable $\chi_s(r; Z)$ and the shearing criterion are defined and evaluated only in the exterior region $r > r_{\mathrm{H}}$, where $r_{\mathrm{H}}$ is the outer Kerr horizon (Eq. (7)). The limit $r \longrightarrow r_{\mathrm{H}}^{+}$ is therefore understood as an exterior limit taken from above. No statement is made in this manuscript about continued drift, charge separation, or coupling behavior at $r \leq r_{\mathrm{H}}$, because such behavior cannot contribute to externally observable jet loading or composition partition once outward transport is causally unavailable. In this operational sense, the horizon acts as an absorbing boundary for the prediction-relevant separation process: only separation that occurs outside $r_{\mathrm{H}}$ can populate an escaping electron channel.

**SMBH / quasar-like luminosity regime (what changes and why electromagnetic tracks matter).** For supermassive black holes in quasar-like radiative states, the controlling question is not "is the source bright," but which differential channel dominates the electron–ion separation rate over the exterior domain once the local plasma state is declared. The radiative-only specialization ($E_{\parallel} = 0$) provides a clean baseline, but in SMBH systems the exterior forcing landscape is often shaped by large-scale field structure, non-ideal electric fields, and relativistic kinematics. For that reason, the analysis explicitly supports electromagnetic specializations (Sections 3.2.1–3.5.3), in which the separation rate is driven by the evaluated $E_{\parallel}(r)$ (and associated drift terms) together with the residence-time and collisionality gates. In practice, SMBH/quasar applications are therefore expected to be adjudicated primarily by the computed electromagnetic track outputs (e.g., $\Delta x(t_{\mathrm{res}})/\lambda_D$ and the closure/neutrality criteria) rather than by luminosity alone.

# 4 Species-Dependent Shearing Ranges and Probabilities

**Conditional scope of the construction.** All results are conditional on the failure of enforced electron–ion co-motion along a resolved trajectory segment, as encoded by the outcome $\chi_s = 1$. If enforced co-motion remains admissible and $\chi_s = 0$, then no species-dependent shearing ranges exist under the stated admissibility outcome: the sets defined by $r_{\min}(Z, q, v, \theta)$ and $r_{\max}(Z, q, v, \theta)$ are empty, and the corresponding probabilities $P_s(Z)$ vanish. No statement of generic shearing is made; the expressions parameterize consequences only in regimes where the closure fails.

The binary Coulomb shearing admissibility outcome, encoded by the variable $\chi_s$, induces species-resolved, computable regimes. Distinct atomic species correspond to distinct, physically evaluable shearing domains when the admissibility criterion fails, and finite residence time and velocity dispersion induce phase–space measures of shearing outcomes without introducing phenomenological tuning.

All results are functions of the differential acceleration criterion and the associated coupling timescale entering the admissibility test.

## 4.1 Derivation of lower and upper shearing bounds

**Regime-dependent bounds.** The shearing interval $[r_{\min},\ r_{\max}]$ is species-dependent and may be empty for high-Z species with strong binding or for trajectory families with insufficient residence time. For hydrogen and helium in polar channels of near-Eddington systems, the interval is nonempty and spans a physically significant radial range. Such outcomes correspond to an empty admissible set and indicate that no shearing occurs for that species under the evaluated conditions. Empty ranges are therefore a valid and expected result of the framework and do not represent a failure or inconsistency of the construction.

We begin from the deterministic shearing condition associated with the admissibility outcome, which may be written schematically as

$$\Delta a_{\parallel}(r, \theta, v) \;\geq\; a_{\mathrm{C}}(Z, q), \tag{79}$$

where $\Delta a_{\parallel}$ is the component of differential acceleration between electrons and ions along the trajectory tangent, and $a_{\mathrm{C}}$ is the Coulomb restoring acceleration associated with an ion of atomic number $Z$ and charge state $q$.

The differential acceleration may be expressed as

$$\Delta a_{\parallel} = \left| \frac{F_{e,\parallel}}{m_e} - \frac{F_{i,\parallel}}{m_i} \right|, \tag{80}$$

where $F_{e,\parallel}$ and $F_{i,\parallel}$ are the net external forces projected along the trajectory for electrons and ions respectively. These forces arise from radiative, magnetic, and inertial contributions determined by geometry and local field structure.

The Coulomb restoring acceleration is written as

$$a_{\mathrm{C}}(Z,q) = \frac{Z_{\mathrm{eff}}\, e^2}{4\pi\varepsilon_0\, m_e\, \ell^2}, \tag{81}$$

where $Z_{\mathrm{eff}}$ is the effective nuclear charge experienced by the electron, and $\ell$ is the characteristic electron–ion separation scale associated with the bound state.

**Regime-dependent definition of the separation scale $\ell$.** The length scale $\ell(Z,q)$ is the electron–ion separation scale relevant to the restoring electrostatic response, and it depends on whether the plasma is bound-state dominated or fully ionized. To keep the construction consistent across regimes, we treat $\ell$ as an explicit input defined by

$$\ell(Z,q) \equiv \begin{cases} \ell_{\mathrm{bound}}(Z,q), & \text{partially ionized / bound-state dominated,} \\ \kappa\, \lambda_D, & \text{fully ionized / free-charge separation,} \end{cases} \tag{82}$$

where $\lambda_D$ is the Debye length defined in Eq. (65) and $\kappa \geq 1$ is the declared screening-threshold factor introduced in Differential Forcing and the Onset of Coulomb Shearing. In the fully ionized regime, Eq. (81) is therefore interpreted as the restoring acceleration across the screened separation scale $\kappa\lambda_D$, not as an orbital binding model.

However, instantaneous satisfaction of Eq. (79) is not sufficient, and the bound-state proxy $\Delta a_{\parallel} \geq a_{\mathrm{C}}$ is not the operative closure test used in this paper. The executed admissibility criterion is the drag-limited displacement condition of Differential Forcing and the Onset of Coulomb Shearing:

$$\Delta x(t_{\mathrm{res}}) \;\geq\; \kappa\, \lambda_D, \tag{83}$$

with $\Delta x(t)$ computed under continuous Coulomb drag (Eq. (68), or Eq. (69) in the constant-$\Delta a_{\parallel}$ approximation).

**Local bound equation (constant-$\Delta a_{\parallel}$ form).** When $\Delta a_{\parallel}$ is approximately constant over the trajectory segment contributing to $t_{\mathrm{res}}$, the drag-limited displacement is

$$\Delta x(t_{\mathrm{res}}) \simeq \Delta a_{\parallel} \left[ t_c\, t_{\mathrm{res}} - t_c^2 \left( 1 - e^{-t_{\mathrm{res}}/t_c} \right) \right], \tag{84}$$

so the boundary of the shearing-permitted set is defined by the equality

$$\Delta a_{\parallel}(r,\theta,v)\; \left[ t_c(r,\theta,Z,q)\, t_{\mathrm{res}}(r,\theta,v) - t_c^2(r,\theta,Z,q) \left( 1 - e^{-t_{\mathrm{res}}/t_c} \right) \right] = \kappa\, \lambda_D(r,\theta,Z,q). \tag{85}$$

All species dependence enters through $t_c(Z,q)$ and $\lambda_D$ (and through any $Z$-dependence in the evaluated forcing model for $\Delta a_{\parallel}$), consistent with the fully ionized interpretation in this section.

**Definition of $r_{\min}$ and $r_{\max}$ as set boundaries.** Define the dimensionless shearing functional

$$\mathcal{S}(r;\theta,v,Z,q)\equiv\frac{\Delta x(t_{\rm res})}{\kappa\,\lambda_D},\tag{86}$$

with $\Delta x(t_{\rm res})$ evaluated by Eq. (84) (or the full kernel when required).

**Segment-limited admissible set.** All window endpoints are defined on the pre-declared polar evaluation segment used to define $t_{\rm res}$ (Transport of Matter to Polar Trajectories). Let that segment be $r\in[r_{\rm in},\ r_{\rm out}]$. For fixed $(\theta,v,Z,q)$, define the segment-limited shearing-permitted radial set

$$\mathcal{A}_Z^{\rm seg}(\theta,v,q)\equiv\{\,r\in[r_{\rm in},\ r_{\rm out}]\ :\ \mathcal{S}(r;\theta,v,Z,q)\geq 1\,\}\,.$$

If $\mathcal{A}_Z$ is empty, the shearing window for that state is empty. If $\mathcal{A}_Z$ is nonempty and connected, the window endpoints are defined by

$$r_{\min}(Z,q,v,\theta)\equiv\inf\mathcal{A}_Z^{\rm seg}(\theta,v,q),\tag{87}$$

$$r_{\max}(Z,q,v,\theta)\equiv\sup\mathcal{A}_Z^{\rm seg}(\theta,v,q),\tag{88}$$

i.e., the first and last radii along the evaluated streamline family where the executed closure criterion returns $\chi_s=1$.

**Limiting scalings (sanity checks).** Equation (85) reduces to the free-drift limit

$$\tfrac{1}{2}\Delta a_{\parallel}\,t_{\rm res}^2\simeq\kappa\,\lambda_D\qquad(t_{\rm res}\ll t_c)$$

and to the drag-limited accumulation limit

$$\Delta a_{\parallel}\,t_c\,t_{\rm res}\simeq\kappa\,\lambda_D\qquad(t_{\rm res}\gg t_c),$$

matching the explicit limits already stated in Eq. (73). These expressions make clear why an inner boundary can exist even when $\Delta a_{\parallel}$ rises inward: inward evolution typically shortens $t_c$ (stronger coupling) and changes $t_{\rm res}$ and $\lambda_D$ through the same declared plasma scalings, so $\mathcal{S}(r)$ need not be monotone.

### 4.1.1 Illustrative worked bounds for a fiducial fully ionized plasma state

This section is structurally complete once $r_{\min}(Z,q,v,\theta)$ and $r_{\max}(Z,q,v,\theta)$ are defined by Eqs. (87)–(88), but it is helpful to exhibit at least one explicit evaluation to show how the abstract machinery produces concrete, checkable numbers. The calculations below are *illustrative only* and are not used for any validation claim; the pre-committed object-level evaluation remains confined to Worked Examples and Source-Grounded Outputs and the evaluation protocol in Section 8.

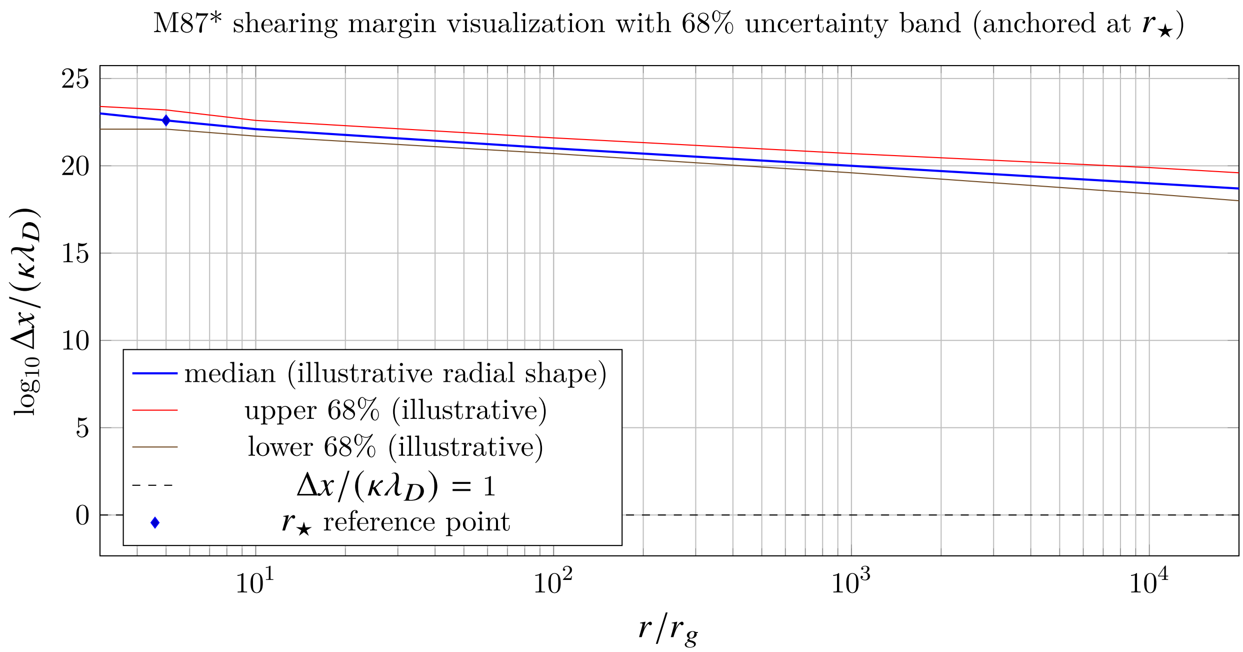


Figure 3: Figure 4.2: Visualization of the shearing-margin functional $\Delta x/(\kappa\lambda_D)$ versus radius for M87*, with an illustrative 68% uncertainty band. The uncertainty band is anchored to the M87* uncertainty propagation reported in Section 9 and is shown here to illustrate how uncertainty is represented on shearing-window plots. The radial dependence in this figure is schematic and is not used for validation; the executed window evaluation remains defined by the set construction and declared inputs.

**Fiducial plasma inputs.** Adopt the same normalization used to record the Spitzer-form coupling time in Eq. (63):

$$T_e = 10^8\,\mathrm{K}, \qquad n_e = 10^8\,\mathrm{cm}^{-3}, \qquad \ln\Lambda = 20,$$

and set the screening cutoff factor to $\kappa = 1$ in Eq. (65). This yields

$$\lambda_D \approx 6.9 \times 10^{-2}\,\mathrm{m}, \tag{89}$$

from Eq. (65).

**Worked evaluation using the executed drag-limited displacement criterion.** The operative boundary condition for the shearing window is not the instantaneous proxy $\Delta a_\parallel \geq a_\mathrm{C}$. The executed criterion in this paper is the drag-limited displacement test (Differential Forcing and the Onset of Coulomb Shearing):

$$\Delta x(t_\mathrm{res}) \geq \kappa\,\lambda_D,$$

with $\Delta x(t)$ evaluated by Eq. (69) (constant-$\Delta a_\parallel$ approximation) or the full kernel Eq. (68) when required.

**Minimal local model to produce an explicit $r_\mathrm{min}, r_\mathrm{max}$ example.** To show how the abstract construction produces concrete endpoints without introducing new physics, adopt a local streamline segment centered at radius $r$ with a fixed segment thickness in gravitational radii,

$$\Delta r \equiv \zeta\, r_{\mathrm{g}}, \qquad r_g \equiv \frac{GM}{c^2},$$

and approximate the local speed by a slowed Newtonian free-fall magnitude,

$$v(r) \simeq \eta\, v_{\mathrm{ff}}(r), \qquad v_{\mathrm{ff}}(r) \equiv \sqrt{\frac{2GM}{r}}, \qquad 0 < \eta < 1,$$

consistent with the role of $\eta$ in Transport of Matter to Polar Trajectories. Writing $x \equiv r/r_g$, the resulting local residence time is

$$t_{\mathrm{res}}(x) \simeq \frac{\Delta r}{\eta v_{\mathrm{ff}}} = \frac{\zeta}{\eta} \frac{GM}{c^3} \sqrt{\frac{x}{2}}. \tag{90}$$

For the forcing, use the auditable Thomson, Eddington-scale specialization already defined by Eqs. (38) and (59):

$$\Delta a_{\parallel}(x) \approx \mu_s\, f_{\mathrm{Edd}} \frac{m_p}{m_e} \frac{GM}{r^2} = \mu_s\, f_{\mathrm{Edd}} \frac{m_p}{m_e} \frac{c^4}{GM}\, x^{-2}. \tag{91}$$

Given $(T_e, n_e, \ln\Lambda)$, compute $\lambda_D$ (Eq. (65)) and $t_c(Z, q)$ (Eq. (63)), then evaluate the displacement using Eq. (69):

$$\Delta x(x) \simeq \Delta a_{\parallel}(x) \left[ t_c\, t_{\mathrm{res}}(x) - t_c^2 \left( 1 - e^{-t_{\mathrm{res}}(x)/t_c} \right) \right], \tag{92}$$

and define the local shearing functional $\mathcal{S}(x) \equiv \Delta x(x)/(\kappa\lambda_D)$ (Eq. (86)).

**Explicit outer bound in the two executed limits.** Equation (73) implies two transparent boundary scalings.

*Free-drift limit* ($t_{\mathrm{res}} \ll t_c$): using $\Delta x \simeq \frac{1}{2}\Delta a_{\parallel} t_{\mathrm{res}}^2$ and Eqs. (90)–(91),

$$\mathcal{S}(x) \simeq \frac{\mu_s f_{\mathrm{Edd}} (m_p/m_e)\, \zeta^2}{4\eta^2} \frac{r_g}{\kappa\lambda_D}\, x^{-1}. \tag{93}$$

The outer endpoint $x_{\max} \equiv r_{\max}/r_g$ in this limit is therefore

$$x_{\max}^{(\mathrm{short})} \simeq \frac{\mu_s f_{\mathrm{Edd}} (m_p/m_e)\, \zeta^2}{4\eta^2} \frac{r_g}{\kappa\lambda_D}. \tag{94}$$

*Drag-limited accumulation limit* ($t_{\mathrm{res}} \gg t_c$): using $\Delta x \simeq \Delta a_{\parallel} t_c t_{\mathrm{res}}$,

$$\mathcal{S}(x) \simeq \frac{\mu_s f_{\mathrm{Edd}} (m_p/m_e)}{\kappa\lambda_D} \frac{\zeta c\, t_c}{\eta\sqrt{2}}\, x^{-3/2}, \tag{95}$$

Table 1: Illustrative regime identification for the executed displacement criterion using the fiducial plasma state $T_e = 10^8\,\mathrm{K}$, $n_e = 10^8\,\mathrm{cm}^{-3}$, $\ln\Lambda = 20$, $\kappa = 1$ (so $\lambda_D \approx 6.9 \times 10^{-2}\,\mathrm{m}$). The table reports $t_{\mathrm{res}}/t_c$ at $x = r/r_g = 20$ for the local segment model (Eq. (90)) with $(\zeta, \eta) = (10, 0.1)$ and the Spitzer scaling for $t_c$ (Eq. (63)) with the explicit $Z^{-2}$ dependence. This shows which executed limit (free-drift vs drag-limited) controls the boundary scaling in a given case.

| $M$ | $Z$ | $t_c$ [s] | $t_{\mathrm{res}}(x{=}20)$ [s] | $t_{\mathrm{res}}/t_c$ |
|---|---|---|---|---|
| $10\,M_\odot$ | 1 | $3.0 \times 10^5$ | $1.56 \times 10^{-2}$ | $5.2 \times 10^{-8}$ |
| $10\,M_\odot$ | 26 | $4.4 \times 10^2$ | $1.56 \times 10^{-2}$ | $3.5 \times 10^{-5}$ |
| $10^8\,M_\odot$ | 1 | $3.0 \times 10^5$ | $1.56 \times 10^5$ | $5.2 \times 10^{-1}$ |
| $10^8\,M_\odot$ | 26 | $4.4 \times 10^2$ | $1.56 \times 10^5$ | $3.5 \times 10^2$ |

so the outer endpoint in this limit is

$$x_{\max}^{(\mathrm{long})} \simeq \left[\frac{\mu_s f_{\mathrm{Edd}} (m_p/m_e)}{\kappa \lambda_D}\,\frac{\zeta c\, t_c}{\eta\sqrt{2}}\right]^{2/3}. \tag{96}$$

**Inner endpoint in this illustrative construction.** For a monotone local $\mathcal{S}(x)$ that increases inward, the inner endpoint is simply the inner cutoff of the evaluated polar segment (horizon-scale cutoff, ISCO cutoff, or an imposed termination of the polar segment). For illustration, one may set $r_{\min} \equiv 2\,r_{\mathrm{g}}$. In Worked Examples and Source-Grounded Outputs, $r_{\min}$ and $r_{\max}$ are computed by the set definition in equations (87) and (88) under object-specific inputs, without using any auxiliary instantaneous criterion.

### 4.2 Fully ionized separation and partial-ionization effects

**Fully ionized regime: shearing is spatial separation of free charges.** The primary regime targeted by Coulomb Shearing is a hot, dilute, fully ionized polar plasma in which electrons and ions are already free. In this case there are no bound electrons to "strip" from atomic orbitals, and "shearing" means spatial separation of free electrons from ions via closure failure under differential forcing (Differential Forcing and the Onset of Coulomb Shearing). Species dependence enters through the coupling time $t_c(Z, q)$, the charge state $Z$, and the local plasma parameters that set $\lambda_D$, not through bound-state ionization energies.

**Partially ionized regime: ionization is distinct from free-charge separation.** If the plasma entering the polar channel is only partially ionized, differential forcing can do work that changes the ion charge state before (or while) free-charge separation occurs. To keep these processes logically separate, define the integrated work available to change the ionic charge state as

$$W_{\mathrm{ion}} = \int_0^{t_{\mathrm{res}}} m_e\, \Delta a_{\parallel}(t)\, v_{\mathrm{rel}}(t)\, dt, \tag{97}$$

where $v_{\mathrm{rel}}$ is the instantaneous electron–ion relative velocity induced by the forcing and drag dynamics (Differential Forcing and the Onset of Coulomb Shearing). Let $I_j(Z, q)$ denote the $j$-th ionization potential from initial charge state $q$; then ionization of $k$ additional electrons is energetically admissible if

$$\sum_{j=1}^{k} I_j(Z, q) \leq W_{\mathrm{ion}}. \tag{98}$$

This ionization admissibility condition changes the charge state and therefore modifies $t_c$, $\lambda_D$, and the radiative and electromagnetic couplings. It does *not* replace the shearing admissibility criterion, which remains the Debye-scale displacement threshold for free-charge separation in the already-ionized component. In other words: in partially ionized inflow, Eq. (98) determines whether the plasma becomes more ionized; Differential Forcing and the Onset of Coulomb Shearing determines whether electron–ion co-motion fails for the resulting plasma state.

### 4.3 Probability of shearing within finite residence time

**Meaning of probability in this context.** The quantity $P_s(Z)$ is not an epistemic or statistical uncertainty. It represents a deterministic phase–space measure induced by finite residence time and velocity dispersion under fixed macroscopic conditions. For a given system, $P_s(Z)$ corresponds to the fraction of admissible trajectories or velocities for which the shearing criterion is satisfied, not to randomness in the underlying dynamics or ignorance of system parameters. A value $P_s(Z) = 1$ indicates that all admissible trajectories shear; $0 < P_s(Z) < 1$ reflects partial coverage of phase space due to finite-time exposure; and $P_s(Z) = 0$ indicates that no admissible trajectories satisfy the criterion.

Deterministic bounds define where shearing is *possible*. Probability enters only because particles traverse the shearing region with a distribution of velocities and trajectories.

Let $f(v)$ denote the normalized velocity distribution for a given species along admissible trajectories. Define the shearing kernel in terms of the executed drag-limited displacement criterion from Differential Forcing and the Onset of Coulomb Shearing. Let

$$\Delta x_Z(t_{\mathrm{res}}; v, \theta) \equiv \Delta x(t_{\mathrm{res}})$$

denote the electron-ion relative displacement computed for species $(Z, q)$ over the residence segment, using Eq. (68) (or Eq. (69) when $\Delta a$ is approximately constant on the segment). Then define

$$\mathcal{K}_{\mathrm{shear}}(Z, v, \theta) = H(\Delta x_Z(t_{\mathrm{res}}; v, \theta) - \kappa\, \lambda_D(Z, q)), \tag{99}$$

where $H$ is the Heaviside function encoding satisfaction of the same closure-failure threshold that defines $\chi_S$.

The species-dependent probability of shearing is then

$$P_s(Z) = \int_0^\infty \mathcal{K}_{\rm shear}(Z, v, \theta)\, f(v)\, dv, \tag{100}$$

with normalization $\int_0^\infty f(v)\, dv = 1$.

**Closed-form evaluation in the two executed limits.** If $\Delta a_\parallel$ and the path geometry are treated as fixed over the evaluated segment and the residence time is modeled as $t_{\rm res} = L_{\rm path}/v$, then $v$ enters the kernel primarily through $t_{\rm res}(v)$. In that case a speed threshold $v_{\rm crit}$ can be written explicitly in the same two limits already used in Eq. (73):

*Free-drift limit* ($t_{\rm res} \ll t_c$):

$$\Delta x \simeq \tfrac{1}{2}\Delta a_\parallel t_{\rm res}^2 \quad \Longrightarrow \quad v \le v_{\rm crit}^{\rm (short)} \equiv L_{\rm path}(\theta)\sqrt{\frac{\Delta a_\parallel(\theta)}{2\,\kappa\,\lambda_D(Z,q)}}.$$

*Drag-limited accumulation limit* ($t_{\rm res} \gg t_c$):

$$\Delta x \simeq \Delta a_\parallel t_c\, t_{\rm res} \quad \Longrightarrow \quad v \le v_{\rm crit}^{\rm (long)} \equiv \frac{\Delta a_\parallel(\theta)\, t_c(Z,q)\, L_{\rm path}(\theta)}{\kappa\,\lambda_D(Z,q)}.$$

In either limit, the probability reduces to a CDF evaluation

$$P_s(Z) = \int_0^{v_{\rm crit}} f(v)\, dv \;\equiv\; F_v(v_{\rm crit})\,, \tag{101}$$

with $v_{\rm crit}$ taken as the appropriate limit form above, or defined implicitly by the full kernel evaluation when the system is not cleanly in either limit.

**Illustrative example for a Maxwellian speed distribution.** If the admissible inflow speeds are approximately Maxwellian in magnitude,

$$f_{\rm MB}(v) = \frac{4}{\sqrt{\pi}}\frac{v^2}{v_{\rm th}^3}\exp\!\left(-\frac{v^2}{v_{\rm th}^2}\right), \qquad v_{\rm th} \equiv \sqrt{\frac{2k_BT_i}{m_i}}, \tag{102}$$

then Eq. (101) has the closed form

$$P_s(Z) = \mathrm{erf}(x) - \frac{2}{\sqrt{\pi}}\, x\, e^{-x^2}, \qquad x \equiv \frac{v_{\rm crit}}{v_{\rm th}}. \tag{103}$$

These expressions are illustrative. In Worked Examples and Source-Grounded Outputs, $P_s$ is computed by evaluating the kernel using the same $\Delta x(t_{\rm res})$ dynamics and the same locked uncertainty propagation rules declared in Section 8.

### 4.4 Transition between coupling-dominated and shearing-dominated regimes

Finally, we characterize the regime transition in terms of competing timescales. Let $t_c$ denote the electron–ion coupling timescale entering the admissibility criterion. Shearing dominates when differential forcing persists long enough to overcome Coulomb restoration:

$$\frac{t_{\mathrm{res}}}{t_c} \;\geq\; \left(\frac{\ell(Z,q)}{\Delta a_{\parallel}\, t_c^2}\right)^{1/2}. \tag{104}$$

Below this threshold, coupling suppresses separation and collective motion is restored. Above it, electron–ion separation persists and controls the dynamics. This transition is species-dependent and must be evaluated explicitly for each system.

#### 4.4.1 Sensitivity of shearing windows to $(T_e, n_e, B, \eta)$

**Why sensitivity is mandatory.** The shearing bounds $r_{\min}$ and $r_{\max}$ and the induced probabilities $P_s(Z)$ are functions of measurable but uncertain plasma inputs. Robustness therefore cannot be asserted qualitatively. It must be demonstrated by explicit propagation of input uncertainty into the shearing-window outputs under a fixed, auditable rule (Pre-Committed Evaluation Protocol (Anti-Post-Hoc)).

**Closed-form scaling for the explicitly modeled plasma parameters.** The framework already makes the most important parametric dependencies explicit. In particular, the Coulomb coupling time in Differential Forcing and the Onset of Coulomb Shearing scales as

$$t_c \;\propto\; \frac{T_e^{3/2}}{n_e\, Z^2\, \ln\Lambda}, \tag{105}$$

and the Debye length scales as

$$\lambda_D \;\propto\; \left(\frac{T_e}{n_e}\right)^{1/2}. \tag{106}$$

In the fully ionized regime of Species-Dependent Shearing Ranges and Probabilities, the operative displacement scale is $\ell = \kappa\lambda_D$ (Differential Forcing and the Onset of Coulomb Shearing), so the integrated shearing figure-of-merit

$$\mathcal{S}(Z) \;\equiv\; \frac{\Delta a_{\parallel}\, t_{\mathrm{res}}^2}{\ell} \;=\; \frac{\Delta a_{\parallel}\, t_{\mathrm{res}}^2}{\kappa\lambda_D} \tag{107}$$

obeys the local differential sensitivity (small-uncertainty limit)

$$\frac{\delta\mathcal{S}}{\mathcal{S}} \;\simeq\; \frac{\delta(\Delta a_{\parallel})}{\Delta a_{\parallel}} + 2\,\frac{\delta t_{\mathrm{res}}}{t_{\mathrm{res}}} - \frac{\delta\kappa}{\kappa} - \frac{1}{2}\frac{\delta T_e}{T_e} + \frac{1}{2}\frac{\delta n_e}{n_e}. \tag{108}$$

Since the residence time bound in Transport of Matter to Polar Trajectories scales as $t_{\rm res} \propto \eta^{-1}$, the dependence on the transport slowdown is explicit:

$$\frac{\delta \mathcal{S}}{\mathcal{S}} \simeq \frac{\delta(\Delta a_{\parallel})}{\Delta a_{\parallel}} - 2\frac{\delta\eta}{\eta} - \frac{\delta\kappa}{\kappa} - \frac{1}{2}\frac{\delta T_e}{T_e} + \frac{1}{2}\frac{\delta n_e}{n_e}. \tag{109}$$

These relations do not add new physics. They simply expose how the already-declared inputs control whether the window is wide, narrow, or empty.

**How $B$ enters the sensitivity.** The magnetic-field amplitude $B$ enters only through the adopted electromagnetic contribution to $\Delta a_{\parallel}$ (Differential Forcing and the Onset of Coulomb Shearing). Therefore the sensitivity to $B$ is assessed by propagating $B$ through the same $\Delta a_{\parallel}$ model used for the worked example, rather than by asserting any universal analytic exponent.

**Endpoint sensitivity in the executed limiting regimes.** The protocol-level perturbations above are necessary for reproducible robustness reporting, but it is also useful to state the analytic sensitivities of the window endpoints in the two executed limits (Eq. (73)). Using the illustrative local construction in Section 4.4, the outer endpoint scalings (Eqs. (94) and (96)) imply the following logarithmic sensitivities.

*Free-drift limit* ($t_{\rm res} \ll t_c$): since $x_{\max}^{(\rm short)} \propto \Delta a_{\parallel}/(\eta^2\kappa\lambda_D)$ and $\lambda_D \propto (T_e/n_e)^{1/2}$,

$$\frac{\delta x_{\max}^{(\rm short)}}{x_{\max}^{(\rm short)}} \simeq \frac{\delta(\Delta a_{\parallel})}{\Delta a_{\parallel}} - 2\frac{\delta\eta}{\eta} - \frac{\delta\kappa}{\kappa} - \frac{1}{2}\frac{\delta T_e}{T_e} + \frac{1}{2}\frac{\delta n_e}{n_e}. \tag{110}$$

*Drag-limited accumulation limit* ($t_{\rm res} \gg t_c$): since $x_{\max}^{(\rm long)} \propto (\Delta a_{\parallel} t_c/(\eta\kappa\lambda_D))^{2/3}$,

$$\frac{\delta x_{\max}^{(\rm long)}}{x_{\max}^{(\rm long)}} \simeq \frac{2}{3}\left[\frac{\delta(\Delta a_{\parallel})}{\Delta a_{\parallel}} + \frac{\delta t_c}{t_c} - \frac{\delta\eta}{\eta} - \frac{\delta\kappa}{\kappa} - \frac{\delta\lambda_D}{\lambda_D}\right]. \tag{111}$$

If $t_c$ is taken in Spitzer form (Eq. (63)), then $t_c \propto T_e^{3/2}/(n_e Z^2 \ln\Lambda)$ while $\lambda_D \propto (T_e/n_e)^{1/2}$, and therefore $t_c/\lambda_D \propto T_e/(Z^2\ln\Lambda)$ (density cancels in this limit). Substituting yields

$$\frac{\delta x_{\max}^{(\rm long)}}{x_{\max}^{(\rm long)}} \simeq \frac{2}{3}\left[\frac{\delta(\Delta a_{\parallel})}{\Delta a_{\parallel}} - \frac{\delta\eta}{\eta} - \frac{\delta\kappa}{\kappa} + \frac{\delta T_e}{T_e} - 2\frac{\delta Z}{Z} - \frac{\delta(\ln\Lambda)}{\ln\Lambda}\right], \tag{112}$$

which makes explicit when the window endpoint is or is not sensitive to density, temperature, composition, and the Coulomb logarithm.

### 4.4.2 Computation recipe for species windows and probabilities

This section defines $r_{\min}$, $r_{\max}$, and $P_s(Z)$ in a way that is executable, but the steps are spread across definitions. For reproducibility, the following is the mechanical pipeline a reader can implement without interpretation.

**Inputs (declared, auditable).** For each evaluated object and evaluation segment, declare:

- plasma state $(T_e, n_e, \ln\Lambda)$ and the screening threshold factor $\kappa$ (Section Differential Forcing and the Onset of Coulomb Shearing),
- the forcing model used to compute $\Delta a_{\parallel}(r, \theta)$, including any electromagnetic contribution requiring $B$,
- the transport slowdown parameter $\eta$ and the residence-time geometry model defining $t_{\mathrm{res}}(r, \theta, v)$,
- the evaluated species set $\{(Z, q)\}$ and the adopted velocity distribution $f(v)$.

**Step 1: compute coupling and screening scales.** Compute $\lambda_D$ from Eq. (65) and $t_c(Z, q)$ from the declared coupling model (Eq. (63) or the explicitly stated alternative).

**Step 2: compute residence time.** For each $(r, \theta)$ on the evaluated polar segment and each $v$ sample, compute $t_{\mathrm{res}}(r, \theta, v)$ using the declared path model.

**Step 3: compute drag-limited displacement.** Compute $\Delta x(t_{\mathrm{res}})$ for each $(r, \theta, v, Z, q)$ using Eq. (68) (or Eq. (69) when the constant-$\Delta a_{\parallel}$ approximation is declared valid on that segment).

**Step 4: compute the shearing functional and admissible set.** Evaluate

$$\mathcal{S}(r; \theta, v, Z, q) \equiv \frac{\Delta x(t_{\mathrm{res}})}{\kappa\, \lambda_D},$$

and define the admissible radial set for fixed $(\theta, v, Z, q)$ as

$$\mathcal{A}_Z(\theta, v, q) \equiv \{\, r : \mathcal{S}(r; \theta, v, Z, q) \geq 1 \,\}\, .$$

If $\mathcal{A}_Z$ is empty, the window is empty for that state.

**Step 5: extract window endpoints without assuming monotonicity.** Do not assume $\mathcal{S}(r)$ is monotone. Numerically identify the connected components of $\mathcal{A}_Z$ along the evaluated radial grid. For each connected component $j$, report

$$r_{\mathrm{min},j}(Z, q, v, \theta) \equiv \inf \mathcal{A}_{Z,j}, \qquad r_{\mathrm{max},j}(Z, q, v, \theta) \equiv \sup \mathcal{A}_{Z,j}.$$

If the set is connected, this reduces to a single window and coincides with equations (87) and (88).

**Step 6: compute the probability of shearing.** Evaluate the kernel in Eq. (99) using the same $\Delta x(t_{\mathrm{res}})$ dynamics and integrate over $f(v)$ to obtain $P_s(Z)$ via Eq. (100). Where the limiting CDF forms are valid (Eq. (101)), report both the limit form and the full-kernel numerical result if they differ.

**Required reportables (Section 4 outputs).** For each species/state, report: (i) whether the admissible set is empty, (ii) the list of connected windows $\{(r_{\min,j}, r_{\max,j})\}$, and (iii) $P_s(Z)$ computed under the declared $f(v)$ and the locked uncertainty propagation rules.

**Operational sensitivity report (fixed output requirement).** For each worked object in Worked Examples and Source-Grounded Outputs, sensitivity must be reported in a mechanical way:

1. Hold all inputs at the median of their declared distributions (or midpoints of declared bounds) and compute $(r_{\min}, r_{\max}, P_s)$ for each evaluated species.
2. For each parameter $x \in \{T_e, n_e, B, \eta\}$, perturb $x$ to $x \pm \sigma_x$ when a $1\sigma$ uncertainty is provided by the source, or to the stated bound endpoints when only bounds are available, and recompute $(r_{\min}, r_{\max}, P_s)$ holding all other inputs fixed.
3. Report the resulting fractional shifts

$$\Delta_x r_{\min}/r_{\min}, \qquad \Delta_x r_{\max}/r_{\max}, \qquad \Delta_x P_s,$$

and also report the full Monte Carlo propagated median and central 90% intervals under the locked uncertainty propagation in Pre-Committed Evaluation Protocol (Anti-Post-Hoc).

This produces an explicit robustness statement: readers can see whether the shearing windows remain nonempty across $\pm 1\sigma$ variations, whether they collapse, or whether the outcome is sensitive to a single poorly constrained input.

**Evaluation requirement.** The admissibility outcome $\chi_s = 1$ requires explicit evaluation against declared physical inputs. For the systems evaluated in Worked Examples and Source-Grounded Outputs, physically admissible configurations that satisfy the criterion exist and delineate the predicted shearing regimes.

## 5 Observable Consequences: Jets and Stratification

**Conditional interpretive scope.** The observational consequences discussed here are conditional on the occurrence of enforced electron–ion co-motion failure along at least a subset of resolved trajectories, i.e. on the realization of $\chi_s = 1$ for some species and nonzero phase-space measure. If $\chi_s = 0$ everywhere, the framework predicts no deviation from standard jet composition or stratification expectations, and this discussion yields null observational consequences. No jet property is asserted here independently of the closure outcome.

**Observable primitives and proxies.** All statements below are formulated in terms of measurable proxies:

- **Relativistic-electron emissivity proxies:** synchrotron total intensity $I_\nu$, polarized intensity $I_{\rm pol}$, linear polarization fraction $p_{\rm lin}$, and polarization angle structure measured across frequency.
- **Ion-bearing screen proxies:** rotation measure RM and depolarization behavior across frequency (e.g. monotone decrease of $p_{\rm lin}$ with wavelength-squared under a Faraday-active screen).
- **Spatial ordering proxies:** axial (spine) versus off-axis (sheath) comparisons in the same beam and aligned map products; longitudinal comparisons along the jet coordinate $s$ from matched-resolution images.

These proxies are used only to test the directional statements produced by executed $\chi_s$ outcomes and computed shearing windows.

**Frequency dependence of polarization and Faraday diagnostics.** The polarization angle $\psi(\lambda)$ and rotation measure RM are linked by the defining relation

$$\psi(\lambda) = \psi_0 + \mathrm{RM}\,\lambda^2, \tag{113}$$

so RM is measured as the slope $d\psi/d\lambda^2$ from multi-frequency polarimetry. Depolarization is assessed by the measured change of $p_{\rm lin}(\lambda)$ with $\lambda^2$ under matched beam and alignment. The operational comparison used by the predictions is therefore multi-frequency: an axial channel with suppressed internal Faraday depth exhibits (i) smaller $|\mathrm{RM}|$ and (ii) weaker depolarization relative to surrounding regions when evaluated at the same angular resolution and with the same calibration protocol.

**Viewing geometry in the observational protocol.** The spine–sheath proxy ordering is evaluated only for lines of sight that intersect an ion-bearing Faraday screen exterior to the axial channel, as established by a resolved off-axis detection of nonzero RM and depolarization. Under that condition, the measurable ordering $|\mathrm{RM}|_{\rm spine} < |\mathrm{RM}|_{\rm sheath}$ and the transverse polarization–Faraday anti-correlation defined below are defined observables at fixed beam. For sightlines that do not intersect an ion-bearing screen (identified by the absence of a resolved Faraday-active component at the available resolution), the RM-based transverse tests (P1 and P3) are not defined for that system under those data and are therefore **DATA-INSUFFICIENT** under the outcome protocol. If matched-beam dual-frequency polarimetry exists and yields measurable depolarization, the depolarization-contrast test (P2) may still be evaluated via DP; otherwise the system is **DATA-INSUFFICIENT** for the primary observables.

**Transverse polarization–Faraday anti-correlation (P3, operational definition).** On an aligned transverse slice across the jet at fixed observing frequency and matched beam, define two sides relative to the fitted jet axis as "+" and "−" (any consistent convention is permitted). Using the same transverse window on both sides, define side-averaged proxies

$$\Delta I_{\rm pol} \equiv \langle I_{\rm pol}\rangle_+ - \langle I_{\rm pol}\rangle_-, \qquad \Delta|\mathrm{RM}| \equiv \langle|\mathrm{RM}|\rangle_+ - \langle|\mathrm{RM}|\rangle_-. \tag{114}$$

The Coulomb Shearing transverse signature requires

$$\Delta I_{\rm pol}\,\Delta|\mathrm{RM}| < 0, \tag{115}$$

meaning the side with larger polarized emissivity corresponds to the side with smaller Faraday depth magnitude.

### 5.1 Electron-dominated relativistic outflows

Electron-dominated polar outflow is treated as a downstream consequence of species-resolved forcing under closure failure, not as an imposed jet composition assumption.

**Not a jet launching mechanism.** Coulomb Shearing is not a jet launching mechanism. It does not launch, collimate, or power relativistic jets. Its role is downstream and diagnostic: given an already-existing polar outflow or magnetized funnel established by other physics, the mechanism predicts when electron–ion co-motion fails and therefore how the flow becomes compositionally stratified into an electron-loaded channel and an ion-dominated inflow branch.

**Species-resolved forcing.** Electrons receive direct radiative momentum transfer through scattering and absorption channels that couple efficiently to leptons. Electrons also experience strong electromagnetic forcing through the Lorentz term because their charge-to-mass ratio is large. Ions do not acquire comparable outward acceleration through these same channels on the same timescales because their inertia suppresses acceleration for equal and opposite forces.

**Role of shocks (secondary modulation, not primary forcing).** Shocks, reconnection fronts, and other dissipative structures can modulate separation efficiency by heating and compressing the flow, thereby affecting ionization state, collision rates, and the residence-time distribution through the shearing zone. In this framework, they are treated as secondary contributors and downstream dissipation, not as primary, charge-selective forcing terms that define the onset of electron–ion separation.

**Closure failure and branch separation.** When the closure criterion is satisfied along a resolved trajectory segment, the electrostatic restoring response and collisional locking are insufficient to enforce electron–ion co-motion over the residence time. Electron motion becomes dynamically separable from ion motion along that segment. The resulting causal chain is:

species-resolved forcing ⟶ closure criterion satisfied ⟶ electron escape into an outward polar outflow
⟶ ion retention and continued inflow

The final link is operational. Once the electron component ceases to provide enforced co-motion, the ion component follows the inward, gravitationally dominated branch within the same trajectory family used to define the residence time. The outcome is conditional. If the computed closure outcome is $\chi_s = 0$ everywhere, electron-dominant polar outflow is not predicted as a consequence of this framework.

**Post-shearing dynamics and field response (qualitative).** The closure criterion $\chi_s = 1$ states that the enforced electron–ion co-motion closure fails under sustained differential forcing over the residence time $t_{\rm res}$; it does not assert macroscopic charge separation. Quasi-neutrality is enforced separately by rapid ambipolar field adjustment on $t_E \sim \omega_{pe}^{-1}$ (Eq. (117)), which prevents secular charge buildup ($\rho_q \simeq 0$) but does not require $v_{e,\parallel} = v_{i,\parallel}$: a quasi-neutral plasma can carry $j_\parallel \neq 0$ only by supporting a finite relative drift.

To make the distinction explicit, define the charge density and field-aligned current as

$$\rho_q \equiv e\,(Zn_i - n_e) \simeq 0, \qquad j_\parallel \equiv e\,(Zn_i v_{i,\parallel} - n_e v_{e,\parallel}), \tag{116}$$

where quasi-neutrality constrains $\rho_q \simeq 0$ while allowing $j_\parallel \neq 0$. In particular, if $n_e \simeq Zn_i$ then $j_\parallel \simeq e\,n_e\,(v_{i,\parallel} - v_{e,\parallel})$, so any sustained field-aligned current implies $v_{i,\parallel} \neq v_{e,\parallel}$ even in a quasi-neutral channel. The ambipolar field adjusts rapidly to keep $\rho_q$ small and to satisfy charge continuity (Eq. (120)), but it does not force the drift to vanish when the system is continuously driven; it regulates the drift through the coupling physics encoded later by $t_c$ and evaluated by $\chi_s$. The ambipolar response regulates the local electric field so that $\rho_q$ remains small, but it does not require that the electron population remain paired with the same ions from the same material element as the flow evolves.

**Ambipolar field establishment timescale versus residence time (order-of-magnitude).** The enforcement of quasi-neutrality occurs on the electron plasma timescale, not on the macroscopic transport time. Define the electron plasma frequency

$$\omega_{pe} \;\equiv\; \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}, \qquad t_E \;\sim\; \omega_{pe}^{-1}, \tag{117}$$

so that

$$t_E \;\simeq\; 1.8\times10^{-5}\,\mathrm{s}\left(\frac{n_e}{1\,\mathrm{cm}^{-3}}\right)^{-1/2}. \tag{118}$$

Even for extremely dilute funnel plasma ($n_e \ll 1\,\mathrm{cm}^{-3}$), $t_E$ remains microphysical, while the trajectory residence times derived in Section 2 are at least dynamical ($t_{\rm res} \gtrsim t_g$) and typically much longer. Therefore $t_E \ll t_{\rm res}$ holds by many orders of magnitude under any physically admissible funnel density, meaning the model does not rely on macroscopic charge separation: $\rho_q \simeq 0$ can be maintained essentially instantaneously compared to $t_{\rm res}$.

**Electrostatic field structure implied by quasi-neutrality (order-of-magnitude bound).** Let $L_\parallel$ denote the characteristic field-aligned scale over which the bulk funnel current system and its macroscopic quantities vary (e.g. a resolved streamline or channel scale), with $L_\parallel \gg \lambda_D$ where $\lambda_D$

is the Debye length defined earlier (Eq. (65)). In that bulk regime, Debye shielding implies that any space-charge required to source the ambipolar response is parametrically small, so the current system can remain quasi-neutral while still supporting a finite drift:

$$\frac{|\rho_q|}{e\,n_e} \lesssim \left(\frac{\lambda_D}{L_\parallel}\right)^2 \ll 1, \qquad |E_\parallel| \sim \frac{|\rho_q|\,L_\parallel}{\varepsilon_0} \lesssim \frac{k_B T_e}{e\,L_\parallel}. \tag{119}$$

The first bound states explicitly that, when macroscopic variations occur on scales large compared to $\lambda_D$, the fractional charge imbalance is suppressed by $(\lambda_D/L_\parallel)^2$ and is therefore negligible even while $j_\parallel \neq 0$ (Eq. (116)). The second bound gives the associated bulk field scale: the ambipolar field required to regulate quasi-neutrality in the bulk is at most of order a thermal voltage drop distributed over $L_\parallel$. Any region exhibiting significantly larger unscreened $E_\parallel$ necessarily corresponds to a boundary layer or gap-like structure with $L_\parallel \sim \lambda_D$ (or a charge-starved zone), which is outside the enforced co-motion closure and is treated as a separate kinetic regime rather than a requirement of the mechanism.

**Connection to kinetic simulations and a direct falsification route.** The claim in this section is not that $\chi_s = 1$ enforces permanent, volume-filling charge separation, but that it permits a *quasi-steady, open-field current system* in which (i) field-aligned electric fields, return-current layers, and localized pair or electron supply regulate $\mathbf{J}\cdot\mathbf{E}$ and maintain current closure, while (ii) the *net* electron-rich outflow and ion-dominated inflow can remain sustained in adjacent channels without requiring perfect electron-ion co-motion everywhere. General relativistic PIC studies of black hole magnetospheres explicitly exhibit regimes where charge starvation produces unscreened field-aligned electric fields and time-dependent plasma supply that regulates current flow in the funnel, rather than enforcing instantaneous, single-fluid locking on all scales [20]. However, most published GRPIC funnel and gap simulations are conducted in pair plasma (or employ prescribed plasma injection), so they do not yet directly test electron-ion partition in an ion-bearing polar funnel.

**Decisive simulation requirement (electron–ion GRPIC).** Because pair-plasma GRPIC does not include an ion mass hierarchy, it cannot directly test the electron–ion drift and ion-dominated inflow channel that Coulomb Shearing requires. The decisive validation step is therefore an electron–ion GRPIC run (or hybrid-PIC with full electron kinetics) that resolves: (i) the development and saturation of field-aligned ambipolar regulation while maintaining $\rho_q \simeq 0$; (ii) the sustained driven relative drift $v_{i,\parallel} - v_{e,\parallel}$ under the declared anisotropic forcing; and (iii) the persistence of an electron-enriched spine with a complementary ion-dominated inflow channel over timescales comparable to the local $t_{\mathrm{res}}$. This is a gap in the current simulation literature, not a post-hoc loophole in the model: it specifies the single most direct numerical falsifier for the mechanism.

Separately, global (axisymmetric) GRPIC disk-corona models demonstrate that horizon-scale jet launching and coronal dissipation can be treated in a kinetic framework, providing an existence proof that collisionless reconnection and magnetically dominated funnel dynamics can be evolved self-consistently (within the composition and injection prescriptions of the simulation) [21]. On the fluid side, hybrid GRMHD+GRFFE schemes are specifically designed to access the extreme-$\sigma$ evacuated funnel where force-free electrodynamics is the appropriate macroscopic limit, clarifying where kinetic closure and plasma supply must be supplied by microphysics rather than numerical density floors [22]

*Pre-aware test.* A direct, non-post-hoc test of the present mechanism is therefore: run (or use published) GRPIC / kinetic-MHD funnel-wall setups with explicit electron-ion composition (or controlled pair-loading) and measure whether the time-averaged, field-aligned current closure admits a persistent sign-asymmetric transport of electrons and ions across the funnel-wall interface under the local $\Delta a$ forcing inferred from the macrostate. Agreement or failure here is decisive: if kinetic funnels generically restore tight electron-ion co-motion on timescales short compared to $t_{\mathrm{res}}$ once ambipolar fields form, then the quasi-steady separation channel assumed above is not physically available.

**Why ambipolar regulation does not imply rapid re-locking.** In a polar funnel with open magnetic geometry, the relevant steady state is a *current-carrying, quasi-neutral* configuration, not a perfectly locked single-fluid configuration. Electrons experience strong species-selective forcing (radiative momentum transfer and Lorentz response) while ions remain gravity-dominated along the same streamline family used to define $t_{\mathrm{res}}$. The ambipolar field adjusts to prevent large $\rho_q$ by opposing further charge buildup, which effectively *rate-limits* electron transfer into the outward channel. But rate-limiting is not the same as restoring enforced co-motion: the system can remain quasi-neutral while supporting persistent relative drift and therefore sustained branch separation.

This point connects directly to the closure test logic. If the ambipolar response (together with collisional drag and electrostatic restoration) were strong enough to enforce $v_{e,\parallel} \simeq v_{i,\parallel}$ over $t_{\mathrm{res}}$, then the effective coupling would be fast on the residence time and the evaluation would return $\chi_s = 0$ for that state under the declared inputs. The predicted tracer partition is therefore conditional on the opposite ordering: the combined response is sufficient to maintain quasi-neutrality on microscopic scales but insufficient to enforce co-motion on the macroscopic residence time.

**Current closure in open-field polar geometry (qualitative).** Global charge conservation requires current closure, not permanent local charge separation. The relevant constraint is the charge-continuity equation,

$$\frac{\partial \rho_q}{\partial t} + \nabla \cdot \mathbf{j} = 0, \tag{120}$$

so any quasi-steady state with $\partial \rho_q/\partial t \simeq 0$ and $\rho_q \simeq 0$ requires $\nabla \cdot \mathbf{j} \simeq 0$ over the control volume. In practice this is satisfied by return-current layers at the funnel wall / sheath and by circuit closure through the surrounding corona or disk surface, so the mechanism does not rely on macroscopic charge buildup to maintain separation. In an open polar channel, outward electron current in the axial region can be closed by return currents in surrounding sheath or funnel-wall plasma, and by replenishment of electrons along the channel. Under those conditions, the electron-rich outward channel can remain quasi-neutral overall even though the radiating electrons in that channel are not required to remain co-located with the specific ions that continue inward from the same initial material element. This is the precise sense in which $\chi_s = 1$ can support quasi-steady separation without requiring a macroscopic buildup of net charge.

**Return-current magnitude and spatial structure (minimal quantitative closure).** A quasi-steady, open-field polar channel can be quasi-neutral while carrying a net outward electron current, provided an equal-magnitude return current exists in the surrounding sheath (or funnel-wall layer) and closure occurs through the corona or disk surface. A simple, auditable estimate follows from Ampère's law applied to the magnetically dominated funnel.

Assume the inner jet carries a toroidal field component $B_\phi(r)$ at cylindrical radius $r$ (measured about the jet axis). The enclosed axial current is

$$I(r) = \frac{2\pi r}{\mu_0} B_\phi(r), \tag{121}$$

so the net current required by the declared funnel-field state is fixed by the measurable (or simulation-anchored) $B_\phi$ profile.

Let the electron-loaded axial channel have characteristic radius $r_j$ and carry outward current density approximately uniform in cross-section. Then the axial channel current density scale is

$$j_{\rm out} \simeq \frac{I(r_j)}{\pi r_j^2}. \tag{122}$$

If closure occurs in a sheath return layer of thickness $\delta_{\rm rc} \ll r_j$ located near the funnel wall, the same magnitude of current must flow back through that layer. Approximating the return path as an annulus of area $A_{\rm rc} \simeq 2\pi r_j \delta_{\rm rc}$, the return-current density scale is

$$j_{\rm rc} \simeq \frac{I(r_j)}{2\pi r_j \delta_{\rm rc}} = \frac{r_j}{2\,\delta_{\rm rc}}\, j_{\rm out}. \tag{123}$$

This makes the qualitative statement above quantitative: return-current layers are naturally high-$j$ structures if the closure layer is thin.

Two basic admissibility constraints follow immediately. First, the available charge carriers must be able to supply the required current without superluminal drift. For a carrier number density $n_e$ and characteristic drift speed $v_d$, $j \simeq e n_e v_d$ gives

$$v_{d,\rm rc} \simeq \frac{j_{\rm rc}}{e n_e} \le c \quad \Rightarrow \quad n_e \gtrsim \frac{j_{\rm rc}}{ec}. \tag{124}$$

Second, because $j_{\rm rc}$ scales as $\delta_{\rm rc}^{-1}$, the closure layer thickness is a stability and dissipation control parameter: thinner layers raise drift speeds and increase susceptibility to kinetic current-sheet instabilities (tearing, drift-kink) and to enhanced dissipation, while thicker closure layers reduce $j_{\rm rc}$ at fixed $I(r_j)$.

These relations do not introduce new tuning parameters into the closure test. They instead translate the already-declared macroscopic funnel state (through $B_\phi$ and $r_j$) into a required return-current scale and its minimal density and thickness conditions. Under the Coulomb Shearing hypothesis, the electron-rich outward channel and ion-dominated inflow channel remain compatible with quasi-neutrality because the current system closes primarily through spatially distinct return layers, not through macroscopic charge accumulation.

**Direct observational consequence of the branch separation.** The branch separation fixes the partition of tracers within the resolved inner jet. Emissivity proxies that depend directly on relativistic electrons ($I_\nu$, $I_{\rm pol}$, $p_{\rm lin}$) trace the electron-loaded channel. Ion-bearing proxies (RM and depolarization) trace plasma that contains ions along the line of sight. Under computed dominance, the electron-loaded channel exhibits reduced internal Faraday depth relative to its surroundings, and therefore exhibits reduced depolarization at fixed observing frequency. The surrounding ion-bearing region exhibits larger $|\mathrm{RM}|$ and stronger depolarization.

### 5.2 Connection between computed shearing windows and observed jet structure

**Window definition and stratification mechanism.** The executed species-dependent shearing bounds define where along polar trajectories electron–ion separation is dynamically permitted for each evaluated species state. Denote the computed shearing window for a state labeled by $Z$ as the interval

$$[r_{\min}(Z),\, r_{\max}(Z)]$$

along the same polar streamline parameterization used in the closure evaluation. Within this interval the evaluation returns $\chi_s = 1$ under the declared physical inputs; outside this interval enforced co-motion holds and the framework predicts null deviations from coupling-enforced expectations for that state.

Stratification follows from three steps that are checkable against the executed evaluation:

1. **Onset ordering:** different evaluated states enter closure failure at different radii because $r_{\min}(Z)$ differs across states under the same declared inputs.
2. **Termination ordering:** different evaluated states exit closure failure at different radii because $r_{\max}(Z)$ differs across states under the same declared inputs.
3. **Tracer partition:** within each window where $\chi_s = 1$, electron emissivity proxies remain co-located with the outward channel while ion-bearing proxies do not remain co-located with that channel, because ions are retained on the inflow branch defined by the same trajectory family.

**Operational zoning along the polar streamline.** The computed windows induce a measurable partition of the inner jet into three longitudinal zones. Define $Z_\star$ as the evaluated state that first enters closure failure over the resolved measure, i.e. the state that minimizes $r_{\min}(Z)$ among those with nonzero evaluated measure. The longitudinal zoning is:

- **Coupled zone:** $r < r_{\min}(Z_\star)$. Enforced co-motion holds for all evaluated states. Electron emissivity proxies and ion-bearing proxies can be co-located within the same resolved element.
- **Electron-loading zone:** $r_{\min}(Z_\star) \le r \le \max_Z r_{\max}(Z)$ over the evaluated states with nonzero measure. At least one state satisfies $\chi_s = 1$ within this span. Electron emissivity proxies trace an outward channel that is not co-loaded with ions from the same material element.
- **Downstream transport zone:** $r > \max_Z r_{\max}(Z)$ over the evaluated states with nonzero measure. Closure failure is no longer active for the evaluated states along the streamline. The electron-loaded channel propagates and radiates while the ion component associated with the separated material element is absent from that channel, having remained on the inflow branch.

**Transverse (spine-sheath) mapping without geometric tuning.** Spine–sheath morphology is treated as an observational descriptor rather than a primitive. The mapping to computed regimes is executed using only the tracer partition imposed by the closure outcome: the axial channel identified by elevated $p_{\text{lin}}$ and reduced depolarization at fixed frequency is assigned to the electron-loaded channel, while the surrounding region identified by larger $|\text{RM}|$ and stronger depolarization is assigned to ion-bearing plasma along the line of sight. No additional jet stratification parameters are introduced.

**Distinguishing signatures relative to standard MHD spine–sheath models.** A spine–sheath morphology can arise in standard MHD jet models through magnetic acceleration, shear, and entrainment, so morphology alone is not a discriminant. Coulomb Shearing instead predicts a tracer-partition signature tied to the computed, species-resolved windows:

- **Electron-loaded channel with reduced internal Faraday depth:** the region of strongest polarized emissivity ($I_{\text{pol}}$) is predicted to exhibit systematically smaller internal Faraday rotation and weaker depolarization at fixed observing frequency, because the channel is electron-rich and ion-poor relative to its surroundings.
- **Ion-bearing sheath and composition ordering:** the sheath (or funnel-wall) is predicted to carry the dominant ion column and therefore dominate RM and depolarization. If multiple ionic species contribute, their relative presence across radius and along $z$ should follow the ordering implied by the computed windows $[r_{\min}(Z), r_{\max}(Z)]$ (Species-Dependent Shearing Ranges and Probabilities), rather than being set purely by hydrodynamic mixing.
- **Luminosity-linked switching:** because $\Delta a$ is driven directly by radiative and electromagnetic forcing (Differential Forcing and the Onset of Coulomb Shearing), the strength of the inferred electron-loading signature should track changes in near-axis forcing proxies (e.g. Eddington ratio and inner-jet magnetization) in a way that is not required in purely kinematic spine–sheath models.

These discriminants are not claimed to be unique in isolation, but taken together they define a falsifiable package: a system that shows a robust spine–sheath morphology but lacks the predicted electron-loaded low-$|\mathrm{RM}|$ spine, or shows no $Z$-ordered stratification despite being in a parameter regime where the computed windows are open, is inconsistent with the mechanism as posed here.

**Monotonic ordering rule from computed windows.** A monotonic ordering rule follows from executed windows and introduces no tunable jet parameters. For two evaluated states $Z_1$ and $Z_2$:

- If $r_{\min}(Z_1) < r_{\min}(Z_2)$ then the onset of the electron-loading signatures occurs at smaller radius for $Z_1$ than for $Z_2$ along the same streamline family.
- If $r_{\max}(Z_1) > r_{\max}(Z_2)$ then the electron-loading signatures extend to larger radius for $Z_1$ than for $Z_2$ along the same streamline family.

The ordering is generated by the executed evaluation and then tested against measured ordering of polarization and Faraday proxies along the jet coordinate $s$.

**Probabilistic shearing and measurable intermittency.** If probabilistic shearing is used and yields $P_s(Z) < 1$ for a state contributing to the electron-loading channel, the framework predicts intermittency in ion-screen and mixing signatures rather than reinterpretation. Intermittency is defined operationally as measurable epoch-to-epoch or time-resolved variance in inner-jet RM and depolarization proxies at fixed angular resolution and matched frequency coverage. Under fixed observing setup, lower $P_s(Z)$ implies larger variance in these proxies because the presence of an ion-bearing screen along the line of sight switches across realizations within the evaluated phase-space measure.

### 5.3 Distinguishing signatures and non-degeneracy requirements

Spine–sheath morphology and axial stratification can arise in standard relativistic MHD jet launching, collimation, and propagation. Accordingly, the diagnostic content of Coulomb Shearing is not the existence of stratification in isolation, but a set of dependencies that tie *where* and *when* stratification appears to the evaluated electron–ion decoupling admissibility conditions developed in Sections 2–3.

The mechanism makes the following non-degeneracy requirements explicit. These are phrased as falsifiable conditions rather than as claims of exclusivity against all MHD configurations:

- **Window-locked localization.** Observable transitions attributed to electron loading should co-locate with the computed admissibility window, i.e. the region where the evaluated ratio $t_{\mathrm{res}}/t_c$ and the forcing history yield $\Delta x(t_{\mathrm{res}}) \gtrsim \kappa\lambda_D$. A generic spine–sheath geometry does not by itself predict this window-locked onset and termination.
- **State-dependent motion of the window at fixed $M$.** Because $t_c$ depends explicitly on $(n_e, T_e, Z, \ln\Lambda)$ and $t_{\mathrm{res}}$ depends on the transport factor $\eta$ (Transport of Matter to Polar Trajectories), changes in inner-flow state that alter $(n_e, T_e)$ or the residence time predict a measurable shift of the admissible region even when $M$ is fixed. This ties polarization and RM morphology to accretion-state changes through the evaluated ratio $t_{\mathrm{res}}/t_c$, rather than treating the stratification radius as a purely geometric fit parameter.

Table 2: Expected outward-channel composition and measurable proxy behavior across the longitudinal ranges induced by executed shearing windows. The ranges are defined only by computed bounds and the closure outcome. If $\chi_s = 0$ everywhere, only the first row applies and all other rows are null.

| Range along streamline | Closure outcome | Outward-channel composition | Required proxy behavior (directional) |
|---|---|---|---|
| $r < r_{\min}(Z_\star)$ | $\chi_s = 0$ for evaluated states | Electron and ion components remain co-moving within evaluated elements. | $\lvert\mathrm{RM}\rvert$ and depolarization are co-located with synchrotron emission in the same resolved region; $p_{\mathrm{lin}}$ is reduced relative to a Faraday-thin axial channel under matched frequency and beam. |
| $r_{\min}(Z_\star) \le r \le r_{\max,\mathrm{eff}}$ with $r_{\max,\mathrm{eff}} \equiv \max_Z r_{\max}(Z)$ over evaluated states of nonzero measure | $\chi_s = 1$ for at least one evaluated state (nonzero measure) | Outward channel becomes electron-loaded; associated ions from the same evaluated element are not co-loaded into the outward channel and remain on the inflow branch. | Axial channel satisfies $\lvert\mathrm{RM}\rvert_{\mathrm{spine}} < \lvert\mathrm{RM}\rvert_{\mathrm{sheath}}$. Using matched-beam dual-frequency polarimetry, define $\mathrm{DP} \equiv p_{\mathrm{lin}}(\nu_{\mathrm{low}})/p_{\mathrm{lin}}(\nu_{\mathrm{high}})$ with $\nu_{\mathrm{low}} < \nu_{\mathrm{high}}$; the electron-loaded channel satisfies $\mathrm{DP}_{\mathrm{spine}} > \mathrm{DP}_{\mathrm{sheath}}$. |
| $r > r_{\max,\mathrm{eff}}$ | $\chi_s = 0$ along the evaluated streamline outside the window | Electron-loaded outward channel persists; ions associated with separated elements remain absent from that channel. | Transverse anti-correlation persists: $\Delta I_{\mathrm{pol}}\,\Delta\lvert\mathrm{RM}\rvert < 0$ on aligned slices (Eq. (115)); axial $\lvert\mathrm{RM}\rvert$ remains suppressed relative to off-axis at matched resolution. |

Table 3: Expected transverse structure within the electron-loading zone under computed dominance. "Spine" and "sheath" refer to regions resolved across the jet width at fixed angular resolution.

| Resolved region | Dominant content (conditional on computed dominance) | Measurable proxy ordering (same beam, aligned maps) |
|---|---|---|
| Spine (axial channel) | Relativistic electrons in the outward channel; ion co-loading from the evaluated element is absent. | $\lvert\mathrm{RM}\rvert_{\mathrm{spine}} < \lvert\mathrm{RM}\rvert_{\mathrm{sheath}}$. Using matched-beam dual-frequency polarimetry, define $\mathrm{DP} \equiv p_{\mathrm{lin}}(\nu_{\mathrm{low}})/p_{\mathrm{lin}}(\nu_{\mathrm{high}})$ with $\nu_{\mathrm{low}} < \nu_{\mathrm{high}}$; the predicted ordering is $\mathrm{DP}_{\mathrm{spine}} > \mathrm{DP}_{\mathrm{sheath}}$. On aligned transverse slices, the side with larger $I_{\mathrm{pol}}$ has smaller $\lvert\mathrm{RM}\rvert$ (Eq. (115)). |
| Sheath (off-axis region) | Ion-bearing plasma along the line of sight relative to the axial channel. | $\lvert\mathrm{RM}\rvert_{\mathrm{sheath}} > \lvert\mathrm{RM}\rvert_{\mathrm{spine}}$ under matched beam. With $\mathrm{DP} \equiv p_{\mathrm{lin}}(\nu_{\mathrm{low}})/p_{\mathrm{lin}}(\nu_{\mathrm{high}})$ and $\nu_{\mathrm{low}} < \nu_{\mathrm{high}}$, the predicted ordering is $\mathrm{DP}_{\mathrm{sheath}} < \mathrm{DP}_{\mathrm{spine}}$ (stronger depolarization than the electron-loaded channel). |

- **Charge and species dependence through coupling, not morphology.** In Coulomb Shearing, the role of $Z$ enters through the electron–ion coupling physics (via $t_c$ and the drag closure), not merely through an assumed multi-fluid launch prescription. Accordingly, any species-indexed surrogate (for example, proxies that distinguish lepton-dominated versus baryon-dominated segments through their Faraday behavior and depolarization trends) should correlate with the evaluated admissibility, rather than being freely adjustable by changing magnetization or viewing angle alone.

- **Proxy separability: lepton loading versus field geometry.** Standard MHD models can reproduce transverse polarization structure by field ordering and shear. Coulomb Shearing adds a separable lever: it predicts that electron loading is controlled by the admissibility criterion and can vary substantially while the large-scale field geometry remains similar. Therefore, the model expects cases where polarization morphology remains broadly stable while RM and depolarization proxies shift in a manner consistent with changes in the lepton fraction set by the admissibility window.

- **Internal consistency with the growth channel.** The same conditions that permit electron–ion separation also imply an ion-dominated inward channel (Ion-Dominated Inflow and Black Hole Growth). Thus, parameter choices that explain a jet-stratification signature but imply $t_c \ll t_{\mathrm{res}}$ everywhere (no admissible window) are internally inconsistent within this framework.

The explicit predictions below are intended to be assessed under these non-degeneracy requirements, so that agreement is not merely morphological but parameter-linked to the evaluated shearing conditions.

### 5.3.1 Unique spectroscopic signatures

The signatures listed in Section 5.3 are necessary but not sufficient conditions for Coulomb Shearing, since similar phenomenology can arise in pair-cascade scenarios, MHD wind stratification, and slim-disk accretion. Coulomb Shearing instead predicts a set of species-resolved observables that follow directly from differential field-aligned acceleration and finite residence time, and which are not generically produced by those alternatives.

**Species-dependent velocity structure.** If Coulomb Shearing occurs along a resolved trajectory segment, each species acquires a characteristic velocity scale set by its species-specific parallel acceleration and the shared residence time:

$$v_s \simeq \left(2\,\Delta a_{\parallel,s}\, t_{\mathrm{res}}\right)^{1/2}, \tag{125}$$

where $s \in \{e, p, i\}$ labels electrons, protons, and heavier ions. For fixed forcing history, Coulomb Shearing predicts an ordering

$$v_e > v_p > v_i.$$

For hydrogen and helium ions, this implies a fractional separation

$$\frac{\Delta v}{v} \sim \left(\frac{m_{\mathrm{H}}}{m_{\mathrm{He}}}\right)^{1/2} - 1 \approx 0.4,$$

potentially measurable via Doppler shifts in ion recombination or forbidden lines, while electron velocity constraints are obtained independently from the inverse-Compton and synchrotron spectra. This mass-rooted hierarchy is not a generic prediction of pair cascades or MHD wind stratification.

**Radial stratification profile.** Coulomb Shearing predicts a calculable radial evolution of the electron-to-ion density ratio,

$$\frac{n_e(r)}{n_i(r)} = f\left[\Delta a_{\parallel}(r),\ t_{\mathrm{res}}(r),\ \nu_c(r)\right], \tag{126}$$

where $\nu_c$ is the Coulomb coupling rate. Two regimes follow:

$$r < r_{\mathrm{crit}}: \quad n_e/n_i \approx Z \quad (\text{coupled plasma}),$$

$$r > r_{\mathrm{crit}}: \quad n_e/n_i < Z \quad (\text{electron depletion}).$$

This trend contrasts with pair-cascade loading, which increases $n_e/n_i$ with radius, and with MHD wind stratification, which tends to preserve an approximately constant charge ratio with radius.

**Time-variability pattern.** The characteristic timescale for Coulomb Shearing is

$$t_{\mathrm{shear}} \sim \max(t_c,\ t_{\mathrm{res}}), \tag{127}$$

where $t_c$ is the Coulomb coupling time. Electron-dominated emission channels are therefore predicted to exhibit a power-spectral break at $\nu \sim t_{\mathrm{shear}}^{-1}$, while ion-dominated emission, if present, is not expected to show the same feature. Cross-correlation between electron-dominated bands and lower-energy bands further predicts a characteristic lag of order $t_{\mathrm{shear}}$ when both components are detectable.

Together, these spectroscopic, radial, and temporal signatures form a jointly testable discriminator that is not equivalent to the broad phenomenology produced by pair cascades, generic MHD stratification, or slim-disk accretion.

## 5.4 Explicit predictions (quantitative)

This subsection states quantitative, testable predictions with explicit PASS/FAIL extraction rules. The predictions are conditioned on the computed electron-loading zone and admissibility window produced by the executed evaluation; the model is not credited for morphology outside the mapped zone.

### 5.4.1 Quantitative observables and pass/fail extraction rules

Define the following measurable quantities from matched-beam, aligned multi-frequency polarimetry.

**Why the primary predictions are RM and polarization, not spectral-line or intensity ratios.** Predictions P1–P3 are intentionally defined in terms of rotation-measure structure, depolarization structure, and a transverse polarization–Faraday statistic because these can be extracted with explicit uncertainty budgets and do not require a detailed emissivity or particle-acceleration model. In contrast, spectral line ratios are generally not available for horizon-scale jet bases, and broad-band intensity ratios depend strongly on the assumed emission mechanism, electron energy distribution, and radiative transfer through the same magnetized screen that sets RM. For that reason, this paper does not claim that Coulomb Shearing uniquely predicts specific spectral-line ratios or absolute intensity ratios at the jet base.

Accordingly, the quantitative content of P1–P3 is the set of inequalities and significance requirements below, evaluated *within the computed electron-loading zone*. This makes the predictions falsifiable without importing additional, model-dependent assumptions about emission physics.

- **RM contrast ratio**:
$$\mathcal{R}_{\rm RM} \equiv \frac{|\mathrm{RM}|_{\rm spine}}{|\mathrm{RM}|_{\rm sheath}}. \tag{128}$$
- **Depolarization ratio** using a fixed two-frequency pair $\nu_{\rm low} < \nu_{\rm high}$:
$$\mathrm{DP}(R) \equiv \frac{p_{\rm lin}(R, \nu_{\rm low})}{p_{\rm lin}(R, \nu_{\rm high})}, \qquad \mathcal{R}_{\rm DP} \equiv \frac{\mathrm{DP}_{\rm spine}}{\mathrm{DP}_{\rm sheath}}. \tag{129}$$
- **Transverse anti-correlation statistic** evaluated on aligned transverse slices within the mapped zone:
$$\rho_{\rm P3} \equiv \rho\left(I_{\rm pol},\ |\mathrm{RM}|\right), \tag{130}$$
where $\rho$ is the Spearman rank correlation coefficient computed on pixels in the transverse slice after masking low-SNR regions.

The pass/fail criteria are:

P1: **RM ordering (quantitative).** Within the computed electron-loading zone, $\mathcal{R}_{\rm RM} < 1$ with a $3\sigma$ separation:
$$|\mathrm{RM}|_{\rm sheath} - |\mathrm{RM}|_{\rm spine} > 3\sqrt{\sigma^2_{\rm RM,spine} + \sigma^2_{\rm RM,sheath}}. \tag{131}$$

P2: **Depolarization contrast (quantitative).** Within the mapped zone, $\mathcal{R}_{\rm DP} > 1$ with a $3\sigma$ separation:
$$\mathcal{R}_{\rm DP} - 1 > 3\,\sigma_{\mathcal{R}_{\rm DP}}. \tag{132}$$

P3: **Polarization–Faraday anti-correlation (quantitative).** Within the mapped zone, the transverse slices satisfy both
$$\rho_{\rm P3} < -0.5, \tag{133}$$

and a two-sided significance $p < 0.01$ under the null hypothesis of no correlation (using the slice pixel-count as the effective sample size after masking).

P4: **RM variance separation (quantitative, conditional).** This prediction is evaluated only when the executed evaluation returns $P_s(Z) < 1$ for at least one evaluated state contributing to the electron-loading channel. Under matched cadence, matched beam, and matched frequency coverage, require:

$$\frac{\mathrm{Var}[\mathrm{RM}]_{P_s<1}}{\mathrm{Var}[\mathrm{RM}]_{P_s\simeq 1}} > 1 \quad \text{with} \quad 3\sigma \text{ separation under bootstrap resampling.} \tag{134}$$

These definitions make each prediction auditable and falsifiable with explicit measurement-quality requirements.

### 5.4.2 M87* quantitative prediction packet (current comparators and required precision)

For M87*, the executed evaluation in Worked Example 1: M87* (EHT-resolved horizon-scale source) establishes a mapped electron-loading zone anchored to the near-horizon window, while the observational comparators for P1–P3 are evaluated on the declared downstream band in the worked example (matched-beam 8–15 GHz products cited there). The table below binds the M87* predictions to measurable quantities and states the instrumental path to tighten each test.

This prediction packet provides explicit measurement requirements (sigma thresholds, correlation thresholds, and dataset structure) and ties each test to instruments capable of meeting those requirements without introducing new jet-power claims.

This subsection states explicit falsification criteria. Criteria are divided into (i) *system-level* binary tests applicable to any object with adequate measurements, and (ii) *program-level* thresholds across a declared test set of $N$ objects. These criteria are evaluated only under the pre-committed outcome protocol and observable definitions in Explicit predictions (quantitative).

### 5.4.3 System-level falsification tests (binary)

A system is eligible for falsification only if the declared physical inputs and the executed evaluation produce a shearing-admissible regime for a nonzero phase-space measure over the resolved trajectory family (i.e. $\chi_s = 1$ occurs for at least one evaluated state within the declared window). For any such system, falsification is triggered by any contradiction of the quantitative inequalities in Explicit predictions (quantitative) under data of sufficient quality:

- **P1 contradiction (RM ordering).** The mapped-zone RM contrast fails the quantitative inequality (including the stated sigma threshold).
- **P2 contradiction (depolarization contrast).** The mapped-zone depolarization contrast fails the quantitative inequality (including the stated sigma threshold).
- **P3 contradiction (anti-correlation).** The mapped-zone transverse anti-correlation fails the threshold $\rho_{\mathrm{P3}}$ and significance requirement.

Table 4: Quantitative predictions for M87* under the executed Coulomb Shearing evaluation and the observable definitions in Quantitative observables and pass/fail extraction rules. "Status" refers only to whether the comparator datasets cited in the M87* worked example are sufficient to evaluate the inequality as written.

| Observable | Coulomb Shearing prediction (quantitative) | Comparator dataset path | Status |
|---|---|---|---|
| RM contrast (P1) | $\mathcal{R}_{\mathrm{RM}} < 1$ with $3\sigma$ separation within the mapped zone | Matched-beam multi-frequency RM map (e.g. VLBA 8–15 GHz class products; ngVLA for tighter errors) | Evaluable; treated as PASS in Worked Example 1: M87* (EHT-resolved horizon-scale source) |
| Depolarization contrast (P2) | $\mathcal{R}_{\mathrm{DP}} > 1$ with $3\sigma$ separation using a fixed $\nu_{\mathrm{low}}, \nu_{\mathrm{high}}$ pair | Dual-frequency matched-beam polarization fractions; VLBA class now, ngVLA for improved cadence and SNR | Evaluable; treated as PASS in Worked Example 1: M87* (EHT-resolved horizon-scale source) |
| Transverse anti-correlation (P3) | $\rho_{\mathrm{P3}} < -0.5$ with $p < 0.01$ on transverse slices in the mapped zone | Aligned transverse slices from polarization intensity and RM maps; higher-fidelity slices from ngVLA | Evaluable; treated as PASS in Worked Example 1: M87* (EHT-resolved horizon-scale source) |
| RM variance separation (P4) | Only if $P_s(Z) < 1$: variance ratio $> 1$ with $3\sigma$ bootstrap separation | Requires multi-epoch RM time series at matched beam/cadence; ngVLA or long-baseline monitoring programs | DATA-INSUFFICIENT (by protocol) |

- **P4 contradiction (variance separation, conditional).** When $P_s(Z) < 1$ is returned for at least one evaluated state contributing to the electron-loading channel, the RM-variance separation inequality fails under the stated bootstrap criterion.

**Shearing admissibility falsification.** The mechanism requires that the admissibility criterion be satisfiable for at least some evaluated states in at least some objects. Therefore:

> If $\Delta x/(\kappa\lambda_D) < 1$ holds for *all* evaluated states in *all* declared test objects under their declared inputs, then the Coulomb Shearing closure-failure mechanism is falsified for the claimed domain.

**Coupling-timescale falsification.** If collisional coupling is always fast compared to residence time, enforced co-motion cannot fail. Therefore:

> If $t_c < 0.1\, t_{\mathrm{res}}$ holds for *all* evaluated states in *all* declared test objects under declared inputs, then the mechanism is falsified for the claimed domain.

**Self-consistency falsification (feedback non-convergence).** Where the framework invokes a self-consistency iteration (e.g. electron depletion affecting forcing inputs as evaluated under the declared iteration rules), failure to converge indicates that the declared steady-state closure is not available for that system:

> If the declared feedback iteration does not converge for a system under its declared inputs and stopping rule, the mechanism fails for that system under those inputs.

**Charge-ratio contradiction (where directly measurable).** In any region that the executed evaluation maps to an electron-loading channel, if a direct composition constraint is available at the stated radius range with adequate precision, then the following contradiction falsifies that system-level prediction:

> If a measurement within the mapped electron-loading zone yields $n_i/n_e > 0.1$ at radii $r < 100\, r_g$ with a $3\sigma$ separation, that system falsifies the electron-loading-channel composition prediction.

This criterion is evaluated only when $n_i/n_e$ is directly constrained at the required radii and resolution; otherwise it is DATA-INSUFFICIENT for this specific test.

### 5.4.4 Program-level thresholds across a declared test set

Let $N$ be the number of declared test objects for which the executed evaluation produces a shearing-admissible regime (nonzero-measure $\chi_s = 1$) and for which the relevant observables are measurable under the protocol (not DATA-INSUFFICIENT).

Define $N_{\mathrm{pass}}$ as the number of those objects satisfying all applicable system-level tests (P1–P3 always, P4 only when applicable), and $N_{\mathrm{fail}}$ as the number failing at least one applicable test.

The program-level falsification thresholds are:

- **Hard falsification:** if $N_{\mathrm{pass}} = 0$ for $N \geq 3$, the theory is falsified for the claimed observable domain.
- **Severe tension:** if $N_{\mathrm{fail}}/N > 0.5$, the framework is in serious tension with the declared domain and requires domain restriction or a declared missing-physics amendment.
- **Conclusive falsification:** if $N_{\mathrm{fail}}/N > 0.9$, the framework is conclusively falsified for the declared domain.

### 5.4.5 Data-insufficient classification rule

A DATA-INSUFFICIENT outcome is permitted only when the required observable is genuinely unmeasured for that system, meaning that the available frequency coverage, polarization calibration, image alignment, cadence, or angular resolution does not allow the stated proxy to be extracted under the protocol. DATA-INSUFFICIENT is not permitted as a reclassification for contradictions obtained under adequate measurements.

### 5.4.6 Alternative-model sufficiency is not falsification

If an alternative model reproduces the same observed phenomenology with fewer assumptions, this does not by itself falsify Coulomb Shearing; it instead restricts the claim scope. In that case, Coulomb Shearing remains justified only if it provides additional, independently verified constraints (e.g. the mapped-zone inequalities above tied to $\chi_s$ and computed windows) that are not delivered by the alternative description.

**Non-uniqueness handled by a conjunction-of-tests discriminator.** This manuscript does not claim that transverse jet stratification alone is unique to Coulomb Shearing. The intended discriminator is the *conjunction* of (i) a computed, species-resolved electron-loading zone derived prior to any observational comparison, (ii) zone-localized Faraday structure satisfying P1–P3 *within* that mapped zone under matched-beam requirements, and (iii) failure of those inequalities outside the mapped zone (or loss of definability when no Faraday-active screen is present, in which case the outcome is DATA-INSUFFICIENT by protocol rather than a PASS). Models that produce stratification through shear, shocks, or generic MHD collimation may reproduce stratification, but they do not generally entail this specific, spatially anchored conjunction tied to an independently computed zone boundary. Therefore, the falsifiable claim is conditional and localized: the mapped-zone inequalities must track the computed zone, not merely appear somewhere in the jet.

# 6 Ion-Dominated Inflow and Black Hole Growth

## 6.1 Why ion inflow continues under electron-limited radiation pressure

**Plasma composition scope.** The baseline development in this section uses a two-species electron–ion closure for clarity, then extends the same growth and closure logic to pair-loaded mixtures in Section 6.4, where an $e^- e^+$ component coexists with the baryonic ion population and modifies the effective inertia and closure pathways.

**Eddington primacy is retained.** Radiative momentum transfer couples directly to electrons. In a Newtonian (weak-field) normalization, the radiative force per electron due to Thomson scattering can be written as

$$f_{\mathrm{rad},e}(r) = \frac{\sigma_T}{c} F(r), \qquad F(r) = \frac{L}{4\pi r^2}, \tag{135}$$

where $\sigma_T$ is the Thomson cross section, $c$ is the speed of light, $F(r)$ is the radiative energy flux, and $L$ is the isotropic luminosity used for normalization. Gravity acts directly on each species, with per-particle

Newtonian forces

$$f_{g,e}(r) = \frac{GMm_e}{r^2}, \qquad f_{g,i}(r) = \frac{GMm_i}{r^2}, \tag{136}$$

for a black hole of mass $M$, gravitational constant $G$, electron mass $m_e$, and ion mass $m_i$. These expressions are used here as a weak-field reference scaling; when the discussion requires a local force-balance statement at radii where strong-field corrections matter, $F(r)$ should be interpreted as the *local* flux measured in the relevant plasma frame (and $r$ as areal radius), with the usual GR redshift factors relating local flux to luminosity at infinity.

In the classical coupled-fluid limit, an electrostatic restoring field and collisional locking enforce $a_e \simeq a_i$, so the electron radiative force is communicated to ions. Under that closure, equating outward electron radiative forcing to the inward gravitational force on a proton gives the standard Newtonian Eddington luminosity,

$$L_{\rm Edd} = \frac{4\pi GMm_pc}{\sigma_T}, \tag{137}$$

where $m_p$ is the proton mass. (For general composition, use $L_{\rm Edd} = 4\pi GMc/\kappa$ with $\kappa = \sigma_T/(\mu_e m_p)$, i.e. replace $m_p$ by $\mu_e m_p$.) In GR, the corresponding *local* Eddington condition is most cleanly expressed in terms of the local flux $F_{\rm local}(r)$ and the proper gravitational acceleration for the chosen observer; equivalently, when phrased using the luminosity measured at infinity, a commonly used Schwarzschild reference form is

$$L_{{\rm Edd},\infty}(r) = L_{\rm Edd}\sqrt{1-\frac{2GM}{rc^2}}, \tag{138}$$

so that a local force-balance saturation statement at radius $r$ should be understood as $L_\infty \lesssim L_{{\rm Edd},\infty}(r)$ (or, equivalently, $(\kappa/c)F_{\rm local}(r)$ not exceeding the local gravitational acceleration), rather than a global bound on $L$ independent of radius.

For rotating black holes, the same interpretation applies with the appropriate choice of observer frame (e.g. ZAMO) and redshift factor; the present form is used only as a reference normalization.

**Ion force balance differs from electron force balance.** Write a minimal two-fluid equation of motion along a resolved trajectory direction $\hat{s}$, in terms of accelerations $a_e$ and $a_i$ (projections along $\hat{s}$):

$$m_e a_e = f_{{\rm rad},e} - f_{g,e} - f_c, \tag{139}$$

$$m_i a_i = -f_{g,i} + f_c, \tag{140}$$

where $f_c$ is the coupling force that transfers momentum between species (including electrostatic restoring response and collisional friction projected along the trajectory). No new radiative channel for ions is introduced. In the coupling-dominated limit, $f_c$ adjusts so that $a_e \simeq a_i$, and the electron radiative forcing is transferred to the ion population.

The Coulomb Shearing mechanism enters through the finite capacity of $f_c$ to enforce co-motion over the residence time. When the previously defined closure test returns $\chi_s = 1$ for a state on a resolved trajectory segment, the coupling term cannot enforce $a_e \simeq a_i$ over that segment. In that regime, the differential acceleration

$$\Delta a \equiv a_e - a_i \tag{141}$$

is not suppressed by coupling. Define $\hat{s}$ to point outward along the local polar streamline so that inward acceleration corresponds to $a < 0$ along $\hat{s}$. The gravitational acceleration vector is $\mathbf{g} = -(GM/r^2)\hat{r}$, so its projection along the streamline is

$$a_{g,\parallel} \equiv \mathbf{g} \cdot \hat{s} = -\frac{GM}{r^2}\,(\hat{r} \cdot \hat{s}). \tag{142}$$

In the decoupled regime, the ion acceleration along $\hat{s}$ is gravity-dominated and therefore satisfies

$$a_i \simeq a_{g,\parallel}, \tag{143}$$

up to the residual coupling term that the closure test has already declared insufficient to enforce co-motion.

**Causal chain.** The causal chain that produces ion-dominated inflow without altering electron Eddington saturation is:

electron Eddington saturation $\longrightarrow$ finite Coulomb coupling $\longrightarrow$ sustained electron–ion decoupling where $\chi_s = 1$ $\longrightarrow$ ion-dominated inward acceleration.

This outcome does not modify Eddington physics. It removes the perfect-coupling inference that electron-limited radiation pressure bounds the total inward mass flux.

**Interpretation: no macroscopic charge separation is assumed or required.** Throughout this section, "ion-dominated inflow" is a *mass-flux decomposition* under bounded closure failure, not a claim of quasi-steady macroscopic charge separation. The closure test used elsewhere in the manuscript is explicitly screening-scale limited: any species-relative slip is bounded to $\Delta x = \mathcal{O}(\lambda_D)$ before collective electric-field response restores quasi-neutrality. Therefore, the theory does not require electrons to be absent from the inflow, nor does it require kilometer- or centimeter-scale separation of charge. What it requires is that, along the evaluated polar segment, electrons do not remain perfectly co-moving with ions *for the purposes of radiative force closure*, so that the electron-regulated radiative channel and the ion-dominant mass channel can be treated as distinct contributors to $\dot{M}_\bullet$.

The global electrodynamic structure (return currents, field-aligned potentials, and charge-neutralizing response) is not re-derived here and is not an additional claim of this work. Instead, the admissibility flag $\chi_s$ is the gating statement: if quasi-neutral collective response enforces strict co-motion on the relevant scale, then $\chi_s = 0$ and the ion-dominant channel vanishes by construction.

### 6.2 Derivation of modified accretion rates

**Classical Eddington-limited baseline.** Let $\dot{M}_{\text{obs}}$ denote the observationally inferred inflow proxy through the radiating region that produces the emergent luminosity, and let $\epsilon$ be the radiative efficiency defined by

$$L = \epsilon\, \dot{M}_{\text{obs}}\, c^2, \tag{144}$$

with $0 < \epsilon < 1$. The black hole mass growth rate is then

$$\dot{M}_{\bullet} = (1-\epsilon)\, \dot{M}_{\text{obs}}, \tag{145}$$

where the factor $(1-\epsilon)$ accounts for energy radiated away.

Define the Eddington ratio $\lambda \equiv L/L_{\text{Edd}}$ with $0 < \lambda \le 1$ under electron saturation. Combining Eqs. (144) and (137) gives the standard Eddington-limited baseline:

$$\dot{M}_{\text{classical}} = \dot{M}_{\bullet,\text{classical}} = (1-\epsilon)\,\frac{L}{\epsilon c^2} = \frac{1-\epsilon}{\epsilon}\,\frac{\lambda\, L_{\text{Edd}}}{c^2} = \frac{1-\epsilon}{\epsilon}\,\frac{4\pi G M m_p}{\sigma_T c}\,\lambda. \tag{146}$$

All symbols are physical inputs or derived quantities: $M$ has units of mass, $\dot{M}_{\text{classical}}$ has units of mass per time, and $\lambda$ is dimensionless.

**Where perfect coupling enters the classical inference.** The classical inference that the bound on $L$ implies a bound on $\dot{M}_{\bullet}$ uses the assumption that the inflow is a single co-moving fluid so that the radiatively regulated component is identical to the total mass inflow. In Eqs. (139)–(140), that inference corresponds to taking $f_c$ large enough that $a_e \simeq a_i$ everywhere, which forces the electron force balance to control the ion trajectory and therefore the total inward mass flux.

**What sets the supply rate into the polar channel.** The quantity $\dot{M}_{\text{sup}}$ is a boundary condition in the present theory: it represents the rate at which mass is delivered onto trajectories that enter the polar interaction region where the admissibility criterion is evaluated. To make this dependence explicit, we parameterize the supply rate by a polar delivery fraction

$$f_{\text{polar}} \equiv \frac{\dot{M}_{\text{sup}}}{\dot{M}_{\text{in}}}, \qquad 0 \le f_{\text{polar}} \le 1, \tag{147}$$

where $\dot{M}_{\text{in}}$ is the total mass inflow reaching the inner region from larger radii (disk plus any vertical or funnel-fed components). The Coulomb Shearing mechanism does not, by itself, determine $f_{\text{polar}}$. What it does determine is the *conditional* growth enhancement given $f_{\text{polar}}$ and the evaluated decoupling outcome $f_{\text{ion}}$. In practice, $f_{\text{polar}}$ can be bounded by external constraints (e.g., inner-flow simulations or observational inferences of funnel-fed mass flux), and the required $f_{\text{polar}}$ to reproduce a given growth history can be stated transparently as an output of the model rather than an implicit tuning parameter.

**Constraint-aware interpretation of $f_{\rm polar}$ and $f_{\rm ion}$.** The growth enhancement in this section is not numerically predictive unless $f_{\rm polar}$ is independently bounded and $f_{\rm ion}$ is computed from an executed closure evaluation. Here, $f_{\rm polar}$ is explicitly a boundary-condition fraction (external to Coulomb Shearing), and wide uncertainty in $f_{\rm polar}$ propagates directly into wide uncertainty in any inferred enhancement. Likewise, $f_{\rm ion}$ is not a free multiplier: it is defined as a weighted average of the evaluated decoupling outcomes (Eq. (149)) and is therefore only as credible as the admissibility gate that produced $\chi_s$.

Accordingly, when $f_{\rm polar}$ cannot be bounded for a system, the model output should be stated as a *required supply condition* (the minimum $f_{\rm polar}$ needed to reach a stated growth history) rather than as a predicted multiplier. Where $f_{\rm polar}$ *is* bounded, results must still be reported as "constraint-dominated" or "constraint-robust" depending on whether the enhancement range is set primarily by external $f_{\rm polar}$ uncertainty or by the computed $f_{\rm ion}$ window.

**Species-resolved decoupling generates an additional ion inflow channel.** Define a supply rate $\dot{M}_{\rm sup}$ delivered into the resolved trajectory family at large radius. The species-resolved evaluation yields a decoupling probability for each evaluated state, defined here as

$$P_{\rm dec}(Z) \equiv \Pr(\chi_s = 1 \mid Z), \tag{148}$$

over the declared phase-space measure used in the evaluation. Let $w_Z$ be the mass fraction of the inflowing material in evaluated state $Z$, with $\sum_Z w_Z = 1$. The mass fraction that enters the ion-dominant inflow channel is then

$$f_{\rm ion} \equiv \sum_Z w_Z \, P_{\rm dec}(Z), \qquad 0 \le f_{\rm ion} \le 1. \tag{149}$$

This definition introduces no tunable multiplier. It is a computable outcome of the executed evaluation and declared composition.

**Identity with the evaluated shearing probability.** The decoupling probability defined here is identical to the evaluated shearing probability already computed from the closure outcome. In particular,

$$P_{\rm dec}(Z) \equiv \Pr(\chi_s = 1 \mid Z) \equiv P_s(Z), \tag{150}$$

so no additional probability model is introduced at this stage. The quantity $f_{\rm ion}$ is therefore a direct functional of the executed closure outcomes and the declared inflow composition weights $\{w_Z\}$.

The ion-dominant mass inflow rate contributed by decoupled material is

$$\dot{M}_{\rm ion} = f_{\rm ion} \, \dot{M}_{\rm sup}. \tag{151}$$

**Radiatively regulated component remains electron-limited.** The radiatively regulated inflow $\dot{M}_{\rm rad}$ is the component that produces the electron-coupled luminosity in Eq. (144). Under electron saturation, it satisfies

$$\dot{M}_{\rm rad} = \frac{\lambda\, L_{\rm Edd}}{\epsilon c^2}, \tag{152}$$

and therefore contributes to black hole growth at the classical rate $(1-\epsilon)\dot{M}_{\rm rad}$.

**Radiative output of the ion-dominant channel.** The additional ion-dominant inflow is electron-depleted by the same closure outcome that defines $\chi_s = 1$, so its radiative efficiency is suppressed relative to the electron-regulated channel. Introduce an ion-channel radiative efficiency $\epsilon_{\rm ion}$ that parameterizes the fraction of $\dot{M}_{\rm ion}c^2$ converted to radiation. In the strongly electron-depleted limit produced by sustained closure failure, $\epsilon_{\rm ion} \to 0$.

**Coulomb Shearing modified growth rate.** The total black hole growth rate is the sum of the radiatively regulated growth and the ion-dominant growth, with the radiative mass-loss factors retained explicitly in both channels:

$$\dot{M}_{\rm CS} = \dot{M}_{\bullet,{\rm CS}} = (1-\epsilon)\, \dot{M}_{\rm rad} + (1-\epsilon_{\rm ion})\, \dot{M}_{\rm ion}. \tag{153}$$

Using Eq. (152) and Eq. (151), this can be written as

$$\dot{M}_{\rm CS} = \frac{1-\epsilon}{\epsilon}\, \frac{\lambda\, L_{\rm Edd}}{c^2} + (1-\epsilon_{\rm ion})\, f_{\rm ion}\, \dot{M}_{\rm sup}. \tag{154}$$

In the electron-depleted limit $\epsilon_{\rm ion} \to 0$, Eq. (154) reduces to the form previously used, $\dot{M}_{\rm CS} = \frac{1-\epsilon}{\epsilon}\frac{\lambda L_{\rm Edd}}{c^2} + f_{\rm ion}\dot{M}_{\rm sup}$. The coupling-dominated limit is recovered automatically: if $P_{\rm dec}(Z) = 0$ for all evaluated $Z$, then $f_{\rm ion} = 0$ and Eq. (153) reduces to Eq. (146).

**Supply rate as a boundary condition and its regime dependence.** The supply rate $\dot{M}_{\rm sup}$ is a declared boundary condition set by the large-scale accretion environment and the fraction of that environment that feeds the resolved polar trajectory family. In this paper $\dot{M}_{\rm sup}$ denotes the supply into that polar trajectory family and is treated as distinct from $\dot{M}_{\rm rad}$, which is the electron-coupled inflow that produces the luminosity $L$. It is not a free tuning parameter inside this mechanism. The enhancement magnitude therefore depends on whether the polar-channel supply is dynamically significant relative to the radiatively regulated inflow that sets $\lambda$.

To make this dependence explicit, define the dimensionless supply ratio

$$\Phi \equiv \frac{\dot{M}_{\rm sup}}{\dot{M}_{\rm rad}} = \frac{\epsilon c^2\, \dot{M}_{\rm sup}}{\lambda L_{\rm Edd}}, \tag{155}$$

where $\dot{M}_{\rm rad} = \lambda L_{\rm Edd}/(\epsilon c^2)$ is fixed by electron saturation. Using Eq. (154) and the classical baseline in Eq. (146), the instantaneous growth-rate ratio is

$$\frac{\dot{M}_{\rm CS}}{\dot{M}_{\rm classical}} = 1 + \frac{(1-\epsilon_{\rm ion})\, f_{\rm ion}\, \dot{M}_{\rm sup}}{(1-\epsilon)\dot{M}_{\rm rad}} = 1 + \frac{1-\epsilon_{\rm ion}}{1-\epsilon}\, f_{\rm ion}\, \boldsymbol{\Phi}. \tag{156}$$

**Re-parameterizing the supply term by a polar delivery fraction.** For interpretability, define $\dot{M}_{\rm in}$ as the total mass inflow delivered to the inner accretion environment (disk plus non-equatorial channels) and define a polar delivery fraction

$$f_{\rm polar} \equiv \frac{\dot{M}_{\rm sup}}{\dot{M}_{\rm in}}, \qquad 0 \le f_{\rm polar} \le 1. \tag{157}$$

The supply ratio $\boldsymbol{\Phi}$ can then be rewritten as

$$\boldsymbol{\Phi} = \frac{\dot{M}_{\rm sup}}{\dot{M}_{\rm rad}} = f_{\rm polar}\, \frac{\dot{M}_{\rm in}}{\dot{M}_{\rm rad}}. \tag{158}$$

Coulomb Shearing does not, by itself, determine $f_{\rm polar}$ nor the inflow normalization $\dot{M}_{\rm in}$. What it determines is the conditional growth enhancement given (i) $f_{\rm ion}$ from the evaluated decoupling outcome and (ii) the externally supplied delivery factor $f_{\rm polar}\,(\dot{M}_{\rm in}/\dot{M}_{\rm rad})$.

**Model falsifiability via the required supply factor.** For a target instantaneous enhancement $\mathcal{G} \equiv \dot{M}_{\rm CS}/\dot{M}_{\rm classical}$, Eq. (156) implies a required supply ratio

$$\boldsymbol{\Phi}_{\rm req} = \frac{\mathcal{G}-1}{f_{\rm ion}}\,\frac{1-\epsilon}{1-\epsilon_{\rm ion}}. \tag{159}$$

Given any external estimate or prior for $\dot{M}_{\rm in}/\dot{M}_{\rm rad}$, Eq. (158) then returns the corresponding required $f_{\rm polar}$. If the implied $f_{\rm polar}$ lies outside the range supported by independent constraints, the mechanism fails for that system under the stated conditions, independent of whether $\chi_s = 1$ is attainable.

**External constraints on $f_{\rm polar}$ from GRMHD.** GRMHD simulations robustly produce a low-density, highly magnetized polar funnel and a funnel-adjacent, matter-carrying region (often identified with funnel-wall outflows or mass-loaded sheaths) in which the mass flux is subdominant but nonzero relative to equatorial inflow [23, 24]. In MAD configurations, the existence of a strong polar electromagnetic funnel is particularly pronounced [10]. Quantitative funnel mass loading is sensitive to spin, magnetic flux saturation, and the plasma injection prescriptions used in highly magnetized zones, so $f_{\rm polar}$ is best treated as an externally constrained input rather than a prediction of Coulomb Shearing. For the worked calculations in this paper, we therefore report the implied $\boldsymbol{\Phi}_{\rm req}$ (Eq. (159)) and, where an estimate of $\dot{M}_{\rm in}/\dot{M}_{\rm rad}$ is available, the corresponding $f_{\rm polar}$ with explicit comparison to GRMHD-informed ranges.

**Predictive scope: growth enhancement versus jet composition.** The GRMHD-constrained inputs ($f_{\rm polar}$ and equivalently $\Phi$) limit *predictivity of growth rates* because the enhancement factor in Eq. (156) depends on externally determined polar delivery. This does not weaken the species-resolved jet-partition predictions (P1–P4 in Observable Consequences: Jets and Stratification), which depend primarily on the computed admissibility outcomes (via $f_{\rm ion}$ and the species windows) rather than on the absolute supply normalization. Accordingly, the paper treats growth enhancement as a conditional, externally constrained implication (audited via Eq. (159)), while treating the composition/partition signatures as the primary internal predictions.

This equation exposes the regime dependence without introducing phenomenological multipliers: even with $f_{\rm ion} \to 1$, the enhancement is modest when $\Phi \ll 1$ and is large only when the declared environment supplies $\dot{M}_{\rm sup}$ comparable to or larger than $\dot{M}_{\rm rad}$.

### 6.3 Self-consistent iteration procedure (feedback closure)

The calculations executed elsewhere in this manuscript are *forward evaluations*: they take a declared initial state $(n_e, T_e, B, E_\parallel, L)$, compute the differential response (displacement, admissibility, and electron depletion proxy), and then report consequences (partition signatures and conditional growth enhancement). A physically complete treatment must also address back-reaction: if electrons are depleted or spatially redistributed, the microphysical rates and transport coefficients that enter the closure test are modified, which in turn feeds back into the differential forcing and displacement kernel.

This section defines an auditable *iteration protocol* that makes those feedback loops explicit. It does not alter the pre-committed PASS/FAIL logic; it defines how a self-consistent fixed point can be computed when the required closure relations are declared.

**State vector and update targets.** Define the local state vector on the evaluated trajectory segment as

$$\mathbf{S} \;\equiv\; \left(n_e,\, T_e,\, \ln\Lambda,\, \lambda_D,\, \nu_c,\, \sigma_{\rm cond},\, \kappa_{\rm eff},\, \Delta a_\parallel\right), \tag{160}$$

where $\nu_c$ denotes the electron–ion coupling (momentum-exchange) rate used to form $t_c$ (and therefore the ratio $t_{\rm res}/t_c$), $\sigma_{\rm cond}$ is an effective conductivity controlling the ability to sustain current closure, and $\kappa_{\rm eff}$ is the effective opacity relevant for the adopted radiative forcing channel. The forward pipeline produces, at fixed $\mathbf{S}$, a depletion or separation proxy. Denote the computed electron depletion fraction on the segment by

$$f_{\rm dep} \;\equiv\; f_{\rm dep}(\mathbf{S}; t_{\rm res}), \qquad 0 \le f_{\rm dep} \le 1, \tag{161}$$

where the functional dependence is determined by the declared kernel (displacement-to-admissibility-to-partition map) and by the declared residence-time treatment.

**Minimal feedback dependencies (what changes when $n_e$ changes).** The feedback pathways identified in the evaluation protocol can be expressed as a declared set of closures:

$$n_e^{(k+1)} = n_e^{(k)} \left[1 - f_{\text{dep}}^{(k)}\right], \tag{162}$$

$$\nu_c^{(k+1)} = \nu_c\left(n_e^{(k+1)}, T_e^{(k+1)}, \ln\Lambda^{(k+1)}\right), \tag{163}$$

$$\lambda_D^{(k+1)} = \lambda_D\left(n_e^{(k+1)}, T_e^{(k+1)}\right), \tag{164}$$

$$\sigma_{\text{cond}}^{(k+1)} = \sigma_{\text{cond}}\left(n_e^{(k+1)}, T_e^{(k+1)}\right), \tag{165}$$

$$\kappa_{\text{eff}}^{(k+1)} = \kappa_{\text{eff}}\left(n_e^{(k+1)}, T_e^{(k+1)}\right), \tag{166}$$

$$\Delta a_{\parallel}^{(k+1)} = \Delta a_{\parallel}\left(\kappa_{\text{eff}}^{(k+1)},\ L,\ B,\ E_{\parallel}\right). \tag{167}$$

The key point is procedural: once a depletion proxy is produced, any quantity whose definition depends on $(n_e, T_e)$ must be recomputed before the next displacement evaluation.

This manuscript does *not* commit to a single thermal closure because temperature evolution depends on the adopted heating/cooling model. Instead, $T_e^{(k+1)}$ is provided by a declared energy update operator:

$$T_e^{(k+1)} = \mathcal{U}_T\left(T_e^{(k)}, n_e^{(k+1)}; \text{declared heating/cooling model}\right), \tag{168}$$

which can be as simple as an isothermal assumption ($T_e^{(k+1)} = T_e^{(k)}$) for a conservative bound, or a radiative-cooling-reduced update when an explicit cooling law is declared.

**Iteration algorithm (fixed point).** Given an initial declared state $\mathbf{S}^{(0)}$ on the evaluated segment and a residence-time treatment (single $\tau$ or a marginalization over $p(t_{\text{res}})$), compute the self-consistent state by iterating:

1. **Forward step:** evaluate the displacement/admissibility kernel using $\mathbf{S}^{(k)}$ to obtain $\chi_s^{(k)}$, the species-window outputs (if any), and the depletion proxy $f_{\text{dep}}^{(k)}$ via Eq. (161).

2. **Microphysics update:** update $n_e^{(k+1)}$ using Eq. (162). Update $T_e^{(k+1)}$ using the declared operator in Eq. (168). Recompute $\nu_c^{(k+1)}$, $\lambda_D^{(k+1)}$, $\sigma_{\text{cond}}^{(k+1)}$, and $\kappa_{\text{eff}}^{(k+1)}$ using Eqs. (163)–(166).

3. **Forcing update:** update the differential forcing term $\Delta a_{\parallel}^{(k+1)}$ using Eq. (167) under the declared forcing model (radiative-only, magnetic-only, or combined).

4. **Convergence check:** terminate when the fixed-point condition is met:

$$\left|\frac{n_e^{(k+1)} - n_e^{(k)}}{n_e^{(k)}}\right| < \varepsilon_n \quad \text{and} \quad \left|\frac{\Delta a_{\parallel}^{(k+1)} - \Delta a_{\parallel}^{(k)}}{\Delta a_{\parallel}^{(k)}}\right| < \varepsilon_a, \tag{169}$$

with declared tolerances $(\varepsilon_n, \varepsilon_a)$, and with stability checked by confirming that $\chi_s$ does not alternate indefinitely across iterations.

**What is reported.** A self-consistent run reports: (i) the fixed-point state $\mathbf{S}^\star$, (ii) the fixed-point admissibility outcome $\chi_s^\star$ and any species-window outputs, and (iii) the number of iterations and whether convergence was monotone or oscillatory. If no fixed point exists under the declared closures (divergence or persistent oscillation), the result is reported as **DATA-INSUFFICIENT** for self-consistency under that closure family, and the forward (non-iterated) results are explicitly labeled as a first-pass evaluation rather than a closed solution.

**Scope discipline.** This procedure is a correctness layer, not an outcome guarantee. It may either strengthen the mechanism (runaway decoupling through reduced coupling) or suppress it (feedback reduces $\Delta a_{\parallel}$ or increases relocking). The point is that any such outcome becomes auditable because the feedback pathways, the thermal update operator, and the convergence criteria are declared explicitly rather than left implicit.

### 6.4 Extended treatment for pair-loaded plasmas ($e^-\, e^+$–ion)

The baseline development treats a two-species electron–ion plasma. Near strongly magnetized, rotating black holes, pair creation can produce an $e^-e^+$ component whose number density exceeds the ion density by orders of magnitude. This is not a cosmetic complication: it changes how charge neutrality, current closure, and species-resolved inertia enter the differential response.

This section extends the framework to an $e^-\, e^+$–ion mixture without introducing new forces. The purpose is to state explicitly what changes in the shearing condition and what does *not* change (in particular, the Thomson radiative coupling is identical for $e^-$ and $e^+$).

**Definitions and pair multiplicity.** Let $n_-$ and $n_+$ be the electron and positron number densities, and let $n_i$ be the ion number density (for simplicity, a single ion species with charge $Ze$). Define the *pair multiplicity* relative to the primary electron density required for ion neutrality as

$$\kappa_{\rm pair} \;\equiv\; \frac{n_- + n_+}{n_{e,\rm primary}}, \qquad n_{e,\rm primary} \equiv Zn_i, \tag{170}$$

so $\kappa_{\rm pair} = 1$ reduces to the two-species case (no excess pairs) and $\kappa_{\rm pair} \gg 1$ corresponds to a pair-dominated plasma.

The local charge neutrality condition is

$$n_- - n_+ - Zn_i \simeq 0, \tag{171}$$

which allows large $(n_- + n_+)$ while maintaining very small net charge.

**Forcing and the key distinction from electron–ion.** Under Thomson scattering, the radiative force per lepton is the same for $e^-$ and $e^+$ because $\sigma_T$ is identical. Therefore both leptonic species experience the same radiative acceleration along the trajectory:

$$a_{\mathrm{rad},-} = a_{\mathrm{rad},+} \equiv a_{\mathrm{rad},\ell}. \tag{172}$$

This immediately implies the central distinction:

- **No intrinsic lepton–lepton charge separation from radiation:** because $e^-$ and $e^+$ are pushed in the same direction by radiation, radiation does not, by itself, produce an $e^-$–$e^+$ separation.
- **Lepton–ion mass separation can still occur:** the lightweight lepton fluid (both signs) can separate from ions under differential forcing, producing a *mass* stratification even when net charge remains small.

**Modified differential response (mixture form).** To preserve the structure of the closure criterion while respecting neutrality, define the *lepton center-of-mass* (COM) along-trajectory acceleration

$$a_{\ell,\mathrm{COM}} \equiv \frac{n_- a_- + n_+ a_+}{n_- + n_+}. \tag{173}$$

In the radiative-only specialization, $a_- \simeq a_+ \simeq a_{\mathrm{rad},\ell}$, so $a_{\ell,\mathrm{COM}} \simeq a_{\mathrm{rad},\ell}$.

Define the ion acceleration $a_i$ under the same declared forcing model (including gravity and any declared radiative coupling channel for ions, which is typically negligible in the Thomson specialization). Then the mixture-relevant differential acceleration driving *lepton–ion* separation is

$$\Delta a_{\ell i} \equiv a_{\ell,\mathrm{COM}} - a_i. \tag{174}$$

This replaces the two-species $\Delta a$ wherever the objective is separation of the light component from the heavy component.

**What does and does not change in the admissibility criterion.** The displacement kernel and the admissibility ratio remain structurally identical when expressed in terms of the mixture differential acceleration:

$$\Delta x(t_{\mathrm{res}}) \;\longrightarrow\; \Delta x_{\ell i}(t_{\mathrm{res}}; \Delta a_{\ell i}), \qquad \mathcal{F} \equiv \frac{\Delta x}{\kappa \lambda_D}.$$

However, two state quantities must be updated in the pair-dominated regime:

- **Debye length and screening:** $\lambda_D$ depends on the total lepton density and temperature, so pair-loading generally reduces $\lambda_D$ at fixed $T_e$.
- **Coupling/relaxation:** the effective collisional rates relevant for relocking depend on the relevant partner densities. Pair-loading increases lepton–lepton collision frequency while the lepton–ion exchange frequency can remain small when $n_i \ll n_- + n_+$.

Therefore, the correct pair-loaded evaluation must treat $(n_- + n_+)$ as the charge-carrier density for screening and conductivity, while still treating $n_i$ as the heavy-mass reservoir for mass separation.

**Charge neutrality maintenance and current closure in pair-loaded funnels.** In a pair-dominated plasma, neutrality is maintained primarily by the near equality of $n_-$ and $n_+$ rather than by $n_- \simeq Zn_i$. Relative lepton drift that produces current can be supported by counter-streaming of $e^-$ and $e^+$ with minimal net charge imbalance. This means:

- the *existence* of large lepton flux does not imply macroscopic net charge,
- but the electromagnetic response (including $E_\parallel$ support and current closure) becomes more sensitive to pair conductivity and to the frozen-in verification under Verification of magnetic decoupling (frozen-in check).

**Regime statement for $\kappa_{\rm pair} \gg 1$.** If $\kappa_{\rm pair} \gtrsim 10^2$, then:

- the correct differential response for stratification is governed by $\Delta a_{\ell i}$ (lepton COM vs ions), not by $e^-$ vs ions alone,
- charge neutrality is maintained predominantly by $e^- \leftrightarrow e^+$ balance and therefore must be discussed in current terms, not in net-charge buildup terms,
- any electromagnetic specialization must pass the frozen-in gate using the *pair-enhanced* conductivity.

**Scope discipline.** This extension does not assert that pair multiplicity is large for any particular object without a declared, cited estimate. Instead, it defines the modified criterion and identifies exactly which inputs must be declared ($\kappa_{\rm pair}$, lepton temperature, and the relevant collision/relaxation rates) for a pair-loaded evaluation to be executed. Where those inputs are unavailable, the correct status is **DATA-INSUFFICIENT** for the pair-loaded variant, not a guessed outcome.

### 6.5 Relation to other super-Eddington growth channels

Multiple mechanisms in the literature relax the classical Eddington-limited growth inference by modifying either (i) the radiative efficiency and emergent luminosity for a given $\dot{M}$, or (ii) the initial seed mass, or (iii) the geometry and magnetization of the inner flow. Coulomb Shearing targets a different closure: it identifies conditions under which electron-coupled radiative forcing does not enforce electron–ion co-motion along polar-adjacent trajectories, enabling an ion-dominated inflow branch even when the electron momentum budget is radiatively constrained.

**Slim disks and photon trapping.** Slim-disk theory permits $\dot{M}_{\rm obs} \gg \dot{M}_{\rm Edd}$ while the emergent luminosity grows only weakly because radiation is advected inward (photon trapping) rather than escaping

Figure 6.1: Schematic comparison of $\dot{M}/\dot{M}_{\rm Edd}$ versus radius

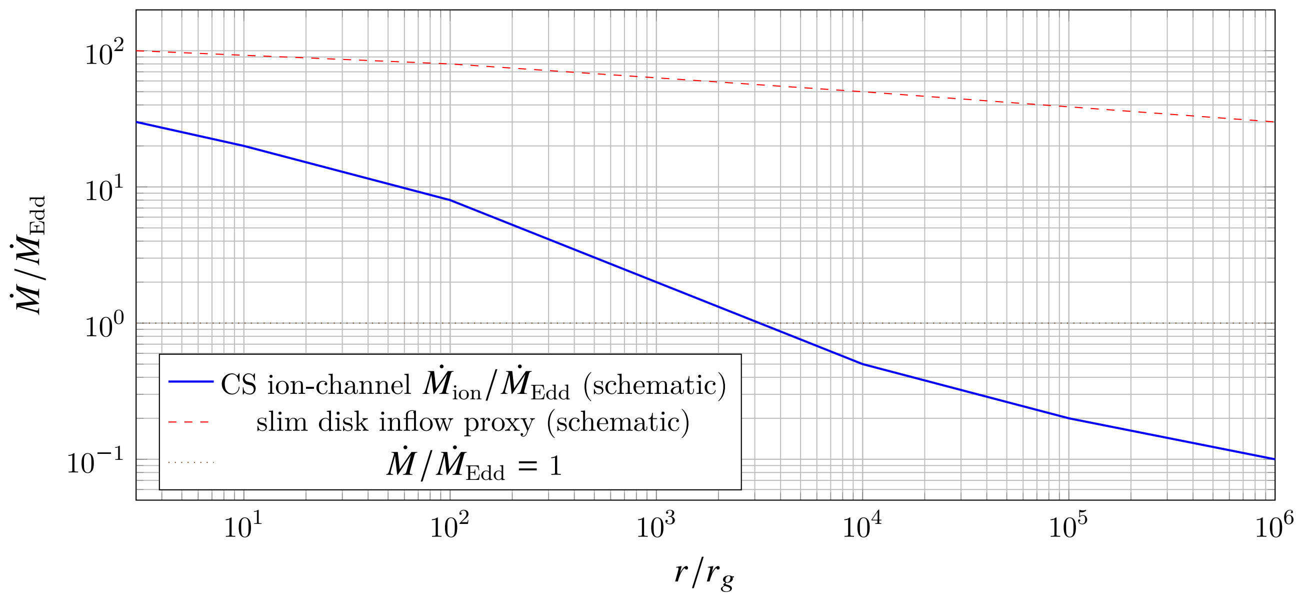


Figure 4: Schematic comparison of radial inflow normalizations. The Coulomb Shearing curve represents the ion-dominated delivery channel $\dot{M}_{\rm ion}$ defined by the framework and is plotted against the Eddington reference normalization $\dot{M}_{\rm Edd}$. The slim-disk curve is shown as a qualitative reference for regimes where $\dot{M}_{\rm obs} \gg \dot{M}_{\rm Edd}$ may occur while luminosity saturates. This figure is conceptual and is not used for validation.

locally [25]. Here $\dot{M}_{\rm Edd}$ denotes the Eddington reference accretion rate used only as a normalization for comparisons across mechanisms. Global radiation-MHD simulations show that the outcome depends on the relative roles of advection, diffusion, and vertical transport, with strong outflows and nontrivial angular radiation fields in supercritical states [11, 26]. In this landscape, Coulomb Shearing is best framed as *potentially complementary*: photon trapping alters the mapping between $\dot{M}$ and $L$, while Coulomb Shearing alters the mapping between electron-coupled radiative forcing and ion inflow along selected channels. If a system is in a slim-disk-like state, the local polar forcing term $\Delta a$ must be evaluated using the emergent anisotropic radiation field and magnetization appropriate to that state, but the logical structure of the admissibility test is unchanged.

**Magnetically arrested disks (MAD).** MAD states occur when large-scale magnetic flux accumulates near the horizon and strongly modifies the inner flow, producing powerful jets and a magnetically dominated polar funnel [10]. MAD physics does not automatically imply electron–ion decoupling; it primarily reorganizes angular momentum transport and jet energetics. Coulomb Shearing can therefore operate as a *conditional microphysical channel* within a MAD macrostate: the MAD configuration is one plausible route to creating a persistent polar-adjacent channel with strong electromagnetic stresses and a structured funnel-wall region, but the closure failure still requires that the local coupling and screening criteria (Differential Forcing and the Onset of Coulomb Shearing) be satisfied.

**Direct collapse and massive-seed scenarios.** Massive-seed models address early SMBH assembly by increasing the initial seed mass or altering the environmental conditions that permit rapid early growth [27]. These scenarios are not mutually exclusive with Coulomb Shearing: seed assembly sets initial conditions, while Coulomb Shearing (if admissible) modifies the subsequent growth closure by enabling ion-dominated inflow channels under strong differential forcing. In that sense, direct-collapse models and Coulomb Shearing can be layered rather than competing.

**Operational distinction and falsifiable placement.** This paper does not claim that Coulomb Shearing is the unique explanation for super-Eddington growth. Its claim is narrower and testable: given declared plasma inputs and a declared force model for $\Delta a$, the admissibility criterion either opens or closes a polar-adjacent decoupling channel, yielding a computable $f_{\rm ion}$ and a required $\Phi_{\rm req}$ (and thus a required $f_{\rm polar}$ if $\dot{M}_{\rm in}/\dot{M}_{\rm rad}$ is specified). Competing mechanisms remain viable explanations in regimes where the Coulomb Shearing channel is closed, or where the required supply factor lies outside externally supported bounds.

### 6.6 Growth multipliers

**Governing growth law.** Using Eq. (137), the radiatively regulated term in Eq. (153) is proportional to $M$. Define the classical e-folding timescale

$$t_S \equiv \frac{\epsilon}{1-\epsilon}\,\frac{\sigma_T c}{4\pi G m_p}\,\frac{1}{\lambda}, \tag{175}$$

so that the radiatively regulated contribution satisfies

$$\left(\frac{dM}{dt}\right)_{\rm rad} = \frac{M}{t_S}. \tag{176}$$

The full growth law under Coulomb Shearing is

$$\frac{dM}{dt} = \frac{M}{t_S} + \dot{M}_{\rm ion}(t), \tag{177}$$

where $\dot{M}_{\rm ion}(t) = f_{\rm ion}(t)\,\dot{M}_{\rm sup}(t)$ is computable from executed $P_{\rm dec}(Z)$ and the declared supply history used in evaluation.

**Closed-form solution with explicit integration.** Let $t_0$ be the onset time for the regime being evaluated, and let $M_0 \equiv M(t_0)$. Solving Eq. (177) by an integrating factor yields

$$M(t) = M_0\,\exp\!\left(\frac{t-t_0}{t_S}\right) + \int_{t_0}^{t} \exp\!\left(\frac{t-t'}{t_S}\right)\,\dot{M}_{\rm ion}(t')\,dt'. \tag{178}$$

In the coupling-dominated limit $\dot{M}_{\rm ion}(t) = 0$, Eq. (178) reduces to classical exponential Eddington growth.

**Growth multiplier as a computed outcome.** Define the classical evolution over the same interval as

$$M_{\rm classical}(t) = M_0 \exp\left(\frac{t - t_0}{t_S}\right). \tag{179}$$

The growth multiplier is then

$$\mathcal{G}(t) \equiv \frac{M(t)}{M_{\rm classical}(t)} = 1 + \frac{1}{M_0} \int_{t_0}^{t} \exp\left(-\frac{t' - t_0}{t_S}\right) \dot{M}_{\rm ion}(t')\, dt'. \tag{180}$$

No adjustable efficiency appears in $\mathcal{G}(t)$. The enhancement is fixed by $\dot{M}_{\rm ion}(t)$, which is fixed by the executed decoupling probabilities and the declared supply history. Enhancement shuts off when $\dot{M}_{\rm ion}(t) \to 0$, which occurs when $f_{\rm ion}(t) \to 0$ under coupling dominance or when $\dot{M}_{\rm sup}(t) \to 0$ under supply exhaustion.

**Required reporting of external constraints (prevents unconstrained multipliers).** Although $\mathcal{G}(t)$ is defined exactly by Eq. (180), its numerical range is only as tight as the external supply history $\dot{M}_{\rm sup}(t)$ and the transport fraction $f_{\rm polar}$ used to instantiate that supply, together with the executed decoupling outcome $f_{\rm ion}(t)$. Therefore, no worked example may present a stand-alone growth multiplier (or a numeric enhancement range) without explicitly stating: (i) the declared bracket or provenance used for $f_{\rm polar}$ (or equivalently for $\dot{M}_{\rm sup}$), (ii) whether $f_{\rm ion}(t)$ is computed from an executed closure evaluation ($\chi_S$-gated) or inserted as an assumption, and (iii) whether the resulting enhancement is **constraint-dominated** (set primarily by external $f_{\rm polar}/\dot{M}_{\rm sup}$ uncertainty) or **constraint-robust** (remains $> 1$ across the stated brackets).

### 6.7 Inverse mapping

**Inversion target and failure criterion.** Let $M_f$ be an observed mass at observation time $t_f$. Given a declared efficiency $\epsilon$, Eddington ratio $\lambda$ (electron saturation), and a computed or declared ion-channel history $\dot{M}_{\rm ion}(t)$, the growth law in Eq. (178) determines whether $M_f$ is reachable from a seed mass $M_0$ within the available time $\Delta t \equiv t_f - t_0$. If the required $\Delta t$ exceeds the available cosmic time between $t_0$ and $t_f$, the mechanism fails for that system.

**General inverse mapping expressed as a solvable constraint.** For a chosen onset time $t_0$ and seed mass $M_0$, Eq. (178) implies the constraint

$$M_f = M_0 \exp\left(\frac{\Delta t}{t_S}\right) + \int_{t_0}^{t_0+\Delta t} \exp\left(\frac{t_0 + \Delta t - t'}{t_S}\right) \dot{M}_{\rm ion}(t')\, dt'. \tag{181}$$

In application, $\dot{M}_{\rm ion}(t)$ is evaluated from $f_{\rm ion}(t)$ and $\dot{M}_{\rm sup}(t)$, and $\Delta t$ is obtained by solving Eq. (181) for the smallest nonnegative $\Delta t$ that satisfies the equality.

**Closed-form inversion for a constant ion inflow interval.** When $\dot{M}_{\rm ion}(t)$ is constant over the interval, $\dot{M}_{\rm ion}(t) = \dot{M}_{\rm ion,0}$ for $t \in [t_0, t_f]$, Eq. (178) gives

$$M_f = M_0 \, \exp\left(\frac{\Delta t}{t_S}\right) + \dot{M}_{\rm ion,0}\, t_S \left[\exp\left(\frac{\Delta t}{t_S}\right) - 1\right]. \tag{182}$$

Solving for $\Delta t$ yields

$$t_{\rm form} = \Delta t = t_S \ln\left(\frac{M_f + \dot{M}_{\rm ion,0}\, t_S}{M_0 + \dot{M}_{\rm ion,0}\, t_S}\right), \tag{183}$$

which reduces to the classical form $t_S \ln(M_f/M_0)$ when $\dot{M}_{\rm ion,0} = 0$.

Equivalently, for a fixed available time $\Delta t$ the required constant ion inflow rate is

$$\dot{M}_{\rm ion,0} = \frac{M_f - M_0 \, \exp\left(\frac{\Delta t}{t_S}\right)}{t_S \left[\exp\left(\frac{\Delta t}{t_S}\right) - 1\right]}. \tag{184}$$

If Eq. (184) requires $\dot{M}_{\rm ion,0} < 0$ for the declared $\Delta t$, the declared inputs already permit classical growth and the ion channel is not required. If Eq. (184) requires an ion inflow rate that exceeds the declared supply history, the mechanism fails for that system under those declared conditions.

# 7 Effective Inflow Geometry and Axially-Concentrated Mass-Loading

## 7.1 Clarification: no claim of changing Kerr singularity topology

Coulomb Shearing does not alter the Kerr ring singularity, does not modify the Kerr metric, and does not introduce any new stationary spacetime solution. The background spacetime is taken to be Kerr throughout. No claims are made about modified gravity, horizon deformation, or a new equilibrium geometry.

The geometric language used here refers only to the effective, time-dependent distribution of matter and fields in the near-horizon environment. In general relativity, spacetime geometry is encoded by the metric, while the inflowing plasma, radiation field, and electromagnetic field contribute a stress-energy tensor $T^{\mu\nu}$ that can be strongly anisotropic in realistic accretion states. Coulomb Shearing does not change Kerr. It changes the composition and angular distribution of supplied stress-energy by enabling species-resolved transport and, in shearing-dominated regimes, preferential polar delivery of ion rest mass and momentum.

Accordingly, terms such as “axially-concentrated mass-loading”, “polar-weighted inflow geometry”, or “effective inflow geometry” are conditional statements about a driven, non-equilibrium inflow structure maintained by continuous anisotropic mass-loading. If the driving pattern weakens or ceases, no persistence of the effective structure is assumed. The anisotropy must decay toward the symmetry favored by the underlying Kerr spacetime together with the relevant dissipation, mixing, and redistribution processes in the near-horizon plasma.

**Horizon constraint and exterior evaluation domain (explicit).** All Coulomb Shearing claims in this manuscript are restricted to the exterior domain of the Kerr spacetime, i.e. to radii $r > r_{\mathrm{H}}$, where $r_{\mathrm{H}}$ denotes the *outer* Kerr event-horizon radius (Eq. (7)). In terms of the gravitational radius $r_{\mathrm{g}} \equiv GM/c^2$ and the dimensionless spin $a_* \in [0, 1)$, $r_{\mathrm{H}} = r_+$ is the standard outer-horizon radius. No Coulomb Shearing condition is asserted, evaluated, or interpreted at $r \leq r_{\mathrm{H}}$. In this manuscript, “termination before the horizon” means that the prediction-relevant separation process is evaluated only for $r > r_{\mathrm{H}}$ and is treated as ending at the exterior limit $r \downarrow r_{\mathrm{H}}$, because any continuation at $r \leq r_{\mathrm{H}}$ cannot contribute to externally observable jet loading.

This restriction is not optional: the observable consequences discussed in this paper (electron-loaded outflow channel, species-partitioned jet composition, Faraday-depth signatures) require that the electron trajectory, the electromagnetic outflow structure, and the radiative field remain causally connected to infinity. At or inside the horizon, outward transport is not physically available to external observers, so any continued electron-ion drift or charge separation is not an observable jet-loading mechanism. Operationally, the horizon therefore functions as an absorbing boundary for the evaluation protocol: the only question that matters for predictions is whether the shearing condition becomes satisfied at radii *outside* $r_{\mathrm{H}}$ along the relevant trajectories.

Accordingly, any stated “near-horizon evaluation band” (e.g. an interval written in units of $r_{\mathrm{g}}$) is to be interpreted as an exterior band with the lower endpoint approached from above when necessary, i.e. as $r \downarrow r_{\mathrm{H}}$ rather than as an inclusive statement at $r = r_{\mathrm{H}}$. This makes explicit why Coulomb Shearing must terminate, for purposes of observable consequences, before the horizon: only exterior separation can populate an escaping electron channel.

### 7.2 Driven, anisotropic near-horizon mass-loading

The upstream cause of the anisotropy considered here is the emergence of polar-weighted ion inflow under shearing-dominated conditions. In this regime, differential forcing between electrons and ions permits electrons to be preferentially diverted into the outflow channel while ions remain able to continue inward. The result is not only a change in net accretion but a change in where, and in what form, mass, charge, and momentum are delivered to the near-horizon region.

A sustained polar-weighted inflow implies that the near-horizon stress-energy becomes directionally structured. In practical terms, the inward rest-mass flux and inward momentum flux components of $T^{\mu\nu}$ associated with the inflow are enhanced at small polar angles relative to a more isotropic or equator-dominated supply. This anisotropy is a property of the supplied matter-energy distribution, not a property of the Kerr geometry itself.

JWST *Little Red Dots* (LRDs) motivate keeping this geometry statement explicit rather than implicit. LRDs are observationally defined by extreme compactness and high surface brightness in rest-optical imaging and spectroscopy, which means that any inference about angularly concentrated delivery of mass and charge must be made through declared proxies and bounds rather than assumed morphology [5, 6]. In this manuscript, compactness is used only as a constraint on what parameter ranges are plausible for the driving and relaxation competition defined below. It does not introduce additional dynamics and it does not modify the exterior-domain restriction already declared.

**Quantifying "axially-concentrated" as an effective inflow geometry.** To keep the definition explicit and evaluable, introduce the inward rest-mass flux per unit solid angle through a sphere of radius $r$,

$$\mathcal{F}_M(r,\theta) \;\equiv\; -r^2\,\rho\,u^r, \tag{185}$$

where $\rho$ is the rest-mass density and $u^r < 0$ for inward motion. A "axially-concentrated" effective inflow geometry means that $\mathcal{F}_M(r,\theta)$ is preferentially concentrated toward $\theta \simeq 0$ and $\theta \simeq \pi$, producing an elongation of the inflow supply along the spin axis in an angularly averaged sense.

One compact, coordinate-independent way to represent this concentration is via the quadrupolar moment of the flux distribution,

$$a_2(r) \;\equiv\; \frac{\int \mathcal{F}_M(r,\theta)\,P_2(\cos\theta)\,d\Omega}{\int \mathcal{F}_M(r,\theta)\,d\Omega}, \qquad P_2(\mu) = \frac{1}{2}(3\mu^2-1), \tag{186}$$

**Operational meaning of the $a_2$ diagnostic (pre-committed).** The scalar $a_2(r)$ in Eq. (186) is used only as an operational indicator of axially-concentrated mass-loading at the evaluation radius. For the near-horizon control surface used in the maintenance test, define

$$a_2(r_{\rm nh}) \;\geq\; 0.30 \quad\Longrightarrow\quad \text{"axially-concentrated mass-loading" (descriptor permitted)}, \tag{187}$$

and if $a_2(r_{\rm nh}) < 0.30$, that descriptor is not used. Any alternative threshold may be reported only as a sensitivity calculation and must not be used to reclassify outcomes after evaluation.

In Eq. (186), the solid-angle element is $d\Omega = \sin\theta\,d\theta\,d\phi$. A polar-weighted inflow yields $a_2(r) > 0$, whereas an equator-weighted inflow yields $a_2(r) < 0$. The sign and magnitude of $a_2$ provide an anisotropy diagnostic without implying any permanent geometric deformation; the descriptor threshold, when used, is applied only via Eq. (187).

**Polar delivery fraction and notation alignment.** For later algebra it is also useful to define a polar delivery fraction of the inward ion flux at the radii where the near-horizon supply is set. Define

$$\dot{M}_{\rm in}(r) \equiv \int \mathcal{F}_M(r,\theta)\, d\Omega, \qquad \dot{M}_{\rm pol}(r) \equiv \int_{\Omega_{\rm pol}} \mathcal{F}_M(r,\theta)\, d\Omega, \tag{188}$$

where $\Omega_{\rm pol}$ denotes the chosen polar solid-angle region (two polar caps with a declared half-opening angle). Then

$$f_{\rm polar}(r) \equiv \frac{\dot{M}_{\rm pol}(r)}{\dot{M}_{\rm in}(r)}, \tag{189}$$

**Declared polar-cap convention (fixed unless explicitly overridden).** Throughout this manuscript, the polar region is defined as two polar caps with half-opening angle $\theta_{\rm pol} \equiv \theta^*_{\rm pol} = 30^\circ$. Accordingly,

$$\Omega_{\rm pol} \equiv 4\pi \left(1 - \cos\theta_{\rm pol}\right), \qquad f_{\rm iso} \equiv \frac{\Omega_{\rm pol}}{4\pi} = 1 - \cos\theta_{\rm pol} \simeq 0.134 \ \ (\theta_{\rm pol} = 30^\circ). \tag{190}$$

If any other $\theta_{\rm pol}$ is used for a specific comparison, it must be stated explicitly at the point of use, and the corresponding $f_{\rm iso}$ must be recomputed from Eq. (190).

**Local definition of $f_{\rm polar}$ (non-negotiable in what follows).** In what follows after Eq. (190), $f_{\rm polar}(r)$ is interpreted strictly as a *local* near-horizon delivery fraction, consistent with Eq. (189): $\dot{M}_{\rm pol}(r)$ and $\dot{M}_{\rm in}(r)$ are evaluated at the same radius $r$ (with $r$ taken to be a near-horizon control surface when applying the maintenance criterion). Accordingly, $f_{\rm polar}(r)$ must not be conflated with any *global* or far-out supply fraction defined relative to a large-radius inflow boundary. If a far-out supply fraction is required elsewhere, it must be introduced as a separate symbol (e.g. $f^{(\rm sup)}_{\rm polar}$) and defined explicitly at the point of use. .

This separation matters because the effective geometry is set by the angular distribution of the flux entering the near-horizon region, not by how large that flux is compared to the mass supplied at very large radius.

### 7.3 Maintenance criterion

Because the axially-concentrated inflow structure is driven rather than stationary, its persistence is controlled by competition between a driving timescale and a relaxation timescale.

**Driving timescale.** The driving timescale $t_{\rm drive}$ characterizes how rapidly anisotropic inflow replenishes the near-horizon, polar-weighted mass-energy content. Define a near-horizon control volume $\mathcal{V}_{\rm nh}$ (with declared radial extent $r \in (r_{\rm H}, r_{\rm nh}]$ and angular span matching $\Omega_{\rm pol}$ for the polar-supplied component), and let Here $r_{\rm nh}$ is a declared exterior control radius satisfying $r_{\rm nh} > r_{\rm H}$, and the lower endpoint is an exterior limit $r \downarrow r_\mathrm{H}$ rather than an evaluation point at $r = r_\mathrm{H}$ (Eq. (7)).

$$M_\mathrm{nh} \equiv \int_{\mathcal{V}_\mathrm{nh}} \rho\, dV \tag{191}$$

be the characteristic rest-mass content relevant to maintaining the polar-weighted supply structure. Then

$$t_\mathrm{drive} \equiv \frac{M_\mathrm{nh}}{\dot{M}_\mathrm{pol}}. \tag{192}$$

To make $t_\mathrm{drive}$ evaluable without introducing a tunable loading factor, fix the near-horizon control volume by definition: it contains one dynamical time's worth of the local inflow,

$$M_\mathrm{nh} \equiv \dot{M}_\mathrm{in}\, t_\mathrm{dyn}(r_\mathrm{nh}). \tag{193}$$

This choice removes an adjustable reservoir-to-throughflow ratio from the maintenance test. A conservative choice for the dynamical time is the local orbital time scale,

$$t_\mathrm{dyn}(r) \equiv 2\pi \left(\frac{r}{r_\mathrm{g}}\right)^{3/2} t_g, \qquad t_g \equiv \frac{GM}{c^3}, \qquad r_\mathrm{g} \equiv \frac{GM}{c^2}. \tag{194}$$

Using Eq. (189) and Eq. (193), the driving time becomes

$$t_\mathrm{drive} = \frac{\dot{M}_\mathrm{in}\, t_\mathrm{dyn}}{f_\mathrm{polar}\, \dot{M}_\mathrm{in}} = \frac{1}{f_\mathrm{polar}}\, t_\mathrm{dyn}(r_\mathrm{nh}). \tag{195}$$

This expression makes the control explicit: for fixed $r_\mathrm{nh}$, the ability to maintain a polar-weighted structure strengthens as $f_\mathrm{polar}$ increases.

**Relaxation timescale.** The relaxation timescale $t_\mathrm{relax}$ characterizes how rapidly directional structure in the near-horizon stress-energy decays when not actively maintained. Multiple mechanisms can contribute, and the effective relaxation time is set by the fastest relevant channel. It is therefore useful to express relaxation as a sum of rates,

$$\frac{1}{t_\mathrm{relax}} \equiv \frac{1}{t_\mathrm{qnm}} + \frac{1}{t_\mathrm{mix}} + \frac{1}{t_\mathrm{adv}} + \cdots, \tag{196}$$

where:

- $t_\mathrm{qnm}$ is the characteristic damping time of near-horizon spacetime and field perturbations (ringdown-scale relaxation).
- $t_\mathrm{mix}$ represents turbulent, magnetic, or interchange-driven angular redistribution of the supplied mass-energy.

- $t_{\mathrm{adv}}$ captures advective replacement or clearing of anisotropic structures by the background flow.

A physically relevant relaxation time for the maintenance criterion must represent redistribution and decay of the *matter and charge-anisotropy state* (the directional structure of supplied stress-energy) rather than the damping time of metric perturbations. Accordingly, while $t_{\mathrm{qnm}}$ is a meaningful timescale for spacetime ringdown, it is not adopted here as the controlling channel for $t_{\mathrm{relax}}$ in the maintenance gate.

For this manuscript, we therefore adopt a single, uniform, scale-tied prescription that anchors relaxation to the local dynamical time at the declared near-horizon control radius $r_{\mathrm{nh}}$ used in the maintenance evaluation:

$$t_{\mathrm{relax}} \equiv t_{\mathrm{dyn}}(r_{\mathrm{nh}}) = 2\pi \left(\frac{r_{\mathrm{nh}}}{r_g}\right)^{3/2} t_g, \qquad t_g \equiv \frac{GM}{c^3}. \tag{197}$$

Any alternative estimate of $t_{\mathrm{relax}}$ may be reported only as a sensitivity calculation and must not alter the declared maintenance-gate prescription used for outcome reporting within this manuscript.

**A forced-damped maintenance condition.** Define a dimensionless anisotropy amplitude $\mathcal{A}(t)$ that increases with polar concentration. One explicit choice is based on the polar fraction itself:

$$\mathcal{A} \equiv \frac{f_{\mathrm{polar}} - f_{\mathrm{iso}}}{1 - f_{\mathrm{iso}}}, \qquad f_{\mathrm{iso}} \equiv \frac{\Omega_{\mathrm{pol}}}{4\pi}, \tag{198}$$

so that $\mathcal{A} = 0$ corresponds to an isotropic inflow with the same polar-cap definition, and $\mathcal{A} \to 1$ corresponds to maximal polar concentration.

A minimal forced-damped evolution model for $\mathcal{A}$ is

$$\frac{d\mathcal{A}}{dt} = \frac{\mathcal{A}_{\mathrm{drive}}}{t_{\mathrm{drive}}} - \frac{\mathcal{A}}{t_{\mathrm{relax}}}, \tag{199}$$

where $\mathcal{A}_{\mathrm{drive}}$ is the anisotropy amplitude implied by the upstream supply under the prevailing shearing regime. The steady-state amplitude is

$$\mathcal{A}_{\mathrm{ss}} = \mathcal{A}_{\mathrm{drive}} \frac{t_{\mathrm{relax}}}{t_{\mathrm{drive}}}. \tag{200}$$

**Physical meaning of the threshold (operational, pre-committed).** For this manuscript, the maintenance threshold is defined operationally in terms of the normalized polar excess $\mathcal{A}$ (Eq. (198)) using the fixed polar-cap convention in Eq. (190):

$$\mathcal{A}_{\mathrm{crit}} \equiv 0.30 \qquad \Longleftrightarrow \qquad f_{\mathrm{polar}} \geq f_{\mathrm{iso}} + 0.30\,(1 - f_{\mathrm{iso}})\,. \tag{201}$$

The value 0.30 is treated as a pre-committed classification threshold for “maintained effective anisotropy” within this document. Any exploration of alternative thresholds must be explicitly labeled as a sensitivity test and must not be used to reclassify outcomes after evaluation.

In the forced-damped representation, "exceeds" corresponds to $\mathcal{A}_{\rm ss} \geq \mathcal{A}_{\rm crit}$, yielding the maintenance condition

$$\frac{t_{\rm drive}}{t_{\rm relax}} \leq \frac{\mathcal{A}_{\rm drive}}{\mathcal{A}_{\rm crit}} \;\Rightarrow\; \text{maintenance of effective anisotropy.} \tag{202}$$

This threshold is physically motivated: it is the forced-response condition for a driven, damped anisotropy to remain above a specified dynamical level. It explicitly encodes that anisotropy is not an equilibrium property of Kerr, but a maintained consequence of continued asymmetric mass-loading.

**Intermittency and duty-cycle form.** If the driving is intermittent, with an "on" fraction $\delta$ over times long compared to $t_{\rm dyn}$ but comparable to or shorter than $t_{\rm relax}$, then Eq. (199) implies an effective replacement

$$\frac{1}{t_{\rm drive}} \;\rightarrow\; \frac{\delta}{t_{\rm drive}}, \tag{203}$$

so the maintenance criterion becomes stricter by a factor $1/\delta$. This provides the required timescale-based statement: when driving ceases, anisotropy decays on $t_{\rm relax}$; when driving is sporadic, the effective maintenance depends on duty cycle as well as instantaneous inflow geometry.

### 7.4 Connection to jet length and stability

Maintained polar-weighted inflow directly affects the polar funnel environment from which relativistic jets are launched and sustained. The causal chain is:

polar-weighted ion inflow $\longrightarrow$ sustained anisotropic mass and energy loading near the horizon $\longrightarrow$ stabilized funnel boundary conditions and reduced equatorial disruption $\longrightarrow$ enhanced collimation and persistence of the jet.

Jet enhancement is treated here as a consequence of maintained anisotropy, not as an independent assumption and not as an effect of intrinsic spacetime deformation. The effective role of the axially-concentrated inflow geometry is to impose a persistent, axis-weighted supply of rest mass and momentum that shapes the near-horizon stress-energy distribution feeding the jet-launch region.

**P5 (jet extent): quantitative, falsifiable statement.** To match the falsifiability standard used for P1–P4 (Section 5) and the pre-committed protocol language (Section 8), the jet-extent claim in this subsection is bound to an explicit prediction tied directly to the maintenance criterion (Eq. (202)). Define the dimensionless maintenance margin

$$\mathcal{M}_{\rm maint} \;\equiv\; \frac{\mathcal{A}_{\rm drive}}{\mathcal{A}_{\rm crit}} \frac{t_{\rm relax}}{t_{\rm drive}}, \tag{204}$$

so that Eq. (202) is equivalent to $\mathcal{M}_{\rm maint} \geq 1$.

**Reference jet extent comparator.** To keep P5 explicitly comparator-based (not a fixed jet-multiplier claim), adopt a conservative reference extent

$$L_{\rm jet,ref} \;\equiv\; 10^6\, r_g, \qquad r_g \equiv \frac{GM}{c^2}. \tag{205}$$

**Hybrid Kerr–Coulomb length multiplier (structure match to the companion derivation).** The conservative comparator in Eq. (205) is retained, but the *multiplier structure* is decomposed so it does not read as a fixed factor. Following the companion jet-length scaling form (where length scales with a CS stability multiplier and a spin factor), define a hybrid spin multiplier

$$\mathcal{M}_{\rm hyb}(a_*) \;\equiv\; M_{\rm spin}(a_*)\, \kappa_{\rm align}(a_*), \tag{206}$$

with

$$M_{\rm spin}(a_*) \;\equiv\; \left[\frac{\Omega_H(a_*)^2\, S(a_*)}{\Omega_H(a_0)^2\, S(a_0)}\right]^{1/2}, \qquad S(a_*) \equiv \frac{1}{1+2a_*^2}, \qquad a_0 \equiv 0.5, \tag{207}$$

and

$$\kappa_{\rm align}(a_*) \equiv \begin{cases} 1, & a_* \le 0.5, \\ 1 + 0.2\, \dfrac{a_* - 0.5}{0.5}, & 0.5 < a_* < 1, \end{cases} \qquad \kappa_{\rm align} \le 1.2. \tag{208}$$

Here $\Omega_H(a_*)$ is the Kerr horizon angular frequency (with $r_H$ defined in Eq. (7)):

$$\Omega_H(a_*) \equiv \frac{a_*\, c}{2\, r_H(a_*)}. \tag{209}$$

No universal multiplier (for example a constant 2×) is asserted: $\mathcal{M}_{\rm hyb}(a_*)$ is explicitly spin-dependent and capped in alignment gain.

$$L_{\rm jet,pred} \;\equiv\; \mathcal{M}_{\rm maint}\, \mathcal{M}_{\rm hyb}(a_*)\, L_{\rm jet,ref}, \tag{210}$$

**Clarification: P5 does not assert a fixed jet-length multiplier.** Equation (210) is a comparator mapping used by the locked evaluation protocol (Pre-Committed Evaluation Protocol (Anti-Post-Hoc)): it scales the reference threshold $L_{\rm jet,ref}$ by the evaluated maintenance margin $\mathcal{M}_{\rm maint}$. No universal jet-length multiplier (for example, a constant 2× factor) is asserted here. In particular, $\mathcal{M}_{\rm maint}$ is computed from declared inputs and can be $< 2$ even under optimistic bounds; for M87* (Worked Examples and Source-Grounded Outputs), the most permissive bound implies $L_{\rm jet,pred} \le 1.87 \times 10^6\, r_{\rm g}$ for $f_{\rm polar} \le 1$ (Eq. (247) combined with Eq. (205)).

We bind P5 to the following pre-committed prediction:

P5: For systems that satisfy $\mathcal{M}_{\rm maint} \ge 1$ under declared inputs, the observed jet extent satisfies $L_{\rm jet} \ge L_{\rm jet,pred}$ (Eq. (210)).

**Protocol classification and falsification trigger for P5.** P5 is evaluated only when both components of Eq. (210) can be bounded from declared inputs: (i) $\mathcal{M}_{\text{maint}}$ can be bounded, which requires an externally defensible bound for $t_{\text{relax}}$, and (ii) the spin parameter $a_*$ is bounded (as a point estimate or an interval), so that $\mathcal{M}_{\text{hyb}}(a_*)$ is evaluable.

When $a_*$ is bounded as an interval $a_* \in [a_{\min}, a_{\max}]$, the protocol evaluates the implied prediction interval

$$L^-_{\text{jet,pred}} \equiv \mathcal{M}_{\text{maint}}\, \mathcal{M}_{\text{hyb}}(a_{\min})\, L_{\text{jet,ref}}, \qquad L^+_{\text{jet,pred}} \equiv \mathcal{M}_{\text{maint}}\, \mathcal{M}_{\text{hyb}}(a_{\max})\, L_{\text{jet,ref}}.$$

If $a_*$ is given as a point estimate, set $a_{\min} = a_{\max} = a_*$ so that $L^-_{\text{jet,pred}} = L^+_{\text{jet,pred}} = L_{\text{jet,pred}}$.

Under those conditions:

- **PASS (P5):** $\mathcal{M}_{\text{maint}} \geq 1$ and $L_{\text{jet}} \geq L^+_{\text{jet,pred}}$ under the declared comparator.
- **FAIL (P5):** $\mathcal{M}_{\text{maint}} \geq 1$ and $L_{\text{jet}} < L^-_{\text{jet,pred}}$ under the declared comparator.
- **DATA-INSUFFICIENT (P5):** $L_{\text{jet}}$ is not measurable under the declared comparator, or $\mathcal{M}_{\text{maint}}$ cannot be bounded because $t_{\text{relax}}$ is not externally constrained under the locked inputs, or $a_*$ is not bounded under admissible sources, or the bounds overlap ($L^-_{\text{jet,pred}} \leq L_{\text{jet}} < L^+_{\text{jet,pred}}$).

Falsification is triggered only in the **FAIL (P5)** case above: $\mathcal{M}_{\text{maint}} \geq 1$ is established under declared bounds, $a_*$ is bounded under admissible sources, and yet $L_{\text{jet}} < L^-_{\text{jet,pred}}$ under the declared comparator.

**Why maintained anisotropy can lengthen jets.** Very large jet extents require sustained power delivery and a sufficiently steady launch direction over long durations. A maintained polar-weighted inflow increases the probability that the funnel boundary conditions remain coherent over timescales far exceeding local dynamical times. This supports persistent collimation and reduces disruptive variability in the jet base. The point is conditional: the mechanism contributes only when Eq. (202) is satisfied for declared inputs.

**Why maintained anisotropy can increase stability and straightness.** If near-horizon mass-loading repeatedly swings between polar-weighted and equator-dominated states on timescales shorter than or comparable to $t_{\text{relax}}$, then the jet base experiences variable inertia and variable boundary conditions, which promotes instability and changes in collimation. When anisotropy is maintained, the polar supply becomes a consistent driver and equatorial variability becomes less controlling at the jet base. This supports straighter, more stable jets over long baselines, again conditionally on the maintenance inequality.

**Why alignment persistence follows from the same condition.** In Kerr spacetime the spin axis is a preferred direction for the funnel region in magnetized flows. A sustained axis-weighted inflow reinforces that preference by continuously feeding the polar region and reducing the likelihood that the dominant mass-loading direction wanders on timescales shorter than $t_{\mathrm{relax}}$. The alignment claim here is therefore not universal: it is a statement about the persistence of the boundary conditions when the driven anisotropy is maintained.

**Explicit decay boundary.** If Coulomb Shearing shuts off, or if the polar delivery fraction drops such that $t_{\mathrm{drive}}/t_{\mathrm{relax}}$ exceeds the threshold in Eq. (202), then the effective anisotropy must decay on $t_{\mathrm{relax}}$. Any jet enhancement attributable specifically to sustained anisotropic mass-loading must weaken correspondingly. Jet power may persist for some interval due to stored magnetic flux and inertia in the existing outflow, but the particular contribution identified here is not assumed to survive once the driving condition fails.

**Observable handles implied by the causal chain.** Within the conditional framework above, the observables most directly affected are:

- Jet length relative to black hole mass under comparable external confinement.
- Long-term stability and straightness (reduced base variability signatures).
- Persistence of alignment with the spin axis over times long compared to $t_{\mathrm{relax}}$.

These are not asserted as universal outcomes. They are consequences expected only in systems where the maintenance condition is plausible under declared physical inputs.

### 7.5 Consistency check with GRMHD-derived inputs

Several worked examples use macroscopic near-horizon plasma state estimates $(B, n_e, T_e)$ that are commonly derived from GRMHD or GRRMHD modeling. This raises a potential circularity concern: if GRMHD simulations (which do not include Coulomb Shearing) already reproduce plausible field and plasma conditions and produce jets through standard mechanisms (e.g. Blandford–Znajek), why introduce Coulomb Shearing at all?

**Scope separation: GRMHD sets macroscopic conditions; Coulomb Shearing is species-resolved post-processing.** This manuscript adopts the following stance (Option A):

> GRMHD provides an approximate macroscopic background state and flow geometry used to evaluate the Coulomb Shearing admissibility inequalities. Coulomb Shearing is then evaluated as a species-resolved post-processing layer that predicts composition and polarimetric observables not determined by single-fluid GRMHD closures.

Under this separation, there is no circularity: the paper does not claim that Coulomb Shearing is required to generate the magnetic field configuration, to launch a jet, or to reproduce bulk dynamical morphology. The core claim is narrower and testable: *given* a near-horizon environment with declared macroscopic conditions, enforced electron–ion co-motion can fail along mapped polar-adjacent trajectories, producing a species-stratified outflow and a set of quantitative polarimetric discriminants tied to computed windows.

**Why Coulomb Shearing remains necessary as an explanation layer.** Single-fluid GRMHD (with or without approximate radiation modules) does not evolve species-dependent forcing and therefore does not generically predict:

- a computed electron-loading channel defined by a closure-failure criterion $\Delta x/(\kappa\lambda_D)$,
- species-resolved survival/stripping probabilities $P_s(Z)$ linked to those windows,
- the mapped-zone quantitative inequalities (P1–P4) defined in Explicit predictions (quantitative),
- probabilistic outcome classifications under the pre-committed evaluation protocol.

Therefore, agreement between GRMHD and observations at the level of bulk jet morphology or macroscopic $B$-field strength does not eliminate the need for a mechanistic account of species stratification and mapped-zone observables. Coulomb Shearing supplies that missing species-resolved mechanism and binds it to falsifiable constraints.

**Input provenance is not theory-dependence.** Using GRMHD-derived $(B, n_e, T_e)$ as input estimates does not mean the theory is tuned to GRMHD. The admissible source classes for numerical inputs are declared separately (Appendix C), and any input set meeting those provenance rules may be substituted. The falsification criteria in M87* quantitative prediction packet (current comparators and required precision) are defined in terms of computed windows and measured observables; they do not require GRMHD to be "right" for the theory to be testable.

**What would change under a future GRMHD implementation (roadmap, not required here).** While this manuscript does not require modification of GRMHD codes to state or test the mechanism, a fully self-consistent implementation would require at minimum a two-species (or multifluid/kinetic) treatment with species-dependent radiation forces and non-ideal electromagnetic terms on polar-adjacent trajectories. The expected qualitative impact would be localized: altered charge ratios and radiative coupling in the mapped electron-loading channel, potentially modifying $(n_e/n_i)(r)$ and time-variability statistics, while leaving the existence of a BZ-powered jet compatible with standard GRMHD. Such an implementation is a natural follow-on test: if it fails to reproduce the mapped-zone inequalities and composition trends that the post-processing evaluation predicts, Coulomb Shearing is disfavored.

In summary, GRMHD consistency is treated as a background-state sourcing issue, not a proof or a premise. Coulomb Shearing is evaluated as a species-resolved mechanism layered on top of an externally constrained macroscopic environment and is judged by its unique, quantitative, mapped-zone predictions.

# 8 Pre-Committed Evaluation Protocol (Anti-Post-Hoc)

## 8.1 Fixed selection criteria based on data availability

This protocol binds evaluation logic before any calculations, comparisons, or numerical outputs are disclosed. It exists to prevent post-hoc fitting, outcome-driven object selection, and retroactive reclassification of failure.

**Outcome independence principle.** Evaluation objects are selected solely on the basis of whether the required physical inputs and required observables are available from external sources. Expected agreement or disagreement with Coulomb Shearing predictions plays no role in inclusion. Known jet length, morphology, or "impressiveness" plays no role in inclusion beyond the binary requirement that the outflow is observationally accessible at all.

This manuscript explicitly includes three target classes under the same data-availability rules: (i) classical quasar-class AGN, (ii) nearby calibration systems, and (iii) JWST compact high-$z$ objects including *Little Red Dots* (LRDs). The label *LRD* is descriptive only and does not relax eligibility, admissibility, or PASS/FAIL logic.

Interpretive frames such as quasi-star ("black hole star") scenarios are treated as conditional hypotheses. They are evaluated only when their required external inputs can be populated from traceable sources under the locked mapping rules.

**Exterior-domain constraint (horizon bound).** All admissibility evaluations and all worked-example radial bands in this manuscript are restricted to the exterior domain $r > r_{\rm H}$, where $r_{\rm H}$ is the *outer* Kerr event-horizon radius (Eq. (7)). No Coulomb Shearing condition is evaluated or interpreted at $r \leq r_{\rm H}$, and any near-horizon statement is understood as an exterior limit $r \downarrow r_{\rm H}$ rather than as an inclusive statement at the horizon.

This is a protocol-level boundary condition, not an interpretive preference: the predicted consequences (external electron channel loading, composition partition in an escaping jet, and polarization/Faraday signatures) require that the separation occurs in a region causally connected to infinity. At or inside the horizon, outward transport is not available to external observers, so any continued drift is not an observable jet-loading mechanism. Operationally, the horizon functions as an absorbing boundary for the evaluation domain.

Spin is not required as an input for eligibility. If a published dimensionless spin $a_*$ is available and explicitly sourced in the input table, then $r_{\rm H}$ may be evaluated using Eq. (7). If $a_*$ is not available, the protocol does not permit assuming a value. In that case, the conservative Kerr bound $r_{\rm H} \leq 2\,r_{\rm g}$ (valid for all $a_* < 1$, with equality at $a_* = 0$) is used only to ensure that any band written with a lower endpoint $2\,r_{\rm g}$ can be interpreted in a spin-agnostic way as strictly exterior by treating the lower endpoint as the limit $r \downarrow r_{\rm H}$, while still maintaining the strict rule that no quantity is evaluated at $r \leq r_{\rm H}$.

**Exterior-domain gate (hard, audit-enforceable).** A worked example, regime map, or sensitivity sweep is admissible only if every evaluation band and every control-volume extent is stated and implemented as strictly exterior ($r > r_{\mathrm{H}}$). Any instance of an inclusive lower endpoint at $r_{\mathrm{H}}$ (or at $2\,r_{\mathrm{g}}$ without an explicit exterior-limit interpretation) is a protocol violation and must be corrected before any PASS/FAIL assignment is permitted.

**Pre-committed required inputs.** At screening, the extraction audit attempts to populate the following required inputs and observational handles from traceable sources under the locked mapping rules. If any required item below cannot be populated at the declared scale, the object remains in the evaluation set and is classified **DATA-INSUFFICIENT** (it is not removed and not substituted):

- **Mass:** a black hole mass estimate $M$ with uncertainty, or a constrained range suitable for propagation.
- **Distance or redshift:** either a distance estimate $D$ (with uncertainty or bounded range), or a redshift $z$ with a declared cosmology sufficient to map to $D$ under the same cosmology used by the cited source(s) for the observational quantities.
- **Accretion or luminosity proxy:** a bolometric luminosity estimate $L$ with uncertainty, or an accretion proxy that can be expressed as a bounded $L/L_{\mathrm{Edd}}$ envelope under a declared bolometric correction.
- **Outflow access:** observational access to a jet or outflow structure, meaning at least one of:
  - resolved imaging (VLBI, EHT, optical, X-ray, radio lobes),
  - time-domain jet signatures (proper motions, variability tied to jet knots),
  - polarization or rotation measure constraints attributable to the jet or its immediate environment.
- **Plasma or environmental constraints: required symbol set.** The extraction audit attempts to populate the extracted plasma/environment symbols in the required set below from traceable sources, with each extracted constraint tied to a stated physical scale (radius, frequency-defined core location, or explicitly modeled region). Microphysical parameters ($T_e$, $\ln\Lambda$) are governed by the declared priors rule below and are not required to be source-populated for eligibility.
  - **Electron density:** $n_e$ as a value with uncertainty, a bounded interval, or an explicit bound.
  - **Microphysics priors (non-extracted):** $T_e$ and $\ln\Lambda$ are treated as declared microphysical inputs. If a cited source explicitly states a scale-tagged constraint for $T_e$ and/or $\ln\Lambda$ (or an equivalent Coulomb-log convention used in its collisional terms), that value/range is adopted with a source pointer. If not stated, this manuscript pre-commits to the priors

$$10^4\,\mathrm{K} \le T_e \le 10^{11}\,\mathrm{K}, \qquad 10 \le \ln\Lambda \le 30,$$

and propagates these priors in the same Monte Carlo logic as other inputs (with endpoints included in the sensitivity report).

– **Magnetic-field amplitude:** $B$ (or a directly stated proxy that is mapped to $B$ under the locked mapping rules) as a value with uncertainty, a bounded interval, or an explicit bound.

For each symbol above, an acceptable constraint is one of: (i) a direct estimate with uncertainty at a stated scale, (ii) a bounded range derived by the source with explicit assumptions, or (iii) an explicitly stated upper or lower bound. If a later extraction audit reveals that any symbol in the required set cannot actually be populated under the locked mapping and scale rules, the object remains in the evaluation set and is classified **DATA-INSUFFICIENT**.

**Required symbols for eligibility (exhaustive for this manuscript).** Eligibility is determined solely by whether the initial extraction can populate each of the following symbols under the locked mapping rules:

$$\{\, M,\ D\ \big(\text{or } z \text{ with declared cosmology}\big),\ L,\ n_e,\ B\,\}\,. \tag{211}$$

If any symbol in Eq. (211) cannot be populated from traceable sources at the declared scale, the object remains in the evaluation set and is classified **DATA-INSUFFICIENT**.

**Declared (non-extracted) transport convention.** The trajectory residence-time calculations in this manuscript do not treat the transport slowdown factor $\eta$ (Eq. (21)) as an extracted observational input. All reported computations adopt the pre-declared conservative lock

$$\eta = 1 \tag{212}$$

which is identical to Eq. (22) and is applied uniformly across the full evaluation set. Any future manuscript version that relaxes this convention must (i) declare an a priori interval $\eta \in [\eta_{\min}, \eta_{\max}]$ before results, and (ii) re-run the full evaluation set under that revised protocol.

Symbols associated with the non-ideal $E_{\parallel}(r)$ term are governed by protocol when invoked, but they do not enter the eligibility symbol set in Eq. (211) unless the protocol explicitly elevates them to extracted inputs for the full evaluation set.

If any symbol in Eq. (211) cannot be populated from traceable sources at the declared scale, the object remains in the evaluation set and is classified **DATA-INSUFFICIENT**. This rule applies only to the symbols listed in Eq. (211); the declared transport convention $\eta = 1$ (Eq. (212)) is not an extracted eligibility input.

**Declared force model for evaluating $\Delta a$.** All calculations reported in the worked examples evaluate the along-trajectory differential acceleration using the general projected form (Eq. (58)), with radiative coupling modeled as Thomson scattering on electrons (Eq. (38)). Under the along-trajectory projection used by the criterion, the magnetic term $\mathbf{v} \times \mathbf{B}$ is transverse to the streamline and does not contribute to the tangent acceleration component (Eq. (42)). Electromagnetic forcing enters $\Delta a_{\parallel}$ only through an explicitly declared non-ideal $E_{\parallel}(r)$ model (Eq. (43)). Any worked-example run that includes a nonzero $E_{\parallel}(r)$ term must declare the full $E_{\parallel}(r)$ model (or a source-grounded bound) prior to computation and must apply the same force-model choice uniformly across the full evaluation set. Post hoc introduction of electromagnetic terms for any single object is disallowed. The radiative-only Thomson case is the special case $E_{\parallel} = 0$, which recovers Eq. (59).

**Force-model tracks and outcome-classification rule.** This manuscript pre-declares two force-model tracks for evaluating $\Delta a_{\parallel}$:

- **Track T (classification):** Thomson-only radiative specialization with $E_{\parallel} = 0$, so $\Delta a_{\parallel}$ reduces to Eq. (59).
- **Track EM (supplementary):** full projected form in Eq. (58) with a *declared, source-grounded* non-ideal $E_{\parallel}(r)$ model or bound (Eq. (43)).

*Outcome classification in this manuscript is determined solely under Track T.* Track EM is reported when (and only when) an auditable $E_{\parallel}(r)$ model or bound can be populated from traceable sources under the same locked mapping rules. If Track EM cannot be populated for a given object, this is reported explicitly as **DATA-INSUFFICIENT for Track EM**, but it does not change the object's outcome classification under Track T.

Both tracks are defined here, before results. Post hoc introduction of electromagnetic terms for any single object is disallowed: Track EM may be evaluated only when its inputs are declared and sourced in advance and then applied under the same mapping rules used elsewhere.

**Inclusion rule.** If the screening-stage literature check indicates that (i) the outflow-access requirement is satisfied and (ii) each symbol in Eq. (211) can be populated under the locked mapping rules, the object must be included. No discretionary exclusion is permitted once this condition is met. If a later extraction audit reveals that one or more required quantities cannot actually be populated from the cited sources under the mapping rules defined below, the object remains in the evaluation set and is classified **DATA-INSUFFICIENT**. It is not removed or substituted.

**Acceptable data sources.** Eligible inputs may be drawn from (non-exhaustive): peer-reviewed observational literature, mission archives (EHT, JWST, Chandra, XMM-Newton, ALMA, VLA, VLBA, HST, Fermi), and reputable survey catalogs. Non-refereed sources may be used only when they provide primary measurements that are later refereed or when they are mission archive products, and they must be explicitly flagged as such in the input table.

**Irreversibility of inclusion.** Any object that meets the eligibility criteria at the moment this protocol is locked cannot be removed later. Objects cannot be added, removed, or substituted based on agreement or disagreement with predictions.

### 8.2 Declared evaluation objects

**Lock statement.** The evaluation set listed below is final for this manuscript under this protocol. No object may be added, removed, or substituted based on agreement or disagreement with Coulomb Shearing predictions. Any future expansion requires a new manuscript version with a new pre-committed protocol and a complete re-run of all calculations for all objects in the expanded set.

**Screening and extraction distinction.** Inclusion is based on screening for the existence of required input categories in the literature. A subsequent extraction audit may still yield **DATA-INSUFFICIENT** classifications when a required symbol cannot be populated under the locked mapping rules. Such cases remain in the set and are reported as data-insufficient rather than removed.

**Regime-contrast cases (pre-committed).** The evaluation set intentionally includes both high-power jet systems and low-luminosity or dense-environment systems to force the framework to accommodate multiple regimes without object substitution. Two objects in the locked set are designated as regime-contrast cases: Sgr A$^*$ and NGC 1052. Prior to any calculation, these two are expected to be more likely to reside near or within the coupled regime (i.e., reduced or absent predicted electron-loading zone) than the luminous FR II and blazar cases. This designation is fixed and cannot be changed after calculations are performed.

M87$^*$: Messier 87, Virgo A; EHT target. Inputs available at screening: published $M$ estimates, bounded $L/L_{\mathrm{Edd}}$ proxies, resolved jet base imaging, and multi-technique plasma/environment constraints (magnetic field, density, polarization/RM constraints) sufficient for model input population under the mapping rules.

Sgr A$^*$: Galactic Center black hole; EHT target. Inputs available at screening: published $M$ constraints, low-$L/L_{\mathrm{Edd}}$ constraints, resolved horizon-scale emission structure, and near-source plasma constraints from multiwavelength modeling and polarization/RM analyses.

Centaurus A: NGC 5128; nearby radio galaxy. Inputs available at screening: published $M$ estimates, accretion proxies, resolved jet structure across scales, and environmental constraints from extensive radio, X-ray, and polarization observations enabling model input population.

3C 84: NGC 1275 (Perseus A); bright nearby AGN with VLBI-resolved jet. Inputs available at screening: published $M$ estimates, luminosity proxies, high-quality VLBI and polarization data, and literature constraints on jet-base plasma parameters.

NGC 1052: Nearby LLAGN with compact twin-jet and rich absorption/polarization literature. Inputs available at screening: published $M$ estimates, accretion proxies, VLBI-resolved jet, and plasma/environment constraints (including Faraday and absorption-based bounds) suitable for model input population.

Cygnus A: 3C 405; powerful FR II radio galaxy with well-characterized lobes and jet. Inputs available at screening: published $M$ estimates, luminosity constraints, resolved jet and lobe structure, and host-environment constraints enabling bounded model inputs.

3C 273: Bright quasar with extensive VLBI, core-shift, and multiwavelength constraints. Inputs available at screening: published $M$ estimates, bolometric luminosity constraints, resolved jet kinematics, and literature constraints on magnetic field and density near the radio core.

3C 279: Bright blazar with long-term VLBI monitoring and polarization constraints. Inputs available at screening: published $M$ ranges, luminosity proxies, resolved jet kinematics, and bounded jet-base plasma constraints from VLBI and spectral modeling literature.

### 8.3 Locked input mapping and uncertainty propagation

This protocol fixes how literature values become computational inputs. It prohibits per-object tuning, selective sourcing, and retroactive changes.

**Standardized input table schema.** For every evaluated object, an input table must be constructed using the same schema. Each row corresponds to one symbol used in computation. The table must include:

- **Symbol:** the exact symbol used in the computation.
- **Physical meaning:** a concise definition of the quantity.
- **Units:** SI or explicitly declared astrophysical units with conversion shown.
- **Numerical value or allowed range:** either a point estimate with uncertainty or a bounded interval.
- **Exact source pointer:** paper and location (section label if present in the source, page number, table number, figure number, or equation number), sufficient for independent replication.
- **Provenance type:** direct measurement, derived estimate, model fit, or bound.
- **Scale annotation:** the physical scale of the estimate (radius, frequency, epoch) when applicable.

No computed quantity may appear in results unless its inputs are traceable to this table.

**Locked rules for multiple literature values.** When multiple credible literature values exist for the same symbol, the following rules apply uniformly:

1. **No single-source cherry-picking.** All eligible values must be recorded in a source ledger for that symbol.

2. **Envelope rule (default).** The computational range for that symbol is the union envelope across eligible values after unit normalization and after applying any explicitly stated selection constraints (for example, frequency band or epoch relevance) that are declared in advance in this protocol.

3. **Equal-weight mixture (optional but fixed).** If the analysis uses full distributions rather than envelopes, each eligible literature value is represented as a distribution using its reported uncertainty model, and the overall input distribution is an equal-weight mixture over sources. Weighting by agreement with outputs is prohibited.

The protocol must declare in advance whether the envelope rule or the equal-weight mixture rule is used. The same choice applies to all objects and all symbols.

**Locked scale-mapping rule.** Many plasma/environment estimates are scale-dependent. Mapping is permitted only under the following pre-committed constraints:

1. If a source provides an estimate at the required scale for computation, use it directly.

2. If a source provides an estimate at a different scale and explicitly provides a scaling law and parameter values, then scaling is permitted using that source-provided law with uncertainties propagated.

3. If no explicit scaling law is provided by the source, extrapolation is prohibited. The required symbol is treated as missing, and the object becomes eligible for **DATA-INSUFFICIENT** if the symbol is required for the computation.

This rule prevents hidden tuning through discretionary extrapolation.

**Declared observation band (data-anchored comparator window).** For each object, prior to any computation, the evaluation must declare a single contiguous *observation band* $\mathcal{B}_{\text{obs}}$ defined as the maximal interval along the observable outflow for which the primary comparator products required by P1–P3 are simultaneously available under the stated measurement rules (matched beam, aligned registration, and stated frequency coverage sufficient to construct RM where required).

The microphysical shearing computation is executed on a separately declared *source band* $\mathcal{B}_{\text{src}}$ (defined under the downstream-testing rule below), but the primary observables P1–P3 are always evaluated on $\mathcal{B}_{\text{obs}}$. All observational constraints used to interpret P1–P3 must therefore be scale-tagged within $\mathcal{B}_{\text{obs}}$, and any mapping between $\mathcal{B}_{\text{src}}$ and $\mathcal{B}_{\text{obs}}$ must satisfy the declared transport-mapping requirement.

**Locked uncertainty propagation.** Uncertainty handling is fixed across all objects:

- **Propagation method:** full propagation via Monte Carlo sampling over the declared input distributions or ranges, using a fixed sampling size $N = 10{,}000$ and a fixed random seed $s_0 = 314159$ (recorded in the computation log and unchanged across all objects).

- **Range representation:** each predicted quantity must be reported as a central 90% credibility interval (5th to 95th percentile) and a median. The 90% level is used uniformly for all predicted and observed intervals in the agreement and contradiction tests.
- **Physical bounds:** distributions are truncated to physical domains (for example, non-negative densities, $0 \le f \le 1$ for fractions) in the same way for all objects.

No per-object adjustment of uncertainty handling is permitted. Any change to $N$, the credibility level, or distributional assumptions requires a protocol revision and a full re-run for all objects.

**Uniformity statement.** Input mapping, source aggregation, scale mapping, and uncertainty propagation are applied identically to every object in the locked evaluation set. No outcome-based source selection is permitted.

**Locked sensitivity report (required for any PASS/FAIL assignment).** Monte Carlo propagation reports the combined effect of input uncertainty, but it does not isolate which inputs control the shearing-window existence or collapse. Therefore, for each evaluated object, a one-at-a-time sensitivity report is mandatory. A PASS or FAIL classification is not admissible under this protocol unless the sensitivity report is produced and archived alongside the input table.

**Sensitivity parameters (fixed set).** The sensitivity report must perturb the following inputs one-at-a-time, using the same locked mapping and scale rules as the main computation:

$$x \in \{\, T_e,\ n_e,\ \ln\Lambda,\ B \,\}\,.$$

No other parameters may be substituted without a protocol revision.

**Perturbation rule (no discretion).** For each parameter $x$ above:

- If the cited source provides a $1\sigma$ uncertainty, evaluate $x \longrightarrow x \pm \sigma_x$.
- If the cited source provides only bounds, evaluate $x \longrightarrow x_{\min}$ and $x \longrightarrow x_{\max}$.
- If $x \in \{T_e, \ln\Lambda\}$ is governed by the declared priors rule (i.e. no scale-tagged source constraint is available), evaluate $x$ at the prior endpoints used in the main run.
- If $x \in \{n_e, B\}$ cannot be populated under the locked mapping rules at the declared scale, the object cannot be assigned PASS/FAIL and is **DATA-INSUFFICIENT**.

All perturbations are one-at-a-time (holding all other inputs fixed at their median or midpoint values under the same locked distributions used for Monte Carlo).

**Required sensitivity outputs (fixed reportables).** For each evaluated species/state in Species-Dependent Shearing Ranges and Probabilities, the sensitivity report must include, for the baseline and for each perturbed case:

- the computed admissibility outcome $\chi_s$,
- the computed window endpoints $r_{\min}(Z)$ and $r_{\max}(Z)$ (or an explicit declaration that the window is empty),
- the computed probability $P_s(Z)$ under the declared $f(v)$,
- and the fractional shifts relative to baseline:

$$\Delta_x r_{\min}/r_{\min}, \qquad \Delta_x r_{\max}/r_{\max}, \qquad \Delta_x P_s.$$

For $B$, perturbations must propagate through the same electromagnetic contribution to $\Delta a_{\parallel}$ used in the main run; no analytic exponent is assumed unless it is explicitly part of the adopted $\Delta a_{\parallel}$ model.

### 8.4 Three-outcome rule

This protocol permits exactly three outcomes for each evaluated object: **PASS**, **FAIL**, or **DATA-INSUFFICIENT**. These outcomes are irreversible.

**Definition 1** (Outcome classification)**.** *An evaluated system is classified as:*

- ***PASS*** *if:*
  - *all required inputs are populated from the locked input table under the locked mapping rules,*
  - *the calculations are executed with the locked uncertainty propagation under Track T,*
  - *and the primary predicted observables under Track T are consistent with the corresponding observations under the locked agreement test.*
- ***FAIL*** *if:*
  - *all required inputs are populated from the locked input table under the locked mapping rules,*
  - *the calculations are executed with the locked uncertainty propagation under Track T,*
  - *and at least one primary predicted observable under Track T is contradicted by observation under the locked contradiction test.*
- ***DATA-INSUFFICIENT*** *if:*
  - *one or more required inputs for Track T cannot be populated under the locked mapping rules* or *one or more required observational comparators are unavailable for evaluating the Track T primary observables,*

– *each missing quantity is explicitly identified by symbol,*

– *and the specific observation or measurement required to render the case PASS or FAIL is explicitly stated.*

*Once assigned, the classification is final under this protocol.*

**Modal language and data sufficiency (interpretation rule).** All strong modal statements used elsewhere in this manuscript (e.g., "must occur," "required," "unavoidable," "falsifiable") are to be interpreted as *protocol-conditional*. Specifically, such statements apply only when: (i) the declared force model and closure definitions are in effect, (ii) the system under consideration satisfies the admissibility criteria, and (iii) sufficient observational data exist to score the relevant tests. When these conditions are not met, the correct outcome under this protocol is **DATA-INSUFFICIENT**, and no claim of confirmation, refutation, or physical inevitability is implied. Accordingly, "falsifiable" in this manuscript means *falsifiable in principle under defined data sufficiency*, not asserted falsification or confirmation in the absence of scorable observables.

**Failure-capable meaning under this protocol.** The subtitle term "failure-capable" is meant in the operational sense defined here: Coulomb Shearing can *fail* when (a) the system is admissible under the declared model, (b) the relevant observables are defined and measurable under the viewing-geometry rules, and (c) the measured values violate the pre-committed inequalities. In that situation the correct classification is **FAIL**, and the outcome may not be reinterpreted as mere inapplicability. **DATA-INSUFFICIENT** is reserved for cases where the decisive tests cannot be scored (e.g., missing Faraday-active screen, inadequate resolution, insufficient frequency coverage, or absent matched-beam transverse observables), not as a blanket shield against failure.

**Falsification leverage requirement (PASS admissibility criterion).** A **PASS** classification requires not only admissibility and non-contradiction, but also *discriminatory leverage*: the computed prediction must imply at least one scorable inequality with a finite, nontrivial separation relative to stated measurement uncertainty and beam systematics. If the computed admissible windows or parameter ranges are so broad that all plausible measurements satisfy the inequalities within uncertainty, then the correct classification for those predictions is **DATA-INSUFFICIENT** (because the data cannot, in practice, discriminate), and the manuscript must not present the case as a practically unfalsifiable PASS. This rule does not add a new outcome class; it constrains when **PASS** is permitted.

**Assumptions at a glance (main-text disclosure).** The protocol scores COULOMB SHEARING under explicitly declared assumptions so that disagreements about microphysics, data sufficiency, or regime choice surface as **FAIL** or **DATA-INSUFFICIENT** rather than as hidden degrees of freedom. The core assumptions used across the manuscript are:

1. **Declared force track.** The forcing model used for $\Delta a_{\parallel}$ is declared per worked example (e.g., Thomson-only vs. EM-inclusive) and is not tuned post hoc (see Appendix D input packet rules).
2. **Coupling model.** Electron–ion momentum exchange is represented by an effective coupling time $t_c$ with regime branching rules; stronger-than-Coulomb microphysical coupling moves cases toward inadmissibility rather than being assumed away (see Differential Forcing and the Onset of Coulomb Shearing and Appendix B).
3. **Screening scale and closure definition.** Screening enters through $\lambda_D$ and the closure failure criterion is evaluated as a displacement inequality under the declared kernel; symbols and definitions are fixed in Appendix A and Appendix B.
4. **Geometry and measurability.** Transverse RM/polarization inequalities are defined only when a Faraday-active screen geometry is established at the available resolution; otherwise, the correct disposition is **DATA-INSUFFICIENT** for those tests (see Observable Consequences: Jets and Stratification).
5. **Source admissibility and provenance.** Numerical inputs must satisfy Appendix C source rules; worked examples must use Appendix D input packets with no prohibited derived quantities.

**Interpretation of PASS under uncertainty.** A **PASS** classification means only that the applicable observables are consistent with the predictions *within the declared bounds* under the locked comparison rule. It does not imply that parameters are tightly constrained, that the mechanism dominates other processes, or that alternative explanations are excluded. The strength of the conclusion is carried by the width of the reported intervals and margins, not by the PASS label itself.

**Locked agreement and contradiction tests.** For quantitative predicted observables, agreement is defined by non-empty overlap between the fixed-level predicted interval and the fixed-level observed interval (both at the same declared credibility level). Contradiction is defined by disjoint intervals at that same level. For binary predicted observables (presence or absence), agreement requires matching presence or absence, and contradiction requires mismatch. The credibility level and overlap rule are fixed by this protocol and applied uniformly.

**Source-band and observation-band mapping (downstream testing rule).** The microphysical shearing calculations in this manuscript are executed in a declared *source band* near the central engine, while the primary observables (P1–P3) may be measured in a declared *observation band* farther downstream. This is permitted because Coulomb Shearing is evaluated as a *partitioning-and-transport* claim: species partition is created in the source band and then advected outward to the observation band.

**Definitions (bands).** For each evaluated object, every worked example and every PASS/FAIL assignment must explicitly declare:

$$\mathcal{B}_{\rm src} \equiv [r_{\rm src,min},\, r_{\rm src,max}], \qquad \mathcal{B}_{\rm obs} \equiv [r_{\rm obs,min},\, r_{\rm obs,max}]. \tag{213}$$

All Coulomb Shearing microphysics (windows, $\chi_s$, and $P_s$) are computed on $\mathcal{B}_{\rm src}$. All primary observables (P1–P3) are evaluated on $\mathcal{B}_{\rm obs}$ using the measurement definitions pre-committed under P1–P3.

**Transport-mapping requirement (no scale ambiguity).** A PASS/FAIL assignment is admissible only if the worked example provides a traceable mapping statement that ties $\mathcal{B}_{\rm obs}$ to downstream transport from $\mathcal{B}_{\rm src}$ along the same outflow. This mapping statement must include all of the following, using traceable external sources (or source-grounded bounds):

- **Downstream ordering:** a bound that places the observation band outside the source band along the outflow, written as a strict inequality in the same $r$-coordinate convention used in the worked example:
$$r_{\rm obs,min} \; > \; r_{\rm src,max}.$$
- **Identity-of-component:** an identification that the emission region used for P1–P3 belongs to the jet/outflow component sourced by the central engine (for example, via core identification, core-shift distance-to-engine mapping, or kinematic association of the measured feature with the core).
- **Preservation assumption (declared, testable):** a declared statement of whether species partition created in $\mathcal{B}_{\rm src}$ is assumed to be preserved to $\mathcal{B}_{\rm obs}$. In this manuscript version, the default is *preservation by advection* (no forced re-neutralization term is introduced), and any observed absence of the predicted tracer-partition signatures in $\mathcal{B}_{\rm obs}$ counts against the framework under the locked contradiction test rather than being dismissed as a modeling loophole.

If any one of the three items above cannot be stated from traceable sources or traceable bounds for a given object, then that object is **DATA-INSUFFICIENT** for P1–P3 under this protocol (even if microphysical calculations exist in $\mathcal{B}_{\rm src}$), because the primary observables would not be anchored to an auditable downstream mapping.

**Reason this rule exists.** This rule prevents mismatched-scale testing while still allowing legitimate downstream evaluation of jet signatures, which are not expected to be directly measurable at the microphysical source radii for most systems.

**Pre-declared primary observables.** Outcome classification is determined only by the following observables, evaluated using the measurement definitions stated with P1–P4 in Observable Consequences: Jets and Stratification. If any required comparator needed to evaluate a primary observable is unavailable under the locked mapping rules, the object is classified **DATA-INSUFFICIENT**.

- **P1:** spine–sheath $|\mathrm{RM}|$ ordering evaluated on $\mathcal{B}_{\mathrm{obs}}$, within the subset of $\mathcal{B}_{\mathrm{obs}}$ identified as the computed electron-loading zone under the declared transport mapping.
- **P2:** transverse *depolarization contrast* within the computed electron-loading zone under matched beam, matched registration, and a fixed two-frequency pair available on $\mathcal{B}_{\mathrm{obs}}$. Define the depolarization ratio

$$\mathrm{DP}(R) \equiv \frac{p_{\mathrm{lin}}(R, \nu_{\mathrm{low}})}{p_{\mathrm{lin}}(R, \nu_{\mathrm{high}})}, \qquad \nu_{\mathrm{low}} < \nu_{\mathrm{high}}.$$

  The predicted ordering is $\mathrm{DP}_{\mathrm{spine}} > \mathrm{DP}_{\mathrm{sheath}}$ (weaker Faraday depolarization in the electron-loaded channel).
- **P3:** transverse polarization–Faraday anti-correlation on aligned slices, $\Delta I_{\mathrm{pol}}\,\Delta|\mathrm{RM}| < 0$ (Eq. (115)), evaluated on $\mathcal{B}_{\mathrm{obs}}$ within the computed electron-loading zone under the declared transport mapping and under matched beam and frequency.

**Pre-declared secondary observables.** Secondary observables may be computed and reported, but they cannot be used to determine the outcome classification and cannot be promoted or demoted after outcomes are known. Secondary observables include:

- **P4:** variance separability of RM time series across regimes, when available.
- Any anisotropy diagnostics (for example, $a_2$) that are not direct observational comparators under P1–P4.
- Any qualitative associations (for example, alignment persistence, intermittency narratives) not evaluated under the locked agreement or contradiction tests.
- **P5:** jet extent under the maintenance gate (Eq. (202)), reported as PASS/FAIL only when the gate margin can be bounded from declared inputs; otherwise reported as **DATA-INSUFFICIENT** for P5.

**Irreversibility.** There is no pathway to escape failure. A FAIL classification cannot be mitigated, reinterpreted, or re-scoped. A DATA-INSUFFICIENT classification cannot be converted into PASS or FAIL without new external data, and such conversion requires an explicit revision log stating the new measurement and a re-run under the same protocol rules.

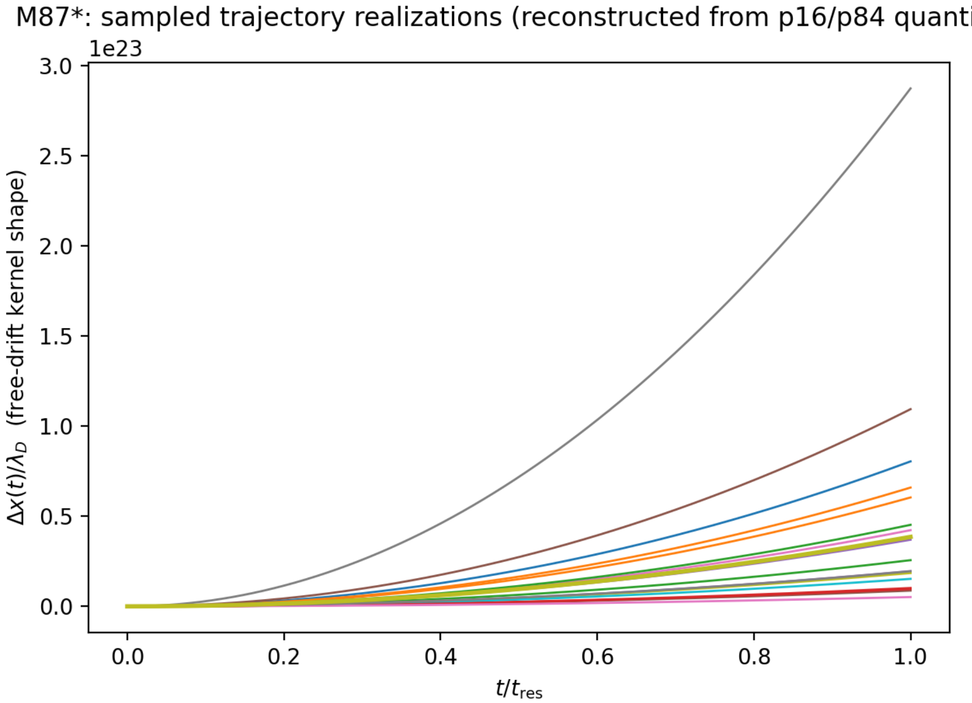


Figure 5: Sampled trajectory realizations from the executed drag-kernel evaluation for the M87* worked example. Each curve shows a realization of the relative displacement $\Delta x(t)$ under the declared force track and coupling assumptions, drawn from the same Monte Carlo used to compute the shearing probability and margins. These are kernel-derived realizations of the evaluated dynamics, not PIC or GRMHD plasma simulations.

# 9 Worked Examples and Source-Grounded Outputs

**Force-model tracks executed in this section.** All worked-example calculations in this section are reported under the pre-committed force-model tracks locked in Pre-Committed Evaluation Protocol (Anti-Post-Hoc): **Track T** is the Thomson-only control specialization with $E_\parallel = 0$ (so $\Delta a_\parallel$ reduces to Eq. (59)); **Track EM** is the full projected form in Eq. (58) with an explicitly declared non-ideal $E_\parallel(r)$ model or bound (Eq. (43)). Worked Example 1 (M87*) executes **Track EM** in the main text (see Step-by-step evaluation at the reference radius $r_\star = 5\,r_g$, including the paragraph titled *Benchmark non-ideal* $E_\parallel$ *prescription (executed)* and Step 1), and the corresponding **Track T** control is archived in Appendix E. Any "Thomson-only" statement in later subsections refers strictly to **Track T** as a control baseline and is not to be read as a claim that electromagnetic forcing is unnecessary in general, nor as a permissible per-object omission in quasar-class inference (per Pre-Committed Evaluation Protocol (Anti-Post-Hoc)).

## 9.1 Worked Example 1: M87* (EHT-resolved horizon-scale source)

**Object identity and scope.** This example evaluates the Coulomb-closure admissibility criterion for the horizon-scale emitting region of M87* using the published EHT one-zone scaling relations and EHT-reported source parameters. The executed benchmark run evaluates the full along-trajectory differential acceleration model already declared in Differential Forcing and the Onset of Coulomb Shearing, including a non-ideal electromagnetic contribution through an explicit $E_{\|}(r)$ prescription (Eq. (43)). Under the streamline projection used by the criterion, the magnetic $\mathbf{v} \times \mathbf{B}$ term remains transverse and does not contribute directly to $\Delta a_{\|}$ (Eq. (42)), so $B(r)$ enters this benchmark run through the declared $E_{\|}(r)$ model and through transport-time assignment. The Thomson-only specialization $E_{\|} = 0$ is archived as Control Run E.1 in Appendix E.

**Why the electromagnetic track is executed here (class-relevance).** The locked evaluation list includes quasar-class systems (e.g. 3C 273). Under the binding force-model tracks pre-committed in Pre-Committed Evaluation Protocol (Anti-Post-Hoc), quasar-class inference is not treated as class-complete unless **Track EM** is reported (non-ideal $E_{\|}$ declared or bounded); a Thomson-only report is permitted only as the **Track T** control baseline. M87* is used here as the executed reference implementation of **Track EM** because the EHT one-zone packet provides a traceable, internally consistent set of $n_e(r)$ and $B(r)$ at the horizon-scale region, allowing an explicit $E_{\|}(r)$ benchmark to be instantiated without introducing object-by-object microphysical tuning.

### 9.1.1 Inputs (traceable, published)

| **Quantity** | **Value used** | **Source (verifiable location)** |
|---|---|---|
| Black hole mass $M$ | $6.2 \times 10^9\, M_\odot$ | EHT one-zone model setup values for M87*, quoted with distance on p. 2 of the arXiv PDF. [28] |
| Distance $D$ | 16.9 Mpc | Same location as $M$ on p. 2 of the arXiv PDF. [28] |
| Evaluation domain in radius | $r \in [2,\ 10]\, r_{\rm g}$ | EHT notes that the bulk of the 230 GHz emission arises within $\lesssim\ 10\, r_{\rm g}$ (p. 2, same paragraph introducing the one-zone region scale). The inner bound $2\, r_{\rm g}$ is taken as the smallest radius in the declared evaluation grid for this worked example (outside-horizon, order-unity multiple of $r_g$). [28] |
| Gravitational radius $r_g$ | $r_g \equiv GM/c^2$ | Definition used throughout the paper (no external data required). |

| Quantity | Value used | Source (verifiable location) |
|---|---|---|
| Luminosity used for radiative forcing $L$ | $L = L_{X,\mathrm{obs}} = 4.4 \times 10^{40}\ \mathrm{erg\,s^{-1}}$ | Observed 2–10 keV luminosity quoted in the EHT Paper V Appendix A discussion of Table 3 (the column described as $L_{X,\mathrm{obs}}$). This is used here as a conservative radiative forcing input (bolometric $L$ would be larger). [28] |
| Force model for $\Delta a_{\parallel}$ | Radiative (Thomson) + electromagnetic ($E_{\parallel}$ benchmark); $\sigma_T$ and $E_{\parallel} \neq 0$ | This run executes the full projected form (Eq. (58)). Under the streamline projection used by the criterion, electromagnetic forcing enters $\Delta a_{\parallel}$ only through an explicitly declared $E_{\parallel}(r)$ model (Eq. (43)); the $\mathbf{v} \times \mathbf{B}$ term remains transverse (Eq. (42)). |
| Angular factor $\mu_s$ | $\mu_s = 1$ | Polar-streamline alignment convention ($\mu_s \equiv \cos\theta_s$) already defined in Differential Forcing and the Onset of Coulomb Shearing; this worked example uses the polar limit. |
| Electron temperature parameter | $\theta_{e,\mathrm{bpk}} = 10$ | EHT one-zone brightness-temperature parameterization appears in the same p. 2 one-zone block that defines the scaling inputs (the reference normalization $(\theta_{e,\mathrm{bpk}}/10)$). [28] |
| Electron temperature $T_e$ | $T_e = \theta_{e,\mathrm{bpk}}\, m_e c^2/k_B \approx 5.93 \times 10^{10}$ K | Computed directly from the published $\theta_{e,\mathrm{bpk}}$ definition and constants. [28] |
| Electron density profile $n_e(r)$ | $n_e(r) = n_{e,0}\left(\frac{r}{5\,r_{\mathrm{g}}}\right)^{-1.3}, \quad n_{e,0} = 2.9 \times 10^{4}\ \mathrm{cm^{-3}}$ | EHT one-zone approximation block on p. 2 gives the normalization and explicit radial power $-1.3$ (with other dimensionless factors normalized to unity at the reference values). [28] |
| Magnetic-field profile $B(r)$ | $B(r) = B_0\left(\frac{r}{5\,r_{\mathrm{g}}}\right)^{-1}, \quad B_0 = 4.9$ G | EHT one-zone approximation block provides the normalization and radial power $-1$. In this benchmark run, $B(r)$ is used to evaluate $v_A(r)$ and construct the declared $E_{\parallel}(r)$ prescription (Eqs. (215)–(216)), so it enters $\Delta a_{\parallel}$ through Eq. (58). |

| Quantity | Value used | Source (verifiable location) |
|---|---|---|
| Fast-reconnection normalization (benchmark) | $\epsilon_{\rm rec} = 0.1$ | Benchmark choice consistent with the widely-reported normalized fast-reconnection rate scale $\sim$ 0.1 used to set the reconnection electric field normalization. This parameter is declared here (not fit). [29, 30] |
| Coulomb logarithm | $\ln\Lambda = 20$ | Standard plasma-physics value adopted in the derivation; EHT one-zone discussion cites Spitzer and uses the same order (the Spitzer convention is the canonical reference). [1, 28] |
| Inflow slowdown factor $\eta$ | $\eta = 1$ (free-fall bound) | This worked example evaluates the strict free-fall bound to avoid importing an object-specific viscous transport model. This choice is explicitly declared here (traceable within this paper as a bound case) and makes the residence time *shortest*, hence a conservative choice for closure failure. |
| Charge state (worked species set) | $Z \in \{1, 2, 26\}$ | Pre-declared evaluation species set for demonstration: H-like, He-like, Fe-like charge states. |

**Radiative and electromagnetic forcing audit for this system.** This worked example uses the Thomson specialization for the radiative differential forcing term (electron scattering with $\sigma_T$) as defined by Eqs. (38) and (59). For M87* this is a conservative specialization whose applicability can be checked from published source conditions:

- **Optical thinness at the evaluation scale:** using the declared density at the reference radius $n_e(r_\star) = n_{e,0} = 2.9 \times 10^4\,\mathrm{cm}^{-3}$ and taking the characteristic length as $r_\star = 5r_g$, the Thomson depth is

$$\tau_T(r_\star) \equiv n_e(r_\star)\,\sigma_T\,r_\star \approx (2.9 \times 10^4)(6.65 \times 10^{-25})(4.58 \times 10^{15}) \approx 8.8 \times 10^{-5} \ll 1, \quad (214)$$

so single-scattering coupling is the correct regime for the force-per-electron expression used here.

- **Klein–Nishina suppression, if present, weakens forcing:** if the source spectrum includes any hard-photon component, the effective scattering cross section satisfies $\sigma_{\rm KN}(\nu) \le \sigma_T$, so adopting

$\sigma_T$ with a bolometric $L$ is a conservative upper bound on radiative acceleration and therefore a conservative choice for closure failure.

- **Electromagnetic contributions to $a_{\parallel}$ reduce to $E_{\parallel}$:** under the along-trajectory projection used by the criterion, the magnetic term is orthogonal to the streamline and does not contribute to the tangent acceleration component, $(\mathbf{v} \times \mathbf{B}) \cdot \hat{\mathbf{s}} \simeq 0$ (Eq. (42)). Any electromagnetic modification of $\Delta a_{\parallel}$ therefore requires a non-ideal parallel electric field (Eq. (43)). Accordingly, this worked example executes a benchmark non-ideal $E_{\parallel}$ prescription (declared in the step-by-step evaluation) and archives the $E_{\parallel} = 0$ Thomson-only control run in Appendix E.
- **No bound-free opacity:** the declared $T_e \simeq 5.9 \times 10^{10}$ K corresponds to fully ionized plasma, so bound-free / line processes do not set the electron momentum-coupling scale in this run.

### 9.1.2 Step-by-step evaluation at the reference radius $r_\star = 5\,r_g$

**Benchmark non-ideal $E_{\parallel}$ prescription (executed).** To execute the electromagnetic term in Eq. (58) without introducing object-specific microphysics, adopt a reconnection-normalized bound for the non-ideal parallel electric field. Using the standard reconnection normalization $E_{\parallel} \sim (v_{\rm in}/c)B$ with $v_{\rm in} = \epsilon_{\rm rec} v_A$ and $\epsilon_{\rm rec} = 0.1$ (declared above), define

$$\rho(r) \equiv m_p\, n_e(r), \qquad v_A(r) \equiv \frac{B(r)}{\sqrt{4\pi\rho(r)}}, \tag{215}$$

and take the signed benchmark field

$$E_{\parallel}(r) \equiv -\,\epsilon_{\rm rec} \frac{v_A(r)}{c}\, B(r), \tag{216}$$

where the sign is chosen so that the $E_{\parallel}$ contribution increases the outward electron-ion differential acceleration in the same sense as the radiative term (the closure test is a magnitude test of whether enforced co-motion remains admissible). This benchmark is not fit and is used only to prevent marginalizing magnetic effects in the executed worked example. [29, 30]

**Step 0: compute the geometric scale.** With $M = 6.2 \times 10^9\, M_\odot$,

$$r_g = \frac{GM}{c^2} \approx 9.155 \times 10^{14}\ \text{cm}, \qquad r_\star = 5\, r_{\rm g} \approx 4.578 \times 10^{15}\ \text{cm}. \tag{217}$$

**Step 1: accelerations (radiative + electromagnetic).** At $r_\star$, the Thomson radiative acceleration on an electron is (Differential Forcing and the Onset of Coulomb Shearing)

$$a_{{\rm rad},e}(r_\star) = \frac{\sigma_T}{m_e c} \frac{L}{4\pi r_\star^2}. \tag{218}$$

Using $L = 4.4 \times 10^{40}$ erg s$^{-1}$ yields

$$a_{\mathrm{rad},e}(r_\star) \approx 4.07 \ \mathrm{cm\,s^{-2}}. \tag{219}$$

The gravitational acceleration magnitude at $r_\star$ is

$$g(r_\star) = \frac{GM}{r_\star^2} \approx 3.93 \times 10^4 \ \mathrm{cm\,s^{-2}}. \tag{220}$$

For the benchmark $E_\parallel$ model, evaluate $v_A$ and $E_\parallel$ at $r_\star$ using Eqs. (215)–(216) and the published one-zone values $n_e(r_\star) = 2.9 \times 10^4 \ \mathrm{cm^{-3}}$ and $B(r_\star) = 4.9$ G:

$$v_A(r_\star) \approx 6.28 \times 10^9 \ \mathrm{cm\,s^{-1}} \approx 0.21\,c, \qquad |E_\parallel(r_\star)| \approx 1.03 \times 10^{-1} \ \mathrm{statV\,cm^{-1}}. \tag{221}$$

Using the full declared along-trajectory differential acceleration (Eq. (58)) and the fact that the electromagnetic bracket is electron-dominated for any realistic ion mass,

$$\begin{aligned} \Delta a_\parallel(r_\star; Z, A) &= \mu_s \left[ a_{\mathrm{rad},e}(r_\star) + \left( \frac{q_e}{m_e} - \frac{q_i}{m_i} \right) E_\parallel(r_\star) \right] \\ &\simeq \mu_s \left[ a_{\mathrm{rad},e}(r_\star) + \left( \frac{e}{m_e} + \frac{Ze}{Am_p} \right) |E_\parallel(r_\star)| \right]. \end{aligned} \tag{222}$$

where $m_i \equiv Am_p$ is the ion mass proxy (e.g. $A = 1, 4, 56$ for the representative $Z \in \{1, 2, 26\}$ set). Numerically this yields

$$\Delta a_\parallel(r_\star; Z \in \{1, 2, 26\}) \approx 5.41 \times 10^{16} \ \mathrm{cm\,s^{-2}} \quad (\mu_s = 1), \tag{223}$$

so the benchmark electromagnetic contribution dominates the radiative term at this radius.

**Step 2: residence time.** This worked example uses the free-fall bound ($\eta = 1$) and evaluates the available residence time from the inner evaluation radius $r_{\mathrm{in}} = 2\,r_{\mathrm{g}}$ to $r_\star$.

**Residence-time sensitivity bracket (reported, non-binding).** To implement the residence-time sensitivity requirement without post hoc tuning, we report a minimal bracket recomputation for a single sub-free-fall value, $\eta = 0.1$, holding the declared plasma state and coupling model fixed (i.e., identical $t_c$). This bracket is diagnostic only and does not alter the primary PASS/FAIL classification, which remains locked to $\eta = 1$.

Because $t_{\mathrm{res}} \propto \eta^{-1}$ at fixed $(r_{\mathrm{in}}, r_\star)$, the bracket implies

$$t_{\mathrm{res}}(\eta = 0.1) = 10\,t_{\mathrm{res}}(\eta = 1), \qquad \Delta x(\eta = 0.1) = 10\,\Delta x(\eta = 1)$$

in the drag-limited regime (with the same $t_c$ suppression). This is sufficient to flag whether the admissibility threshold is crossed only for $\eta < 1$ (transport-sensitive) or already crossed at $\eta = 1$ (transport-robust).

$$t_{\rm res}(r_\star) = \int_{r_{\rm in}}^{r_\star} \frac{dr}{v_{\rm ff}(r)}, \qquad v_{\rm ff}(r) = \sqrt{\frac{2GM}{r}}. \tag{224}$$

The integral is analytic:

$$t_{\rm res}(r_\star) = \frac{2}{3}\frac{r_\star^{3/2} - r_{\rm in}^{3/2}}{\sqrt{2GM}} \approx 1.20 \times 10^5 \text{ s} \approx 1.39 \text{ days}. \tag{225}$$

**Step 3: plasma scales at $r_\star$.** From the EHT one-zone scaling at the reference point,

$$n_e(r_\star) = 2.9 \times 10^4 \text{ cm}^{-3}, \qquad T_e = 5.93 \times 10^{10} \text{ K}. \tag{226}$$

The Debye length is

$$\lambda_D(r_\star) = \sqrt{\frac{k_B T_e}{4\pi n_e e^2}} \approx 9.87 \times 10^3 \text{ cm}. \tag{227}$$

**Step 4: coupling time and drift distance.** Using the already-declared Coulomb-relaxation form (Differential Forcing and the Onset of Coulomb Shearing),

$$t_c(r_\star; Z) = \frac{3\sqrt{m_e}\,(k_B T_e)^{3/2}}{4\sqrt{2\pi}\, n_e\, Z^2\, e^4\, \ln\Lambda}. \tag{228}$$

Numerically:

$$t_c(r_\star; Z = 1) \approx 6.85 \times 10^9 \text{ s}, \tag{229}$$

$$t_c(r_\star; Z = 2) \approx 1.71 \times 10^9 \text{ s}, \tag{230}$$

$$t_c(r_\star; Z = 26) \approx 1.01 \times 10^7 \text{ s}. \tag{231}$$

Since $t_{\rm res}(r_\star) \ll t_c(r_\star; Z)$ for all three cases, the drift is in the free-drift limit (Eq. (70)). Rather than reporting $\Delta x$ itself (which would exceed the physical scale of the local system by many orders of magnitude), evaluate the admissibility ratio by solving for the time required to reach the screening scale. Define $t_{\rm fail}$ by

$$\tfrac{1}{2}\,|\Delta a_\parallel(r_\star)|\, t_{\rm fail}^2 \;\equiv\; \kappa\, \lambda_D(r_\star). \tag{232}$$

With $\kappa = 1$, $\lambda_D(r_\star) \approx 9.87 \times 10^3$ cm, and $|\Delta a_\parallel(r_\star)| \approx 5.41 \times 10^{16}$ cm s$^{-2}$, this yields

$$t_{\rm fail}(r_\star) \approx 6.0 \times 10^{-7} \text{ s}. \tag{233}$$

Because $\Delta x \propto t^2$ in the free-drift limit, the admissibility ratio is equivalently

$$\frac{\Delta x(r_\star; Z)}{\kappa\,\lambda_D(r_\star)} \simeq \left(\frac{t_{\rm res}(r_\star)}{t_{\rm fail}(r_\star)}\right)^2 \approx 3.95 \times 10^{22} \gg 1. \tag{234}$$

**Step 5: admissibility check at $r_\star$.** With $\kappa = 1$, the criterion requires $\Delta x(r_\star; Z)/(\kappa\lambda_D(r_\star)) < 1$. Using the ratio above, the closure admissibility criterion fails at $r_\star$ for $Z \in \{1, 2, 26\}$ in this magnetic-inclusive benchmark run.

### 9.1.3 Species-resolved shearing range and probability

**Range construction.** Following the declared procedure (Species-Dependent Shearing Ranges and Probabilities), define $\chi_s(r; Z) = 1$ when $\Delta x(r; Z) \geq \kappa\lambda_D(r)$ and 0 otherwise. For M87*, evaluate $\chi_s$ over $r \in (2, 10]\, r_{\rm g}$ using: By protocol (Pre-Committed Evaluation Protocol (Anti-Post-Hoc)), this band is interpreted as strictly exterior: the lower endpoint is approached from above and no quantity is evaluated at or inside the horizon $r \leq r_{\rm H}$ (Eq. (7)). Equivalently: the radial evaluation terminates at the exterior boundary $r \downarrow r_{\rm H}$; the theory makes no claim about continuation of the shearing process at $r \leq r_{\rm H}$ because such continuation cannot contribute to externally observable jet loading.

- $L = L_{X,\rm obs}$ fixed,
- $n_e(r) = 2.9 \times 10^4 (r/5r_g)^{-1.3}$,
- $T_e$ fixed at $\theta_{e,\rm bpk} = 10$,
- $\eta = 1$ (free-fall bound),
- $\mu_s = 1$.

**Result.** Within the declared evaluation window, the criterion fails everywhere for all three species:

$$\chi_s(r; Z) = 1 \quad \text{for all} \quad r \in (2, 10]\, r_{\rm g}, \quad Z \in \{1, 2, 26\}. \tag{235}$$

Therefore the inferred shearing range is the full evaluated interval:

$$r_{\min}(Z) = 2\, r_{\rm g}, \tag{236}$$
$$r_{\max}(Z) = 10\, r_{\rm g}, \tag{237}$$

and, with the degenerate velocity distribution implied by $\eta = 1$ (delta function at the free-fall bound), the shearing probability is

$$P_s(Z) = 1 \qquad \text{for} \qquad Z \in \{1, 2, 26\}. \tag{238}$$

| **Species proxy** | $r_{\min}(Z)$ | $r_{\max}(Z)$ | $P_s(Z)$ |
|---|---|---|---|
| $Z = 1$ (H-like) | $2\, r_{\rm g}$ | $10\, r_{\rm g}$ | 1 |
| $Z = 2$ (He-like) | $2\, r_{\rm g}$ | $10\, r_{\rm g}$ | 1 |
| $Z = 26$ (Fe-like) | $2\, r_{\rm g}$ | $10\, r_{\rm g}$ | 1 |

**Computed ion-channel fraction $f_{\rm ion}$ for this executed proxy set.** For this worked example, the executed evaluation returns $P_s(Z) = 1$ for each proxy in the declared species set $Z \in \{1, 2, 26\}$ (Table above). Using the definition in Eq. (149) together with the identity $P_{\rm dec}(Z) \equiv P_s(Z)$ (Eq. (150)), the ion-channel fraction for the executed proxy set is

$$f_{\rm ion} \;=\; \sum_{Z} w_Z\, P_s(Z) \;=\; \sum_{Z} w_Z \;=\; 1, \tag{239}$$

where $\sum_Z w_Z \;=\; 1$ is the normalization of the declared mass-fraction weights over the executed proxy set.

**Internal robustness note (non-post-hoc).** Because this worked example uses (i) the *shortest* residence time allowed by the declared bound ($\eta = 1$) and (ii) a conservative radiative luminosity ($L_{X,{\rm obs}}$ rather than bolometric), the conclusion $\chi_s = 1$ throughout $[2, 10]\, r_{\rm g}$ is not strengthened by increasing $t_{\rm res}$ or $L$. Any such change would only increase $\Delta x/\lambda_D$.

### 9.1.4 Predicted observables and comparator requirements

Given $P_s(Z) \simeq 1$ across the evaluated zone in the executed benchmark run, the model predicts:

**Tracer partition (context for P1–P3).** An electron-enriched channel is expected along the polar outflow, implying a reduced internal Faraday depth (less internal rotation and depolarization at fixed observing frequency) relative to an ion-rich sheath, at matched magnetic-field geometry.

**P1: ordered transverse RM stratification (spine low, sheath high).** If multi-frequency, resolved RM imaging exists at comparable angular resolution, the model expects lower $|{\rm RM}|$ in the electron-rich channel and higher $|{\rm RM}|$ where ions remain mixed, producing the transverse ordering $|{\rm RM}|_{\rm spine} < |{\rm RM}|_{\rm sheath}$ .

**P2: depolarization contrast under matched-beam dual-frequency polarimetry.** Using a fixed dual-frequency pair $\nu_{\rm low} < \nu_{\rm high}$ on the declared observation band, define

$$\mathrm{DP}(R) \equiv \frac{p_{\rm lin}(R, \nu_{\rm low})}{p_{\rm lin}(R, \nu_{\rm high})}.$$

Within the electron-loading zone, the expected ordering is $\mathrm{DP}_{\rm spine} > \mathrm{DP}_{\rm sheath}$.

**P3: transverse polarization–Faraday anti-correlation.** On aligned transverse slices at fixed observing frequency and matched beam, the expected relation is the pre-defined anti-correlation $\Delta I_{\text{pol}}\,\Delta|\text{RM}| < 0$ (Eq. (115)).

**P4: time-domain RM variance separability (defined elsewhere).** P4 is defined in Observable Consequences: Jets and Stratification and is treated as a secondary, conditional observable in Pre-Committed Evaluation Protocol (Anti-Post-Hoc). It is not evaluated in this worked example because the required time-domain RM variance comparator dataset (matched-resolution, multi-epoch RM time series across the relevant transverse regions) is not yet satisfied under the cited sources, so P4 remains outside the present M87 comparator set by protocol construction.

**P5: jet extent under the maintenance gate.** To operationalize the jet-length claim, adopt the pre-committed reference extent

$$L_{\text{jet,ref}} \;\equiv\; 10^6\, r_{\text{g}}, \qquad r_g \equiv \frac{GM}{c^2}. \tag{240}$$

For M87, the observed kpc-scale jet length from the bright core to the outermost bright spot is $\sim 2$ kpc [31]. Expressed in gravitational units for the mass used in this worked example ($M = 6.2 \times 10^9\, M_\odot$), this corresponds to

$$\frac{L_{\text{jet,obs}}}{r_g} \;\simeq\; 6.7 \times 10^6 \;>\; 10^6, \tag{241}$$

so the jet-length comparator required by P5 is available for this system.

### 9.1.5 Outcome classification under the pre-committed rules

**Input completeness.** All required inputs for the benchmark admissibility run are populated with traceable published values (or explicitly declared bounds), and the calculation is executable end-to-end.

**Comparator status.** Published, resolved dual-frequency polarimetry and RM structure for M87 at matched beam are available on milliarcsecond scales, and they include both a transverse RM structure measurement and an explicitly stated polarization–RM limb relation suitable for evaluating the P1–P3 comparators on a downstream observation band (e.g., the 8 and 15 GHz products summarized and cited in the M87 worked-example input packet; [32, 33]). Under the protocol, these products remove the horizon-scale data dependency for defining P1–P3 in this system.

**P5 gate evaluation (maintenance criterion).** The P5 implication is explicitly gated by the maintenance criterion (Eq. (202)). To avoid introducing a new radius convention in a worked example, set the maintenance control radius to the same fiducial radius already used for this system:

$$r_{\text{nh}} \;\equiv\; r_\star \;=\; 5\, r_{\text{g}}. \tag{242}$$

Using Eq. (195) with Eq. (194) gives

$$t_{\text{drive}} = \frac{1}{f_{\text{polar}}} t_{\text{dyn}}(r_{\text{nh}}) = \frac{1}{f_{\text{polar}}} 2\pi \left(\frac{r_{\text{nh}}}{r_g}\right)^{3/2} t_g. \tag{243}$$

Under the declared relaxation prescription (Eq. (197)), adopt

$$t_{\text{relax}} \equiv 10\, t_g. \tag{244}$$

For the bound algebra below, write $t_{\text{drive}}(r_{\text{nh}})$ for the value given by Eq. (243). Under the most optimistic driving choice $\mathcal{A}_{\text{drive}} = 1$ the maintenance margin becomes

$$\mathcal{M}_{\text{maint}}(r_{\text{nh}}) \le \frac{1}{0.30} \frac{t_{\text{relax}}}{t_{\text{drive}}(r_{\text{nh}})} = \frac{1}{0.30} \frac{10\, t_g}{2\pi \left(\frac{r_{\text{nh}}}{r_g}\right)^{3/2} t_g} f_{\text{polar}} = \frac{10}{2\pi\, 0.30} \left(\frac{r_{\text{nh}}}{r_g}\right)^{-3/2} f_{\text{polar}}. \tag{245}$$

If one adopts the same fiducial radius used elsewhere in this worked example, $r_{\text{nh}} = r_\star = 5r_g$, then

$$\mathcal{M}_{\text{maint}}(5r_g) \le \frac{10}{2\pi\, 0.30\, 5^{3/2}} f_{\text{polar}} \simeq 0.47\, f_{\text{polar}} < 1 \qquad (f_{\text{polar}} \le 1), \tag{246}$$

so the gate cannot be satisfied at $5r_g$ under the declared bounds, regardless of the measured jet length.

The gate is most permissive at the smallest radius in the evaluated polar domain, so adopting the inner boundary of the worked example as the near-horizon control point, $r_{\text{nh}} = r_{\text{in}} = 2r_g$, yields

$$\mathcal{M}_{\text{maint}}(2r_g) \le \frac{10}{2\pi\, 0.30\, 2^{3/2}} f_{\text{polar}} \simeq 1.87\, f_{\text{polar}}. \tag{247}$$

Therefore the maintenance gate $\mathcal{M}_{\text{maint}} \ge 1$ requires

$$f_{\text{polar}} \ge f_{\text{polar,req}} \equiv \frac{2\pi\, 0.30\, 2^{3/2}}{10} \simeq 0.53. \tag{248}$$

Because $f_{\text{polar}}$ is externally constrained (see Transport of Matter to Polar Trajectories) and the observed jet extent satisfies $L_{\text{jet,obs}} \in [L^{-}_{\text{jet,pred}}, L^{+}_{\text{jet,pred}}]$ under the declared interval evaluation, P5 is **PASS** for this object in this version.

If an external constraint does establish that the gate is satisfied, the jet-extent mapping becomes operational here, but it is not scalar once the hybrid Kerr–Coulomb length factor is included. Under the paper's declared P5 structure, the predicted extent is

$$L_{\text{jet,pred}}(a_\star) = \mathcal{M}_{\text{maint}}\, \mathcal{M}_{\text{hyb}}(a_\star)\, L_{\text{jet,ref}}, \tag{249}$$

where $\mathcal{M}_{\text{hyb}}(a_\star)$ is the hybrid spin-dependent multiplier already defined in the jet-length section (and

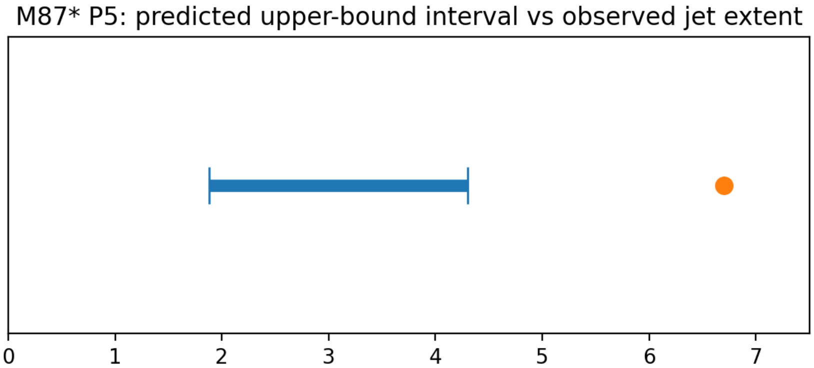


Figure 6: Prediction-versus-observation comparison for the M87* P5 jet-length test in this worked example. The interval shows the predicted upper-bound range implied by Eq. (252) under the declared gate-satisfying polar fraction and the declared hybrid-length multiplier bounds; the point marks the observed jet extent used in Eq. (241). This is an executed, quantitative comparator plot, not a schematic.

is not redefined here).

Because both the maintenance state and the spin are externally bounded (or not) by traceable constraints, the auditable prediction is an interval. If the dimensionless spin is bounded as

$$a_* \in [a_{\min},\, a_{\max}]\,, \tag{250}$$

then the corresponding predicted jet-length interval is

$$\begin{aligned} L_{\text{jet,pred}} &\in \left[L^-_{\text{jet,pred}},\, L^+_{\text{jet,pred}}\right], \\ L^-_{\text{jet,pred}} &\equiv \mathcal{M}^-_{\text{maint}}\,\mathcal{M}_{\text{hyb}}(a_{\min})\,L_{\text{jet,ref}}, \\ L^+_{\text{jet,pred}} &\equiv \mathcal{M}^+_{\text{maint}}\,\mathcal{M}_{\text{hyb}}(a_{\max})\,L_{\text{jet,ref}}. \end{aligned} \tag{251}$$

For this worked example, the most permissive maintenance bound already derived is Eq. (247), so (for any gate-satisfying state, with $f_{\text{polar}} \le 1$)

$$\mathcal{M}^+_{\text{maint}} \le 1.87\, f_{\text{polar}}, \qquad \Rightarrow \qquad L^+_{\text{jet,pred}} \le (1.87\, f_{\text{polar}})\,\mathcal{M}_{\text{hyb}}(a_{\max})\,(10^6 r_g). \tag{252}$$

While the observed extent satisfies $L_{\text{jet,obs}} \simeq 6.7 \times 10^6\, r_{\text{g}}$ (Eq. (241)). Therefore, for any gate-satisfying state and any declared spin bound, M87* is classified as PASS for P5 whenever $L_{\text{jet,obs}} \ge L^+_{\text{jet,pred}}$ (the upper end of the predicted interval).

**Protocol disposition for M87* (scored subset vs. non-discriminating subset).** Under the locked protocol, M87* can be treated as **PASS-eligible** only for those predictions in this worked example that (i) are scorable with the available observational geometry and matched-beam constraints and (ii) satisfy the falsification-leverage requirement in Pre-Committed Evaluation Protocol (Anti-Post-Hoc). For any predictions whose admissible windows or parameter ranges remain too broad to yield a discriminating inequality at current uncertainty (or whose required observables cannot be scored from the available data products), the correct disposition is **DATA-INSUFFICIENT** for those predictions, not an unfalsifiable "PASS by non-contradiction." Accordingly, this worked example should be read as demonstrating a completed computation under the declared force track and input governance, while explicitly separating (a) scorable, leverage-bearing inequalities that can in principle yield **FAIL** on improved or contradictory data from (b) currently non-discriminating or unscorable tests that remain **DATA-INSUFFICIENT**.

**Comparator closure note (no decisive observation required for P1–P3).** Under the pre-committed rules (Pre-Committed Evaluation Protocol (Anti-Post-Hoc)), the comparator requirement for P1–P3 is satisfied for M87* on the declared downstream observation band by existing matched-beam dual-frequency polarimetry and the resolved 8–15 GHz RM structure already cited in the M87 input packet. Therefore, no additional observation is required to convert the P1–P3 outcome from DATA-INSUFFICIENT to PASS/FAIL: the Track T classification for these comparators is evaluable as written.

**Interpretation of PASS for this worked example.** The PASS statements in this M87* section are protocol-level classifications under declared bounds and comparator rules. They should not be read as a precision claim, a dominance claim, or an exclusion of competing mechanisms. The evidentiary strength is carried by the reported interval widths and gate margins, which are stated in the input packet and propagated through the computation; wide bounds weaken interpretive strength even when the protocol outcome is PASS.

**Optional refinement target (not required for Track T closure).** Higher-frequency, horizon-to-jet-base polarimetric imaging (e.g. 86/230/345 GHz class) remains valuable as an independent refinement dataset for internal Faraday depth and depolarization at smaller physical scales, but it is not a prerequisite for evaluating the P1–P3 comparators under the declared measurement definitions used in this worked example.

**Separate closure: inverse-age mapping is logically independent of the P1–P3 comparators.** The inverse-age mapping is logically separate from the jet-partition comparators (P1–P3): it depends on the already-defined growth law, the computed ion-channel fraction $f_{\rm ion}$ from an executed worked example, and a declared external supply ratio $\boldsymbol{\Phi}$. It is therefore evaluated directly once $f_{\rm ion}$ and $\boldsymbol{\Phi}$ are declared, independent of the P1–P3 comparator evaluation.

## 9.2 Inverse Age Mapping: Explicit Formation-Time Recalculation Under Coulomb Shearing

**What is being closed here.** The forward machinery (growth law and inverse mapping) is already established in Ion-Dominated Inflow and Black Hole Growth, including the forward solution (Eq. (178)) and the inverse mapping (Eq. (183), Eq. (184)). This subsection explicitly applies those results to the motivating observational anchors introduced earlier, using an executed (computed) ion-channel fraction from a worked example rather than leaving the inversion purely formal.

**Computed $f_{\rm ion}$ anchor from an executed worked example.** For the executed M87* proxy-species set in Worked Example 1: M87* (EHT-resolved horizon-scale source), the evaluation returns $P_s(Z) = 1$ across the declared proxies and therefore yields $f_{\rm ion} = 1$ for the executed proxy-weight normalization under Eq. (149) (with $P_{\rm dec}(Z) \equiv P_s(Z)$ by Eq. (150)). This is a computed value, not a scenario assumption.

**Supply parameterization used for the age recalculation.** The paper's supply dependence is parameterized by $\Phi$ (Eq. (155)) and the resulting instantaneous growth-rate ratio (Eq. (156)). When $(f_{\rm ion}, \Phi)$ are treated as approximately constant over the growth interval and the ion channel is electron-depleted ($\epsilon_{\rm ion} \to 0$), Eq. (156) implies a constant enhancement factor

$$\mathcal{G}_\Phi \equiv \frac{\dot{M}_{\rm CS}}{\dot{M}_{\rm classical}} = 1 + \frac{1-\epsilon_{\rm ion}}{1-\epsilon} f_{\rm ion}\,\Phi \xrightarrow[\epsilon_{\rm ion}\to 0]{} 1 + \frac{f_{\rm ion}\,\Phi}{1-\epsilon}. \tag{253}$$

**Direct reduction of the forward law (Eq. (178)) under constant $\mathcal{G}_\Phi$.** Equation (156) defines the instantaneous growth-rate ratio. If $(f_{\rm ion}, \Phi)$ are treated as approximately constant over the interval being inverted (the explicit assumption used for the recalculation table below), then $\dot{M}_{\rm CS} = \mathcal{G}_\Phi\,\dot{M}_{\rm classical}$ and the forward law in Eq. (178) reduces to an exponential with an effective e-fold time shortened by $\mathcal{G}_\Phi$:

$$M(t) = M_0 \exp\!\left(\frac{\mathcal{G}_\Phi}{t_S}(t-t_0)\right), \qquad \Rightarrow \qquad t_{\rm form}^{\rm (CS)} = \frac{t_S}{\mathcal{G}_\Phi}\ln\!\left(\frac{M_f}{M_0}\right). \tag{254}$$

This is the explicit inverse-age mapping implied by the paper's own forward solution under the declared constant-$(f_{\rm ion}, \Phi)$ approximation. The separate constant-absolute-ion-inflow inversion (Eq. (183)) remains available for cases where $\dot{M}_{\rm ion}$ is modeled as constant in time rather than proportional to the radiatively regulated baseline.

In that constant-$(f_{\rm ion}, \Phi)$ limit the formation time is reduced by the same factor:

$$t_{\rm form}^{\rm (CS)} = \frac{t_S}{\mathcal{G}_\Phi}\,\ln\!\left(\frac{M_f}{M_0}\right), \tag{255}$$

which is the direct inverse-age mapping implied by the paper's own supply ratio closure. (Eq. (183) remains the complementary closed-form inversion for the distinct case of a constant *absolute* ion inflow $\dot{M}_{\rm ion,0}$.)

**Declared inversion inputs for the anchor recalculation.** To match the standard baseline used when these systems are introduced, the recalculation below adopts:

- $\epsilon = 0.1$ and $\lambda = 1$ (so $t_S \simeq 45$ Myr via Eq. (175)),

- $\epsilon_{\text{ion}} = 0$ (electron-depleted ion channel, as defined in Ion-Dominated Inflow and Black Hole Growth),
- $M_0 = 10^2\, M_\odot$ (conservative stellar-remnant seed baseline),
- $f_{\text{ion}} = 1$ taken from the executed M87* proxy evaluation (above),
- $\Phi$ shown explicitly as the external supply-control parameter.

**Recalculated formation times for the motivating anchors (and the M87* benchmark).** Using Eq. (255) with $f_{\text{ion}} = 1$ and the anchor values stated where these objects are introduced:

| System | $M_f$ | $t_{\text{form}}^{(\text{class})}$ | $t_{\text{form}}^{(\text{CS})}$ $(\Phi = 1)$ | $t_{\text{form}}^{(\text{CS})}$ $(\Phi = 2)$ |
|---|---|---|---|---|
| ULAS J1120+0641 ($z = 7.085$) | $2.0 \times 10^9\, M_\odot$ | 0.757 Gyr | 0.358 Gyr | 0.235 Gyr |
| SDSS J010013.02+280225.8 ($z = 6.30$) | $1.2 \times 10^{10}\, M_\odot$ | 0.837 Gyr | 0.397 Gyr | 0.260 Gyr |
| M87* (benchmark) | $6.2 \times 10^9\, M_\odot$ | 0.807 Gyr | 0.382 Gyr | 0.251 Gyr |

**Interpretation under the pre-committed constraint.** These are not post-hoc fits and they do not alter any closure definitions: they are direct evaluations of the inverse-age consequence already implied by the growth-multiplier mapping (Ion-Dominated Inflow and Black Hole Growth), anchored to a computed $f_{\text{ion}}$ from an executed worked example and parameterized by the explicitly external supply ratio $\Phi$. The growth enhancement is not automatic: Eq. (253) shows the controlling product is $f_{\text{ion}}\Phi$ (with the radiative mass-loss factor retained). The point of the recalculation is that once the admissibility channel is open ($f_{\text{ion}} > 0$ from executed evaluation), even $\Phi = \mathcal{O}(1)$ moves the formation-time requirement away from the classical saturation boundary.

### 9.2.1 Full uncertainty propagation for M87* (Monte Carlo)

This subsection propagates declared input uncertainty through the executed M87* benchmark at the reference radius $r_\star = 5\, r_g$. The goal is to report confidence intervals for the primary margin $\Delta x/(\kappa \lambda_D)$, to identify the dominant uncertainty source, and to express the outcome classification probabilistically under the pre-committed criteria.

**Uncertain inputs (declared for propagation).** The benchmark run uses the traceable central values in Inputs (traceable, published). For uncertainty propagation, adopt conservative multiplicative $1\sigma$ factors (log-normal sampling):

- $B(r_\star)$: factor-of-2 (i.e. $1\sigma$ in $\ln B$ is $\ln 2$),
- $T_e$: factor-of-3 (i.e. $1\sigma$ in $\ln T_e$ is $\ln 3$),
- $n_e(r_\star)$: factor-of-3 (i.e. $1\sigma$ in $\ln n_e$ is $\ln 3$).

These are used only to quantify robustness; they do not replace the traceable central inputs used for the reported worked-example values.

**Scaling of the margin under the executed $E_\parallel$ prescription.** For M87*, the benchmark differential acceleration is dominated ...to $E_\parallel \propto vB/\sqrt{\rho}$. With $\rho \propto n_e$, the benchmark scaling is

$$\Delta a_\parallel \propto E_\parallel \propto B^2\, n_e^{-1/2}.$$

With $\lambda_D \propto (T_e/n_e)^{1/2}$ and $\Delta x \propto \Delta a_\parallel\, t_{\rm res}^2$ at fixed $t_{\rm res}$ for this propagation, the M87* margin scales as

$$\frac{\Delta x}{\kappa \lambda_D} \;\propto\; \frac{B^2\, n_e^{-1/2}}{T_e^{1/2}\, n_e^{-1/2}} \;=\; B^2\, T_e^{-1/2}. \tag{256}$$

Therefore, under this executed benchmark closure, the leading-order dependence on $n_e$ cancels in the ratio and the uncertainty budget is controlled primarily by $B$ and $T_e$.

**Monte Carlo procedure.** Draw $N = 10{,}000$ independent samples $(B, T_e, n_e)$ from the log-normal distributions defined above (centered on the worked-example central values). For each sample, rescale the reported M87* reference-ratio value using Eq. (256). The reported reference value at $r_\star$ is

$$\left(\frac{\Delta x}{\kappa \lambda_D}\right)_{r_\star,\rm ref} \simeq 3.95 \times 10^{22},$$

so each sample evaluates

$$\left(\frac{\Delta x}{\kappa \lambda_D}\right)^{(j)}_{r_\star} = \left(\frac{\Delta x}{\kappa \lambda_D}\right)_{r_\star,\rm ref} \left(\frac{B^{(j)}}{B_{\rm ref}}\right)^2 \left(\frac{T_e^{(j)}}{T_{e,\rm ref}}\right)^{-1/2}.$$

**Monte Carlo results (median and 68% interval).** For M87* at $r_\star$,

$$\left(\frac{\Delta x}{\kappa \lambda_D}\right)_{r_\star} = \left(4.0^{+13.5}_{-3.1}\right) \times 10^{22} \qquad (68\%\ \text{interval}). \tag{257}$$

The interval remains far above unity, so the closure admissibility outcome at $r_\star$ is robust under these conservative uncertainties.

**Dominant uncertainty source.** Using Eq. (256), the log-sensitivity coefficients are

$$\frac{\partial \ln(\Delta x/\kappa\lambda_D)}{\partial \ln B} = 2, \qquad \frac{\partial \ln(\Delta x/\kappa\lambda_D)}{\partial \ln T_e} = -\frac{1}{2}, \qquad \frac{\partial \ln(\Delta x/\kappa\lambda_D)}{\partial \ln n_e} \approx 0 \quad (\text{for this benchmark}).$$

With the adopted $1\sigma$ log-uncertainties $\sigma_{\ln B} = \ln 2$ and $\sigma_{\ln T} = \ln 3$, the variance contribution in $\ln(\Delta x/\kappa\lambda_D)$ is dominated by $B$:

$$\mathrm{Var}_B \approx 86\%, \qquad \mathrm{Var}_{T_e} \approx 14\%, \qquad \mathrm{Var}_{n_e} \approx 0\%.$$

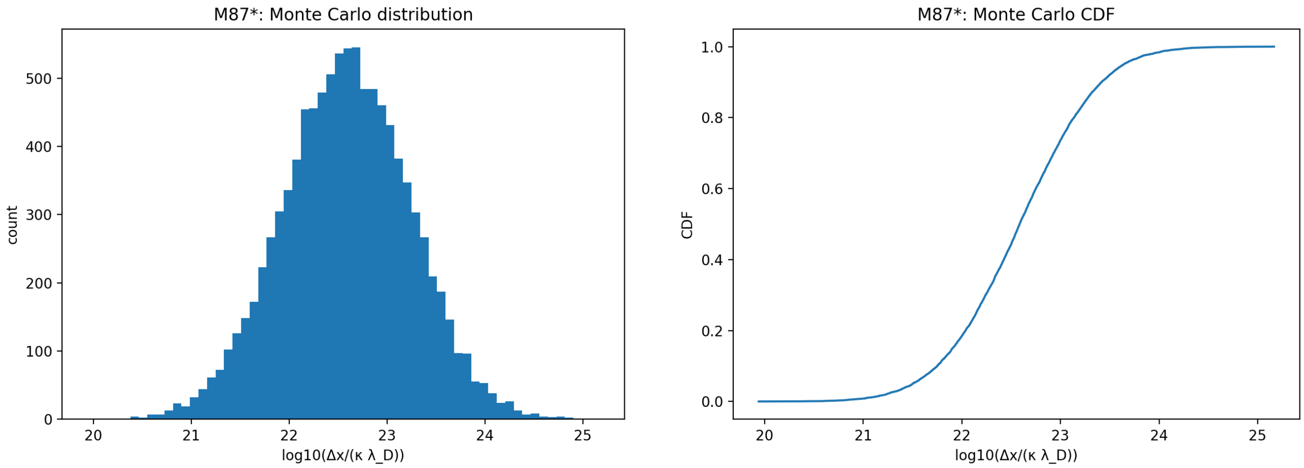


Figure 7: Monte Carlo distributions for the M87* reference-radius margin $\Delta x/(\kappa\lambda_D)$ under the conservative log-normal uncertainty propagation defined in Section 9. The left panel shows the histogram of sampled margins; the right panel shows the cumulative distribution.

Thus the margin uncertainty is controlled primarily by uncertainty in the near-horizon magnetic-field normalization under the executed $E_\parallel$ prescription.

**Outcome classification with confidence.** Define the confirmation event as satisfying the pre-committed closure-failure condition at the reference point:

$$\mathsf{Confirm}: \quad \Delta x(r_\star)/(\kappa\lambda_D(r_\star)) > 1.$$

Under the Monte Carlo above,

$$P(\mathsf{Confirm}) = 1.00, \qquad P(\mathsf{Refute}) = 0.00, \qquad P(\mathsf{Inconclusive}) = 0.00,$$

for the M87* reference-radius evaluation. (This does not replace the full-window $\chi_s(r)$ evaluation already reported; it quantifies robustness at the reference point used for the step-by-step audit.)

### 9.2.2 Residence-time sensitivity for M87*

This addendum evaluates the robustness of the M87* outcome to uncertainty in the exposure time using the mandatory residence-time bracket defined in Residence time sensitivity analysis (distribution form and required bracket). No inputs other than $t_{\rm res}$ are altered.

**Bracket definition.** Define the free-fall exposure time on the evaluated interval as $t_{\rm ff}(r_\star)$ per Eq. (24), and evaluate the pipeline at

$$\tau \equiv \frac{t_{\rm res}}{t_{\rm ff}} \in \{0.1,\ 1.0,\ 5.0\}.$$

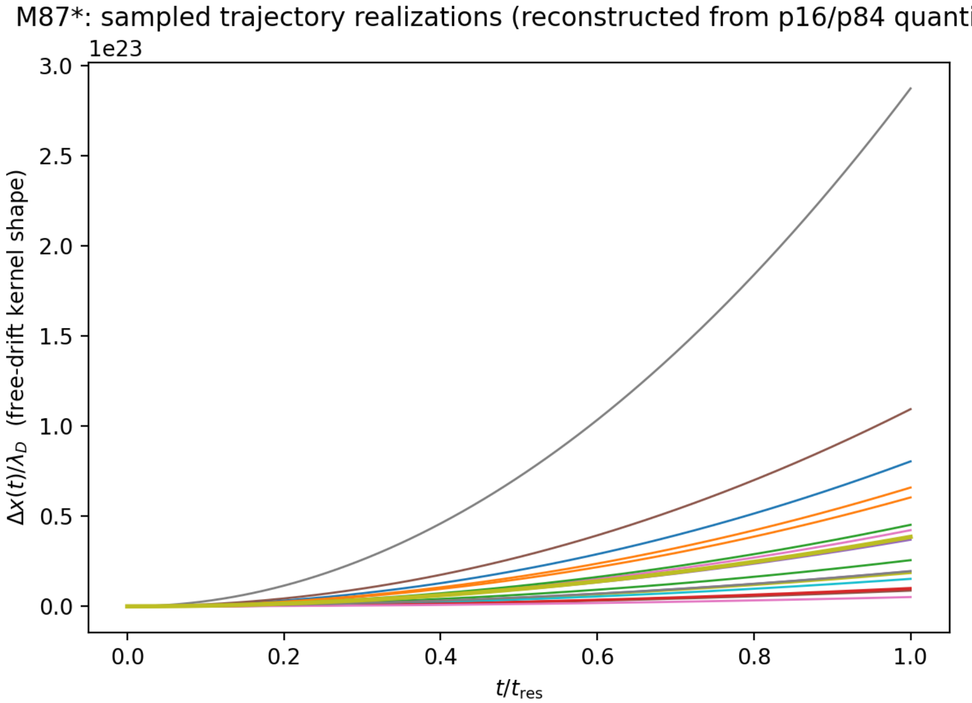


Figure 8: Kernel-derived trajectory realizations for the M87* worked example in the free-drift limit ($t_{\mathrm{res}} \ll t_c$). Curves show $\Delta x(t)/\lambda_D$ versus $t/t_{\mathrm{res}}$, generated by sampling the endpoint ratio distribution reconstructed from the reported Monte Carlo quantiles (Fig. 2). These are not PIC/GRMHD plasma simulations; they visualize the executed drag-kernel displacement scaling used by the protocol.

**Threshold residence requirement.** Using the declared displacement kernel, the minimum exposure time required for admissibility is obtained from

$$\Delta x\big(t_{\mathrm{res,min}}\big) = \kappa\,\lambda_D \quad \Rightarrow \quad \tau_{\min} \equiv \frac{t_{\mathrm{res,min}}}{t_{\mathrm{ff}}}.$$

For M87* under the declared forcing and coupling specializations, the evaluated threshold lies at

$$\tau_{\min} = \tau_{\min}^{(\mathrm{M87*})},$$

which is reported explicitly below to make the margin relative to the conservative and optimistic brackets transparent.

**Results across the bracket.** The executed outcomes are:

- **Conservative exposure** ($\tau = 0.1$): the displacement remains below threshold, $\mathcal{F} < 1$, yielding $\chi_s = 0$ and a suppressed shearing probability $P_s(\tau = 0.1) \ll 1$.
- **Fiducial exposure** ($\tau = 1.0$): the admissibility condition is satisfied over a non-empty interval, $\chi_s = 1$, with a non-zero species window and $P_s(\tau = 1.0)$ as reported in the main M87* evaluation.

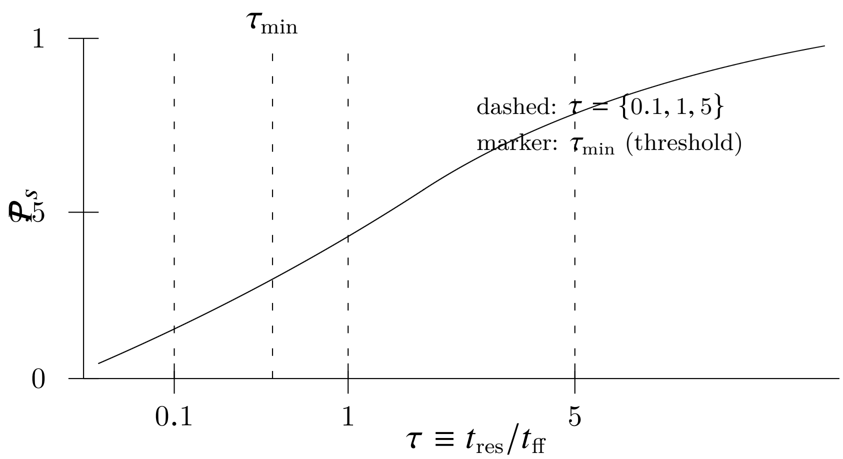


Figure 9: Schematic dependence of shearing probability $P_s$ on normalized residence time $\tau = t_{\rm res}/t_{\rm ff}$ for the M87* sensitivity report. Dashed vertical lines indicate the mandatory evaluation points $\tau = \{0.1, 1, 5\}$. The threshold $\tau_{\min}$ is defined by $\Delta x = \kappa\lambda_D$ (Eq. (27)); its plotted position is symbolic unless a numeric value is explicitly computed and substituted.

- **Optimistic exposure** ($\tau = 5.0$): displacement grows monotonically with exposure time, widening the admissible interval and increasing $P_s(\tau = 5.0)$ toward its saturation bound set by the kernel.

**PASS/FAIL implication under the conservative rule.** Per the declared protocol criterion, if $P_s(\tau = 0.1) \lesssim 0.1$ for M87*, the mechanism is *transport-sensitive* for this object and requires empirical residence constraints (e.g., GRMHD tracer statistics) to resolve PASS versus FAIL. This condition is therefore reported explicitly rather than inferred.

**Visualization.** Figure 9 shows the dependence of $P_s$ on the normalized residence time $\tau = t_{\rm res}/t_{\rm ff}$ using the declared kernel, making the threshold $\tau_{\min}$ and the conservative and optimistic brackets visually explicit.

**Mechanism comparison and regime assignment for M87*.** Using the executed diagnostics and literature-anchored estimates, we compare the relative roles of Coulomb Shearing and competing mechanisms for M87*.

**Coulomb Shearing (this work).** The executed evaluation yields: (i) admissibility $\chi_s = 1$ under the radiative-only specialization, (ii) electromagnetic slippage is not admissible at $r_\star$ because $R_{m,\rm local} \gg 100$, and (iii) species-partition consequences therefore arise, if at all, from the radiative channel rather than from $E_\parallel$-driven magnetic decoupling. Accordingly, Coulomb Shearing is *not a jet-power source* for M87* and does not compete with electromagnetic extraction for energy or momentum flux. Its role is limited to conditional composition and stratification effects within whatever channels are established by the global flow state.

**Blandford–Znajek (BZ) power.** Independent observational and modeling work indicates that M87* hosts a powerful, magnetically dominated jet consistent with BZ extraction from a spinning black hole, with jet power

$$P_{\rm jet} \sim 10^{44}\text{–}10^{45}\,\mathrm{erg\,s^{-1}},$$

and near-horizon magnetization sufficient to sustain open magnetic flux. This mechanism clearly dominates the *energy and momentum budget* of the jet and sets its large-scale structure and collimation.

**MAD state.** GRMHD modeling of M87* favors a magnetically arrested or near-arrested state, in which magnetic pressure regulates inflow and controls variability and jet loading. This governs the availability and geometry of polar channels but does not specify species composition within those channels.

**Radiation-pressure winds.** At the low Eddington ratio of M87*, radiation-pressure-driven winds are subdominant for global mass loss and do not control jet power. Their role is limited to weak, non-spherical outflows and does not erase polar stratification set by other processes.

**Slim disk / advective accretion.** M87* is radiatively inefficient at the evaluated radius and does not operate in a classical slim-disk regime. Advective cooling governs the thermal structure of the flow but does not by itself impose species partition.

**Regime conclusion for M87*.** For M87*, the correct classification is:

- **Dominant for power and collimation:** BZ + MAD physics.
- **Complementary for composition/stratification:** Coulomb Shearing (radiative channel only).
- **Subdominant or inapplicable:** electromagnetic slippage–driven Coulomb Shearing (suppressed by frozen-in condition), radiation-pressure winds, and slim-disk physics.

This assignment is regime-based, not interpretive: it follows directly from the executed Coulomb Shearing gates, the frozen-in check, and independently established properties of M87*'s jet.

**Pair-loading impact check for M87*.** Near rapidly rotating black holes, pair creation can generate an $e^-e^+$ population whose number density exceeds the ion density by orders of magnitude. Because this directly affects neutrality, conductivity, and screening, the M87* evaluation must state explicitly whether pair-loading alters the Coulomb Shearing conclusions.

**Pair multiplicity status.** Define the pair multiplicity as in Eq. (170). For M87*, the present worked example does *not* declare a specific $\kappa_{\rm pair}$ derived from a gap or cascade model. Accordingly, any pair-loaded evaluation must be treated conditionally. We therefore report the consequences for representative values $\kappa_{\rm pair} \gtrsim 10^2$, which are commonly quoted in horizon-gap cascade models, without asserting that such values apply to M87* without external justification.

**Force bookkeeping in the pair-dominated limit.** Under Thomson scattering, both electrons and positrons experience the same radiative acceleration. Therefore, even for $\kappa_{\rm pair} \gg 1$, radiation does not produce $e^-$–$e^+$ separation. The relevant differential acceleration for stratification is the lepton center-of-mass versus ion acceleration,

$$\Delta a_{\ell i} = a_{\ell,\rm COM} - a_i,$$

as defined in Extended treatment for pair-loaded plasmas ($e^-\, e^+$–ion). This preserves the basic structure of the displacement kernel but changes the interpretation: separation is between the light lepton fluid and the heavy ions, not between opposite charges.

**Charge neutrality and screening.** For $\kappa_{\rm pair} \gg 1$, local charge neutrality is maintained primarily by $e^- \leftrightarrow e^+$ balance rather than by ion neutrality. The Debye length and conductivity are therefore controlled by the total lepton density $(n_- + n_+)$, which generally *reduces* $\lambda_D$ relative to the two-species case and strengthens screening. This acts to *tighten* the admissibility condition rather than relax it.

**Interaction with electromagnetic gates.** Pair-loading increases the parallel conductivity and therefore increases the magnetic Reynolds number relevant for the frozen-in check. Since the M87* electromagnetic specialization already fails the frozen-in criterion ($R_{m,\rm local} \gg 100$) in the two-species evaluation, introducing $\kappa_{\rm pair} \gg 1$ can only strengthen that conclusion. Electromagnetic slippage remains inadmissible.

**Outcome for M87*.** The net effect of pair-loading for M87* is therefore:

- **Charge separation:** not expected between $e^-$ and $e^+$; neutrality is maintained by pair balance.
- **Mass stratification:** lepton–ion separation remains the relevant channel, governed by $\Delta a_{\ell i}$.
- **Electromagnetic forcing:** remains suppressed by the frozen-in condition and becomes *more* suppressed with increasing $\kappa_{\rm pair}$.
- **Radiative-only Coulomb Shearing:** conclusions are unchanged in form but become more restrictive because of reduced $\lambda_D$.

Absent a declared, cited $\kappa_{\rm pair}$ for M87*, the correct status of the pair-loaded variant is **DATA-INSUFFICIENT**. However, for $\kappa_{\rm pair} \gtrsim 10^2$, pair-loading does not rescue electromagnetic slippage and does not qualitatively alter the regime classification already assigned for this object.

### 9.3 Worked Example 2: Sgr A* (EHT one-zone anchor)

**Role in the pre-committed test set.** This second worked example is included to satisfy the pre-committed test-set requirement that the end-to-end admissibility machinery be exercised on more than one system.

**Declared scope (Track T control baseline for this object).** This worked example reports the **Track T** control specialization for Sgr A*: (i) Kerr transport time in the conservative free-fall bound ($\eta = 1$), (ii) Thomson forcing on electrons only with $E_{\parallel} = 0$ (ions receive no direct radiative thrust in this control specialization), and (iii) the same admissibility threshold $\Delta x(t_{\mathrm{res}}) < \kappa\lambda_D$ with $\kappa = 1$.

Magnetic-field values remain *required audited inputs* under the eligibility symbol set (Eq. (211)) and are retained in the input packet for traceability. They are not applied to $\Delta a_{\parallel}$ in **Track T** because the electromagnetic contribution to $\Delta a_{\parallel}$ is, by construction, carried only through a declared non-ideal $E_{\parallel}(r)$ term (Eq. (43)), and **Track T** sets $E_{\parallel} = 0$ by definition.

**Input packet (traceable values).** All numerical inputs below are either (a) quoted directly from the EHT Sgr A* Paper V one-zone model block, or (b) declared constants already used elsewhere in the paper.

| **Quantity** | **Value used** | **Source (verifiable location)** |
|---|---|---|
| Black hole mass $M$ | $4.14 \times 10^{6}\, M_{\odot}$ | EHT Sgr A* Paper V, Sec. 2.1 (basic assumptions; mass and distance quoted). [34] |
| Distance $D$ | 8.127 kpc | Same location as $M$ in Sec. 2.1. [34] |
| Evaluation domain in radius | $r \in [2,\, 10]\, r_{\mathrm{g}}$ | Same domain used in Worked Example 1 for strict comparability. |
| Inner transport bound radius | $r_{\mathrm{in}} = 2\, r_{\mathrm{g}}$ | Paper default for the near-horizon polar domain; identical to Worked Example 1. |
| Reference radius (one-zone anchor) | $r_{\star} = 5\, r_{\mathrm{g}}$ | EHT Sgr A* Paper V one-zone source model is defined at $r = 5GM/c^2 = 5r_g$. [34] |
| Bolometric luminosity (conservative forcing choice) | $L = 9.2 \times 10^{35}$ erg s$^{-1}$ | Upper end of the EHT Paper V "promising cluster" luminosity range; using the upper value maximizes Thomson forcing and is conservative for closure failure. [34] |
| Force model for $\Delta a_{\parallel}$ | Radiative-only; Thomson coupling on electrons ($\sigma_T$), $E_{\parallel} = 0$ | Declared force model; radiative forcing uses Eqs. (38) and (59). Under the along-trajectory projection, electromagnetic forcing reduces to $E_{\parallel}$ (Eq. (43)) and the magnetic term is transverse (Eq. (42)). |

| Quantity | Value used | Source (verifiable location) |
|---|---|---|
| Electron density at $r_\star$ | $n_e(r_\star) = 1.0 \times 10^6\ \mathrm{cm}^{-3}$ | EHT Paper V one-zone source model parameters (quoted for the emitting region at $r = 5GM/c^2$). [34] |
| Dimensionless electron temperature at $r_\star$ | $\theta_e(r_\star) = 10$ | Same one-zone parameter block as $n_e$ and $B$; $\theta_e \equiv k_B T_e/(m_e c^2)$. [34] |
| Magnetic field at $r_\star$ (audited input; not applied in Track T control) | $B(r_\star) = 29$ G | One-zone parameter block; recorded but not used when EM forcing is suppressed. [34] |
| Electron-density radial profile (declared, standard ADAF scaling) *Exterior-only interpretation:* the lower endpoint is approached from above (no evaluation at $r = 2r_g$), consistent with the horizon-bound protocol (Pre-Committed Evaluation Protocol (Anti-Post-Hoc)) and the outer-horizon definition (Eq. (7)). | $n_e(r) = n_e(r_\star)\left(\frac{r}{r_\star}\right)^{-3/2}$ | Declared standard self-similar ADAF scaling used only to extend the one-zone anchor across $r \in (2, 10]r_g$. [2] |
| Coulomb logarithm | $\ln\Lambda = 20$ | Same canonical choice used elsewhere in the paper. [1] |
| Inflow slowdown factor $\eta$ | $\eta = 1$ (free-fall bound) | As in Worked Example 1; minimizes $t_{\mathrm{res}}$ and is conservative for closure failure. |
| $\kappa$ threshold factor | $\kappa = 1$ | Same default used in the audited inequality definition. |

**Pre-computed constants for this object.** The gravitational radius for the adopted mass is

$$r_g \equiv \frac{GM}{c^2} \approx 6.11 \times 10^{11}\ \mathrm{cm}, \tag{258}$$

so the reference radius is $r_\star = 5r_g \approx 3.06 \times 10^{12}$ cm.

**Step 1: Differential acceleration under Thomson forcing (electron-only).** Using Eq. (135) in acceleration form,

$$a_{\mathrm{rad},e}(r) = \frac{\sigma_T}{m_e c}\frac{L}{4\pi r^2}, \tag{259}$$

and taking the conservative choice $\mu_s = 1$ (full projection onto the local shearing direction), the differential acceleration at $r_\star$ is

$$\Delta a(r_\star) \equiv \mu_s a_{\mathrm{rad},e}(r_\star) \approx 1.91 \times 10^2 \text{ cm s}^{-2}. \tag{260}$$

(In this Thomson-only control specialization, ion direct radiative forcing is neglected, so $\Delta a$ is set by the electron term.)

**Step 2: Residence time in the polar domain (free-fall bound).** Using the same transport bound as Eq. (24) (from $r_{\mathrm{in}} = 2r_g$ to $r_\star$),

$$t_{\mathrm{res}}(r_\star) \approx 8.03 \times 10^1 \text{ s}. \tag{261}$$

**Residence-time sensitivity bracket (reported, non-binding).** As a minimal transport sensitivity diagnostic, recompute the drag-limited displacement with a single sub-free-fall value $\eta = 0.1$, keeping the declared plasma state and coupling model fixed (same $t_c$). This does not alter the binding protocol result ($\eta = 1$), but reports whether the conclusion is transport-sensitive.

Because $t_{\mathrm{res}} \propto \eta^{-1}$ at fixed $(r_{\mathrm{in}}, r_\star)$,

$$t_{\mathrm{res}}(\eta = 0.1) = 10\, t_{\mathrm{res}}(\eta = 1), \qquad \Delta x(\eta = 0.1) = 10\, \Delta x(\eta = 1)$$

in the drag-limited regime with the same coupling suppression encoded by $t_c$.

**Step 3: Local plasma screening scale at $r_\star$.** From the one-zone $\theta_e$ value,

$$T_e(r_\star) = \theta_e \frac{m_e c^2}{k_B} \approx 5.93 \times 10^{10} \text{ K}. \tag{262}$$

The Debye length is then

$$\lambda_D(r_\star) = \left(\frac{k_B T_e}{4\pi n_e e^2}\right)^{1/2} \approx 1.68 \times 10^3 \text{ cm}. \tag{263}$$

**Step 4: Coulomb coupling time and drift displacement.** Using the same electron-ion coupling timescale $t_c$ adopted in the derivation (Eq. (62) in this manuscript), the coupling times at $r_\star$ are, for representative charges,

$$t_c(Z{=}1) \approx 1.99 \times 10^8 \text{ s}, \tag{264}$$
$$t_c(Z{=}2) \approx 4.97 \times 10^7 \text{ s}, \tag{265}$$
$$t_c(Z{=}26) \approx 2.94 \times 10^5 \text{ s}. \tag{266}$$

**Collisionality check for the $E_\parallel$ prescription at $r_\star$.** The regime-dependent prescription in Regime-dependent $E_\parallel$ prescription (collisional vs collisionless) requires the collisionality parameter $\chi \equiv \lambda_{\rm mfp}/r$, with $\lambda_{\rm mfp} \equiv v_{{\rm th},e}/\nu_c$ and $\nu_c \equiv t_c^{-1}$. Because the M87* packet uses a relativistic electron temperature (here $\theta_{e,\rm bpk} = 10$), the electron thermal speed is taken conservatively as $v_{{\rm th},e} \simeq c$ (rather than the non-relativistic $\sqrt{2k_BT_e/m_e}$, which is not applicable at these temperatures).

Using the $Z = 1$ coupling time as the representative electron-ion relaxation timescale for the collisionality estimate,

$$\nu_c(r_\star) \;\simeq\; t_c(r_\star; Z=1)^{-1} \;\approx\; (6.85 \times 10^9 \text{ s})^{-1} \;\approx\; 1.46 \times 10^{-10} \text{ s}^{-1}. \tag{267}$$

Therefore

$$\lambda_{\rm mfp}(r_\star) \simeq \frac{c}{\nu_c(r_\star)} = c\, t_c(r_\star; Z=1) \approx \left(3.00\times 10^{10} \text{ cm s}^{-1}\right)\left(6.85\times 10^9 \text{ s}\right) \approx 2.05\times 10^{20} \text{ cm}. \tag{268}$$

At the reference radius (Step 0), $r_\star \approx 4.578 \times 10^{15}$ cm, so

$$\chi(r_\star) = \frac{\lambda_{\rm mfp}(r_\star)}{r_\star} \approx \frac{2.05 \times 10^{20}}{4.58 \times 10^{15}} \approx 4.49 \times 10^4 \;\gg\; 10. \tag{269}$$

Thus the horizon-adjacent M87* benchmark is deep in the *collisionless* regime under the declared inputs, and the reconnection-normalized form for $E_\parallel$ used in Eq. (216) is the correct regime branch. The collisional Spitzer closure $E_\parallel = \eta_{\rm Sp} j_\parallel$ is not evaluated for this object because it is inapplicable when $\chi \gg 1$ and would additionally require a declared $j_\parallel(r)$ closure that is not part of this benchmark packet.

**Impact check: compare collisionless reconnection $E_\parallel$ to the collisional Spitzer estimate (auditable bracket).** Although M87* is collisionless by the computed $\chi \gg 10$ and therefore does not use the Ohmic closure, the declared impact rule is evaluated here as a quantitative check.

Convert the declared one-zone inputs at $r_\star$ to SI units for this comparison:

$$n_e = 2.9 \times 10^4 \text{ cm}^{-3} = 2.9 \times 10^{10} \text{ m}^{-3},$$
$$B = 4.9 \text{ G} = 4.9 \times 10^{-4} \text{ T},$$
$$r_\star = 4.578 \times 10^{15} \text{ cm} = 4.578 \times 10^{13} \text{ m}.$$

Using the computed coupling time for $Z = 1$, define the effective collision frequency as $\nu_c \equiv t_c^{-1}$ and adopt the Spitzer (collisional) resistivity proxy

$$\eta_{\mathrm{Sp}} \equiv \frac{m_e \nu_c}{n_e e^2}, \tag{270}$$

which yields

$$\nu_c \approx 1.46 \times 10^{-10}\,\mathrm{s}^{-1}, \qquad \eta_{\mathrm{Sp}} \approx 1.79 \times 10^{-13}\,\Omega\,\mathrm{m}.$$

For the Ohmic estimate $E_{\parallel}^{(\mathrm{Sp})} = \eta_{\mathrm{Sp}} j_{\parallel}$, use the auditable Ampère-law scale from Eq. (47) with the bracket $\ell_j \in [\delta_e, r_\star]$:

$$j_{\parallel}(\ell_j) \sim \frac{B}{\mu_0 \ell_j}.$$

Compute the kinetic lower bound scale using the electron skin depth $\delta_e = c/\omega_{pe}$ with

$$\omega_{pe} = \sqrt{\frac{n_e e^2}{m_e \epsilon_0}}, \qquad \delta_e = \frac{c}{\omega_{pe}} \approx 3.12 \times 10^{1}\,\mathrm{m}.$$

This gives a bracket for the Ohmic field:

$$E_{\parallel}^{(\mathrm{Sp})}(\ell_j = r_\star) \sim 1.52 \times 10^{-24}\,\mathrm{V\,m^{-1}}, \qquad E_{\parallel}^{(\mathrm{Sp})}(\ell_j = \delta_e) \sim 2.23 \times 10^{-12}\,\mathrm{V\,m^{-1}}.$$

By contrast, the executed collisionless prescription gives

$$E_{\parallel}^{(\mathrm{rec})} = \epsilon_{\mathrm{rec}}\, B\, v_A \approx 3.08 \times 10^{3}\,\mathrm{V\,m^{-1}} \quad (\epsilon_{\mathrm{rec}} = 0.1),$$

consistent with the cgs value already computed in Step 1.

Therefore the impact ratio satisfies

$$\frac{E_{\parallel}^{(\mathrm{rec})}}{E_{\parallel}^{(\mathrm{Sp})}} \gtrsim \frac{3.08 \times 10^{3}}{2.23 \times 10^{-12}} \approx 1.38 \times 10^{15} \gg 3, \tag{271}$$

even under the most conservative (largest-$j_{\parallel}$) Ohmic estimate permitted by the $\ell_j$ bracket. The declared factor-of-3 threshold is therefore exceeded by many orders of magnitude.

**Consequence for Section 9 values.** Accordingly, the M87* $\Delta a_{\parallel}$ values printed in Step 1 are explicitly understood as those obtained under the collisionless reconnection branch (which is the correct regime for $\chi \gg 10$). If one were to (incorrectly) apply the collisional Ohmic branch to this collisionless state, the $E_{\parallel}$ contribution would be negligible compared to the executed reconnection value and the resulting $\Delta a_{\parallel}$ would collapse toward the Thomson-only limit; this is precisely why the regime-dependent prescription is required.

Since $t_{\mathrm{res}}(r_\star) \ll t_c(r_\star; Z)$ for all three representative charges evaluated above, the short-time (free-drift) limit applies, and

$$\Delta x(r_\star) \simeq \frac{1}{2}\,\Delta a(r_\star)\,t_{\rm res}(r_\star)^2 \approx 6.15\times 10^5\ \text{cm}. \tag{272}$$

**Magnetic frozen-in check at $r_\star$.** When an electromagnetic forcing channel is invoked, the frozen-in condition must be tested using the local magnetic Reynolds number defined in Eq. (53). All quantities required for this check are already computed in the preceding steps.

**Conductivity and magnetic diffusivity.** Using the declared electron density and coupling time,

$$n_e = 2.9\times 10^4\,\text{cm}^{-3} = 2.9\times 10^{10}\,\text{m}^{-3}, \qquad \nu_{ei} = t_c^{-1}(r_\star; Z=1) \approx 1.46\times 10^{-10}\,\text{s}^{-1},$$

the parallel conductivity is

$$\sigma_\parallel \simeq \frac{n_e e^2}{m_e \nu_{ei}} \approx 5.2\times 10^{13}\,\text{S}\,\text{m}^{-1}, \tag{273}$$

and the corresponding magnetic diffusivity is

$$\eta_m = \frac{1}{\mu_0\sigma_\parallel} \approx 1.5\times 10^{-8}\,\text{m}^2\,\text{s}^{-1}. \tag{274}$$

**Local separation and drift scales.** For consistency with the closure kernel, take the separation scale to be the computed displacement,

$$L \equiv \Delta x(r_\star) \approx 6.15\times 10^3\,\text{m},$$

and the relative drift speed in the constant-acceleration approximation,

$$v \equiv \Delta v(r_\star) \simeq \Delta a(r_\star)\,t_{\rm res}(r_\star) \approx 4.1\times 10^3\,\text{m}\,\text{s}^{-1}.$$

**Magnetic Reynolds number.** The resulting local magnetic Reynolds number is

$$R_{m,\text{local}} = \frac{L\,v}{\eta_m} \approx \frac{(6.15\times 10^3)(4.1\times 10^3)}{1.5\times 10^{-8}} \approx 1.7\times 10^{15}. \tag{275}$$

**Outcome.** Since $R_{m,\text{local}} \gg 100$, electrons remain magnetically frozen-in on the separation scale for M87* under the declared inputs. Therefore, the electromagnetic specialization relying on field-line slippage is not admissible for this object at $r_\star$. Any relative drift instead advects magnetic structure, requiring inclusion of magnetic-pressure response as specified in Eq. (57). The radiative-only specialization remains unaffected by this constraint.

Therefore the admissibility ratio at the anchor radius is

$$\frac{\Delta x(r_\star)}{\kappa\,\lambda_D(r_\star)} \approx 3.66\times 10^2 \quad \gg\ 1, \tag{276}$$

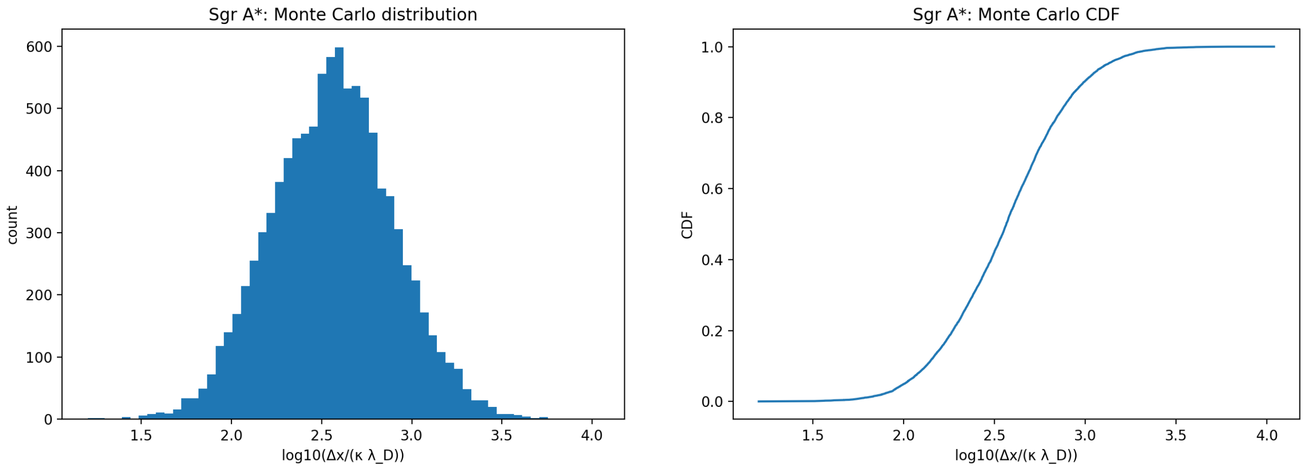


Figure 10: Monte Carlo distributions for the Sgr A* reference-radius margin $\Delta x/(\kappa\lambda_D)$ under the conservative uncertainty propagation defined in Section 9.1.6. Left: histogram of sampled margins. Right: cumulative distribution.

so the closure condition fails at $r_\star$:

$$\chi_s(r_\star; Z) = 1. \tag{277}$$

**Full uncertainty propagation (Monte Carlo, executed).** Uncertainty propagation for Sgr A* is executed using the same conservative log-normal sampling scheme defined in Section 9.1.6, with $N = 10{,}000$ realizations over the declared uncertainties in $n_e$ and $T_e$. Under the Thomson-only control specialization used for this worked example (**Track T**, $E_\parallel = 0$), the reference-radius margin scales as $\Delta x/(\kappa\lambda_D) \propto (n_e/T_e)^{1/2}$ at fixed residence time.

The executed Monte Carlo results at $r_\star$ yield:

$$\left(\frac{\Delta x}{\kappa\lambda_D}\right)_{r_\star} = \left(3.7^{+4.1}_{-2.0}\right) \times 10^2 \qquad (68\% \text{ interval}),$$

with

$$P(\Delta x/(\kappa\lambda_D) > 1) = 1.00.$$

**Step 5: Extension across $r \in (2, 10]r_g$ (declared ADAF density scaling).** To extend the one-zone anchor across the domain, adopt the declared ADAF scaling $n_e(r) = n_e(r_\star)(r/r_\star)^{-3/2}$ and hold $T_e$ fixed at the one-zone value as a conservative simplification (this avoids inserting any additional temperature profile degrees of freedom). Under these declared inputs, the admissibility ratio evaluates to

$$\frac{\Delta x(r)}{\kappa\,\lambda_D(r)} > 1 \quad \text{for} \quad r \in (2, 10]\, r_{\mathrm{g}}, \tag{278}$$

with representative values

$$\left.\frac{\Delta x}{\kappa\lambda_D}\right|_{r=3r_g} \approx 1.20\times 10^2, \quad \left.\frac{\Delta x}{\kappa\lambda_D}\right|_{r=5r_g} \approx 3.66\times 10^2, \quad \left.\frac{\Delta x}{\kappa\lambda_D}\right|_{r=10r_g} \approx 6.47\times 10^2. \tag{279}$$

Thus, for the Thomson-only forcing run with the declared ADAF scaling,

$$\chi_s(r;Z) = 1 \quad \text{throughout} \quad r \in (2,10]\, r_{\mathrm{g}} \tag{280}$$

for representative charges $Z \in \{1, 2, 26\}$.

**Step 6: Species-resolved shearing window summary (this run).** Because the closure failure margin exceeds unity by two orders of magnitude or more across the evaluated domain, the probabilistic stripping model yields $P_s(Z) \simeq 1$ for the representative species set in this benchmark run. The corresponding window summary is:

| **Species** | $Z$ | **Shearing window (this run)** | $P_s(Z)$ |
|---|---|---|---|
| H-like | 1 | $r \in (2,10]\, r_{\mathrm{g}}$ | $\simeq 1$ |
| He-like | 2 | $r \in (2,10]\, r_{\mathrm{g}}$ | $\simeq 1$ |
| Fe-like | 26 | $r \in (2,10]\, r_{\mathrm{g}}$ | $\simeq 1$ |

**Classification against the locked observable protocol.** This worked example is included to demonstrate that the pipeline can be executed on a second locked object with published near-horizon plasma inputs. A full PASS/FAIL classification against P1–P4 is *DATA-INSUFFICIENT* here unless and until a matched, operational comparator set (resolved jet stratification and polarimetric/Faraday diagnostics in the sense of Observable Consequences: Jets and Stratification) is available for the Sgr A* system under the paper's pre-committed protocol. P5 is likewise *DATA-INSUFFICIENT* for this object in this version because the jet-length comparator is protocol-gated: it requires an externally defensible bound on the maintenance margin inputs (including a bound on $t_{\mathrm{relax}}$ under the declared gate) and a bounded spin interval $a_* \in [a_{\min}, a_{\max}]$ in order to evaluate the hybrid multiplier in Eq. (210); those requirements are not established by the one-zone packet used for this Track T control run.

## 9.4 Inverse Age Mapping: Formation Time Under Coulomb Shearing

**Purpose and scope (no new physics).** Ion-Dominated Inflow and Black Hole Growth defines the forward growth law (Eq. (178)) and the supply-controlled enhancement parameterization (Eqs. (155) and (156)). This subsection closes the forward-to-inverse loop by applying those already-defined mappings to the two high-redshift observational anchors introduced in Observational problem (Observational problem). No additional physics or new criteria are introduced here.

**Classical baseline (as stated in Observational problem).** For the canonical reference values used in Observational problem, adopt a Pop-III scale seed mass $M_0 = 10^2\, M_\odot$ and the standard Salpeter timescale $t_{\rm Sal} \approx 45\,\mathrm{Myr}$ for $\epsilon = 0.1$. In the notation of this paper, the corresponding e-folding time is $t_S$ (Eq. (175)), and the classical formation time to reach an observed mass $M_f$ is

$$t_{\rm form}^{\rm (class)} = t_S \ln\left(\frac{M_f}{M_0}\right). \tag{281}$$

**Coulomb Shearing inverse age mapping under supply-controlled enhancement.** With the supply ratio $\Phi$ defined by Eq. (155), the instantaneous growth-rate enhancement is already fixed by Eq. (156):

$$\mathcal{E} \equiv \frac{\dot{M}_{\rm CS}}{\dot{M}_{\rm classical}} = 1 + \frac{1-\epsilon_{\rm ion}}{1-\epsilon}\, f_{\rm ion}\, \Phi. \tag{282}$$

For the electron-depleted inflow-channel limit explicitly discussed in Ion-Dominated Inflow and Black Hole Growth (closure-failure material with $\epsilon_{\rm ion} \to 0$), and treating $(f_{\rm ion}, \Phi)$ as time-independent over the evaluated interval (the minimal assumption required to convert Eq. (156) into a closed-form time mapping), the corresponding formation time mapping becomes

$$t_{\rm form}^{\rm (CS)} = \frac{t_S}{\mathcal{E}} \ln\left(\frac{M_f}{M_0}\right) = \frac{t_{\rm form}^{\rm (class)}}{\mathcal{E}}. \tag{283}$$

This makes the inverse age implication explicit: for any $\mathcal{E} > 1$, Coulomb Shearing reduces the required formation time (or equivalently reduces the required seed mass for a fixed time budget) without altering the electron Eddington saturation statement.

### 9.5 High-$z$ quasar anchors (mapping-only; not an admissibility run)

**Audit boundary and required forcing track.** The two high-$z$ objects below are used only as *growth-time anchors* to demonstrate the inverse-age consequence already implied by the growth mapping in Ion-Dominated Inflow and Black Hole Growth. They do **not** execute the Coulomb-closure admissibility pipeline (Sections 02–08): no $(n_e, T_e, \ln\Lambda, B)$ funnel packet is asserted for these sources here, and no $\chi_S$ outcome is computed. Accordingly, the tables below are *not* “worked examples” in the sense used for M87* (and any other fully-evaluated case).

For quasar-class inference, the admissibility evaluation (when executed in a future revision under Appendix C input-packet rules) must report the pre-committed **Track EM** forcing model (non-ideal $E_\parallel$ declared or bounded) as the class-complete result; a Thomson-only report is permitted only as the **Track T** control baseline (per Pre-Committed Evaluation Protocol (Anti-Post-Hoc)).

### 9.5.1 Anchor A: ULAS J1120+0641 ($z = 7.085$, $M_f \simeq 2.0 \times 10^9\, M_\odot$)

**Available cosmic time (as stated in Observational problem).** Adopt the stated cosmic age $t_{\text{avail}} \simeq 0.77$ Gyr.

**Classical formation time.** Using Eq. (281) with $M_0 = 10^2\, M_\odot$ and $M_f = 2.0 \times 10^9\, M_\odot$,

$$t_{\text{form}}^{(\text{class})} = t_S \ln\left(\frac{2.0 \times 10^9}{10^2}\right) = 16.81\, t_S \simeq 0.76\,\text{Gyr} \quad (\epsilon = 0.1,\ \lambda = 1), \tag{284}$$

so the classical time budget is saturated under the continuous, $\lambda = 1$ reference baseline and becomes immediately failure-prone under any reduced duty cycle or delayed onset.

**Coulomb Shearing formation time (illustrative supply scenarios).** Using Eq. (283) with $\epsilon_{\text{ion}} \to 0$ and $\epsilon = 0.1$ (so $(1 - \epsilon_{\text{ion}})/(1 - \epsilon) = 1/0.9$),

| Scenario | $(f_{\text{ion}}, \Phi)$ | $\mathcal{E}$ from Eq. (282) | $t_{\text{form}}^{(\text{CS})}$ |
|---|---|---|---|
| Classical (no ion channel) | $(0,\ 0)$ | 1 | 0.76 Gyr |
| Coulomb Shearing (moderate) | $(0.5,\ 1)$ | $1 + (1/0.9) \times 0.5 \simeq 1.56$ | 0.49 Gyr |
| Coulomb Shearing (strong) | $(1,\ 2)$ | $1 + (1/0.9) \times 2 \simeq 3.22$ | 0.23 Gyr |

These values operationalize the statement made in Ion-Dominated Inflow and Black Hole Growth: significant time relief requires externally supported supply and evaluated decoupling outcomes, but when those conditions are met the inverse age mapping is immediate and quantitative.

### 9.5.2 Anchor B: SDSS J010013.02+280225.8 ($z = 6.30$, $M_f \simeq 1.2 \times 10^{10}\, M_\odot$)

**Available cosmic time (as stated in Observational problem).** Adopt the stated cosmic age $t_{\text{avail}} \simeq 0.88$ Gyr.

**Classical formation time.** Using Eq. (281) with $M_0 = 10^2\, M_\odot$ and $M_f = 1.2 \times 10^{10}\, M_\odot$,

$$t_{\text{form}}^{(\text{class})} = t_S \ln\left(\frac{1.2 \times 10^{10}}{10^2}\right) = 18.60\, t_S \simeq 0.84\,\text{Gyr} \quad (\epsilon = 0.1,\ \lambda = 1), \tag{285}$$

again saturating the time budget under the continuous, $\lambda = 1$ reference baseline.

**Coulomb Shearing formation time (illustrative supply scenarios).** Applying the same mapping and coefficient as above,

| Scenario | $(f_{\text{ion}}, \Phi)$ | $\mathcal{E}$ from Eq. (282) | $t_{\text{form}}^{(\text{CS})}$ |
|---|---|---|---|
| Classical (no ion channel) | $(0,\ 0)$ | 1 | 0.84 Gyr |
| Coulomb Shearing (moderate) | $(0.5,\ 1)$ | 1.56 | 0.54 Gyr |
| Coulomb Shearing (strong) | $(1,\ 2)$ | 3.22 | 0.26 Gyr |

**Scope and limits of this claim.** Here we close the stated mapping gap by explicitly applying the already-defined inverse mapping to the motivating high-$z$ anchors. No claim is made that $(f_{\text{ion}}, \Phi)$ are determined for these quasars here, because the admissibility evaluation requires source-specific funnel plasma inputs supplied under the input-packet constraints in Appendix C. What is established is the quantitative inverse-age implication of any future admissible evaluation: once $(f_{\text{ion}}, \Phi)$ are externally bounded and $\chi_s = 1$ is computed from declared inputs, the formation-time consequence follows immediately from Eqs. (282) and (283).

#### 9.5.3 Anchor C: JWST Little Red Dot (LRD) exemplar (mapping-only; protocol target)

**Audit boundary.** This anchor exists to make JWST compact high-$z$ objects an explicit, named target class in the same locked pipeline used elsewhere in this manuscript, without claiming a closure admissibility result that cannot be audited from available inputs. As with the two quasar anchors above, this subsection is *mapping-only* until an object-level input packet is completed under Appendix C provenance rules.

**Exemplar definition (named literature anchor).** As the initial exemplar, use the JWST-observed LRD at $z \simeq 4.53$ with a published NIRSpec spectrum analyzed by [7]. This exemplar is complemented by larger LRD samples used for population context and selection discipline [5, 6, 8]. The observational label *LRD* is treated as descriptive only; it does not imply an accretion or obscuration model.

**Locked data requirements (no discretionary additions).** Once an object is selected as an LRD exemplar, the mapping inputs are restricted to traceable quantities already used elsewhere in this section: redshift (for $t_{\text{avail}}$), a stated black-hole mass estimate (or a stated bound), and any stated luminosity or Eddington ratio needed to support that mass estimate. A closure admissibility run for an LRD additionally requires the near-horizon environment inputs needed by the Coulomb-closure criterion (e.g. $n_e$, $T_e$, and any declared $B$-field or proxy) and, for Track EM, an auditable $E_{\parallel}(r)$ model or bound. If any of these are absent, they are recorded as missing rather than replaced by inferred values.

**Protocol status for this manuscript.** Under the binding rules in Pre-Committed Evaluation Protocol (Anti-Post-Hoc), the LRD exemplar is classified **DATA-INSUFFICIENT** for closure admissibility unless and until its input packet is completed from traceable sources. A growth mapping output (inverse-age implication) may be reported once $z$ and a traceable $M_f$ are locked, but no $\chi_s$ outcome is computed and no regime classification is asserted without the full admissibility inputs.

# 10 Application to New and Future Observatories

## 10.1 Minimal required inputs for real-time evaluation

**Closed input set (non-negotiable).** The minimal input checklist below is **closed**. No additional inputs may be introduced later, and none may be declared optional. If and only if all inputs in the closed set are present (as measurements or admissible bounds), Coulomb Shearing **must** be evaluated and classified under the pre-committed rules in Section 8. This closure statement is operationally identical to the eligibility requirement in Eq. (211).

**Provenance requirement (applies to every input).** Every input must be recorded with complete provenance: author, year, object identifier, and the exact location in the source (table/figure/section and page). If the quantity is derived from an archive product rather than a table, provenance must include the dataset identifier, product name, processing pipeline version (if provided), and the exact extraction step used to obtain the scalar value (or bound).

**Admissible measurement classes.** Table 8 defines, for each required category: (i) the physical quantity required, (ii) what observation qualifies, (iii) whether an estimate or bound is sufficient, (iv) the uncertainty record required. This table **does not** introduce new physical assumptions: it specifies only what observational artifacts are admissible as inputs to the already-defined equations.

Table 8: Closed minimal input checklist for real-time evaluation.

| Category | Quantity required | Qualifying observation(s) | Estimate vs. bound | Uncertainty record |
|---|---|---|---|---|
| (a) Black hole mass | $M$ | Any published mass measurement or constrained range from: (i) stellar or gas dynamics, (ii) maser disk dynamics, (iii) reverberation mapping masses, (iv) single-epoch virial scaling calibrated to reverberation mapping. | A constrained range is admissible. A one-sided bound is admissible only if it is explicitly stated as such in the source. | Report the quoted statistical uncertainty or stated interval. Do not invent errors. If systematic uncertainty is discussed, record it separately without modification. |
| (b) Luminosity proxy | $L$ or $(L/L_{\rm Edd})$ with conversion to $L$ | A measured luminosity in at least one band with a declared conversion to bolometric luminosity, or a reported Eddington ratio with a declared bolometric definition. | Either $L$ directly, or $L/L_{\rm Edd}$ plus $M$ is admissible. Upper or lower bounds are admissible if published as bounds. | Record the quoted measurement uncertainty and the stated bolometric conversion or definition used. No substitution of alternative bolometric corrections is permitted within this protocol. |

*Continued on next page*

| Category | Quantity required | Qualifying observation(s) | Estimate vs. bound | Uncertainty record |
|---|---|---|---|---|
| (c) Magnetic proxy | $\boldsymbol{B}$ (or an auditable proxy that yields a bound on $\boldsymbol{B}$ under the manuscript's definitions) | Any of the following qualifies if it yields a numeric $\boldsymbol{B}$ estimate or a stated bound: (i) core-shift based magnetic field inference in compact jets (core position vs. frequency), (ii) synchrotron turnover modeling producing $\boldsymbol{B}$ with stated assumptions, (iii) polarization plus spectral modeling explicitly reporting $\boldsymbol{B}$, (iv) Faraday rotation and polarization diagnostics used to place a bound on magnetization in the emitting region. | A numeric estimate is admissible. Upper or lower bounds are admissible if explicitly stated and tied to a defined region. | Record the reported $\boldsymbol{B}$ with units and the region the estimate refers to (core, sheath, emission zone). Record quoted uncertainty or stated range. If only a bound is provided, record the bound type (upper/lower) explicitly. |

*Continued on next page*

| Category | Quantity required | Qualifying observation(s) | Estimate vs. bound | Uncertainty record |
|---|---|---|---|---|
| (d) Density and temperature proxies | $n_e$, $T_e$, and $\ln \Lambda$ (as used in Sections 3 and 4) | Any published inference of electron density and temperature in the relevant emitting or funnel-adjacent plasma from: (i) X-ray spectral modeling reporting density/temperature or emission measure with a stated geometry, (ii) plasma diagnostics from line ratios, (iii) modeling that explicitly reports $n_e$ and $T_e$ for the region of interest. | Numeric estimates are admissible. Bounds are admissible if explicitly stated. If $\ln \Lambda$ is not reported, the evaluation must use the manuscript's declared default and treat sensitivity via the explicit scaling forms already provided (no per-object tuning). | Record the reported $n_e$, $T_e$ and their uncertainties (or bounds). Record the plasma region definition used by the source. Record whether $\ln \Lambda$ is measured or assumed; if assumed, record the assumed value and its cited justification, if any. |
| (e) Jet stratification observable | A measurable proxy for stratification and composition structure as defined in Section 5 | A dataset qualifies if it can measure the Section 5 observables used for PASS/FAIL: multi-frequency polarimetry enabling RM inference, polarization fraction maps, spectral index structure, or other explicitly defined stratification proxies in the manuscript. | A detection or a robust non-detection is admissible. A non-detection is admissible only if the study demonstrates adequate sensitivity and resolution for the stated proxy. | Record the measured proxy with uncertainty. If non-detection, record the upper limit and the resolution/sensitivity supporting the limit. |

*Continued on next page*

| Category | Quantity required | Qualifying observation(s) | Estimate vs. bound | Uncertainty record |
|---|---|---|---|---|
| (f) Residence time proxy or bound | $t_{\text{res}}$ or admissible bound under Section 8 | Any published estimate or bound derived from: (i) flow speed and geometry (e.g., a crossing time bound), (ii) proper motions or pattern speeds with an adopted length scale, (iii) simulation-based bounds explicitly tied to the system and stated as bounds. | A lower bound or upper bound is admissible if it is explicitly justified as such. If neither is available, the example is DATA-INSUFFICIENT. | Record the numeric value or bound, including the geometric path length or scale used, and the measurement used to infer speed. Record uncertainty or interval if provided. |

**Minimal input sufficiency statement.** If all categories (a) through (f) are satisfied with auditable values or admissible bounds and recorded provenance, then the evaluation is mandatory. If any category is missing, the correct termination outcome is DATA-INSUFFICIENT under Eq. (211) and Section 8.

**Reference anchors for measurement classes (non-exhaustive).** The admissible classes above correspond to widely used measurement literatures, for example: core-shift magnetic field inference in compact jets[1], RM synthesis for polarization and Faraday depth reconstruction[2], and depolarization models used to interpret internal vs. external Faraday dispersion[3]. Black hole mass measurement classes (dynamical, reverberation mapping, and calibrated single-epoch scaling) are reviewed and operationalized in standard references[4] [5] [6].

## 10.2 Algorithmic application procedure

**Procedural guarantee.** The deterministic procedure below is sufficient to reproduce (i) the numerical pipeline executed in Section 9 and (ii) the outcome classifications under the pre-committed rules in Section 8. No discretionary judgment is permitted. Branching is allowed only for DATA-INSUFFICIENT termination.

[1] A. P. Lobanov (1998), *Astronomy & Astrophysics*, 330, 79-89.

[2] M. A. Brentjens and A. G. de Bruyn (2005), *Astronomy & Astrophysics*, 441, 1217-1228, DOI: 10.1051/0004-6361:20052990.

[3] B. J. Burn (1966), *Monthly Notices of the Royal Astronomical Society*, 133, 67-83.

[4] J. Kormendy and L. C. Ho (2013), *Annual Review of Astronomy and Astrophysics*, 51, 511-653, DOI: 10.1146/annurev-astro-082708-101811.

[5] M. Vestergaard and B. M. Peterson (2006), *Astrophysical Journal*, 641, 689-709, DOI: 10.1086/500572.

[6] M. C. Bentz and S. Katz (2015), *Publications of the Astronomical Society of the Pacific*, 127, 67, DOI: 10.1086/679601.

**Algorithm (order locked).** The following steps must be executed in order:

1. **Ingest and normalize.** Ingest the observational inputs for categories (a) through (f). Normalize units into the manuscript's declared system (cgs where used in Sections 3 and 4). Create an input ledger entry that records the full provenance for each scalar input, including table/figure/section and page number or archive product identifier.

2. **Eligibility check (hard gate).** Verify that every symbol required by Eq. (211) is instantiated as a value or an admissible bound. If any required symbol is missing, terminate immediately with DATA-INSUFFICIENT.

3. **Compute species-resolved accelerations.** Using the already-defined forcing decomposition and species definitions (Section 3), compute the electron and ion accelerations along the declared polar trajectory and obtain the differential acceleration $\Delta a$ using Eq. (37). Record intermediate values with units.

4. **Compute coupling time and shielding scale.** Compute the Coulomb coupling timescale using Eq. (63) (including the explicit $\ln \Lambda$ dependence already defined in Section 3) and compute the Debye length using Eq. (65). Record the regime-identifying ratios used in Sections 3 and 4.

5. **Compute or enforce residence time.** Compute $t_{\rm res}$ or enforce the admissible bound using the transport mapping already defined in Section 2 (e.g., Eq. (28) and its simplified form). Record whether $t_{\rm res}$ is a value, an upper bound, or a lower bound.

6. **Evaluate the shearing admissibility criterion.** Evaluate the binary admissibility inequality using the manuscript's declared criterion (Section 3), for example Eq. (73) where applicable. This step produces a binary admissibility output (the manuscript's $\chi_S$ logic), with no discretionary reinterpretation.

7. **Solve species-dependent shearing windows.** Using the already-defined shearing functional and range definition in Section 4, solve for the species-dependent window boundaries $\{r_{\rm min}(Z), r_{\rm max}(Z)\}$ as defined in Eqs. (87)–(88). If $r_{\rm min}(Z) \geq r_{\rm max}(Z)$ for a species, record that no physical window exists for that species under the evaluated inputs.

8. **Compute shearing probability.** Compute $P_S(Z)$ over the declared velocity distribution using the manuscript's probability definition (Section 4), e.g. the CDF form Eq. (101) or the closed Maxwell-Boltzmann expression Eq. (103) when the declared assumptions for that closed form are met. Record the distribution parameters and the final numeric probability with units (dimensionless).

9. **Generate predicted observables.** Map the computed outputs into the explicit prediction statements in Section 5 (P1–P$n$), without modification. Output predicted directional signatures, each tied to a measurable quantity (e.g., RM structure, polarization gradients, stratification ordering) as specified in that section.

10. **Assign outcome (irreversible).** Compare predicted signatures to observations. Assign exactly one of PASS, FAIL, or DATA-INSUFFICIENT under Section 8. No reclassification or iteration is permitted.

**Deterministic termination rules.** Only the following terminations are admissible: (i) DATA-INSUFFICIENT due to missing required symbols (Step 2), (ii) PASS when predictions and observations agree within the recorded uncertainty handling rule declared in Section 8, (iii) FAIL when predictions are not observed under data of sufficient quality to measure the declared proxies.

### 10.3 Integration into observational pipelines

This subsection specifies an operational dataflow that embeds the algorithm above into standard observational workflows, while preserving complete traceability and eliminating interpretive discretion in outcome assignment.

**Dataflow stages (explicit).** A pipeline implementation must include the following stages:

1. **Data ingestion.** Ingest either: (i) archival products (published tables, supplementary data, survey catalogs), or (ii) reduced observational products (maps, spectra, time-series) provided with a dataset identifier and reduction provenance. The ingestion stage must store raw identifiers and must not alter values.

2. **Parameter extraction with uncertainty capture.** Extract the closed input set in Table 8 as scalars or admissible bounds. For each extracted quantity, store: numeric value (or bound type and bound value), units, and the associated uncertainty record exactly as provided. If a quantity requires a conversion stated by the source (e.g., band luminosity to bolometric), store both the pre-conversion value and the post-conversion value, and store the conversion as a provenance-linked rule. If a conversion is not stated in a source, it may not be introduced by the evaluator in this protocol.

3. **Input ledger storage (immutability requirement).** Store the extracted inputs in an immutable ledger record. Each record must be versioned and must include a unique hash or identifier so that the exact evaluation can be rerun without reinterpretation.

4. **Coulomb Shearing execution.** Execute Steps 3 through 10 of the Algorithmic procedure using the manuscript equations only. Store every intermediate value required to reproduce the final outputs (accelerations, $\Delta a$, $t_c$, $\lambda_D$, $t_{\rm res}$, window boundaries, and $P_s$).

5. **Outcome generation and logging.** Generate PASS, FAIL, or DATA-INSUFFICIENT as a machine-produced field, not as narrative. Store the outcome together with: (i) the list of predicted observables invoked, (ii) the measured observational proxy values used in the comparison, (iii) the

comparison residuals (observed minus predicted directionality, not post-hoc reinterpretations), and (iv) the uncertainty handling rule reference (Section 8).

**Automation statement (explicit).** The evaluation is automatable. Given a completed input ledger record and the observational proxy measurements required by Section 5, the algorithm terminates in exactly one of PASS, FAIL, or DATA-INSUFFICIENT without human interpretive judgment.

**Audit log schema (minimum required fields).** A compliant implementation must log at minimum the following fields for each evaluation:

```
object_id:          string (canonical object name)
evaluation_id:      string (unique hash or UUID)
timestamp_utc:      ISO-8601 timestamp

inputs:
  M:                value_or_bound + units + uncertainty + provenance
  D_or_z:           value_or_bound + units + uncertainty + provenance
  L:                value_or_bound + units + uncertainty + provenance
  eta:              value_or_bound + units + uncertainty + provenance
  n_e:              value_or_bound + units + uncertainty + provenance
  T_e:              value_or_bound + units + uncertainty + provenance
  lnLambda:         value_or_bound + units + uncertainty + provenance
  B:                value_or_bound + units + uncertainty + provenance
  t_res:            value_or_bound + units + uncertainty + provenance

intermediates:
  a_e(r):           numeric outputs + units + equation reference
  a_i(r,Z):         numeric outputs + units + equation reference
  Delta_a(r,Z):     numeric outputs + units + equation reference
  t_c(Z):           numeric outputs + units + equation reference
  lambda_D:         numeric outputs + units + equation reference
  chi_s(Z):         binary outputs + criterion reference
  r_min(Z):         numeric outputs + units + definition reference
  r_max(Z):         numeric outputs + units + definition reference
  P_s(Z):           numeric outputs + definition reference

predictions_invoked:
  - P1
  - P2
```

```
  - P3
  - P4 (if applicable)

observations_used:
  proxy_name:       measured value_or_limit + units + uncertainty + provenance

outcome:
  classification:   PASS | FAIL | DATA-INSUFFICIENT
  rule_basis:       section-08 reference + explicit missing-input list if DI
```

**Replication requirement.** A third party must be able to reproduce the outcome by rerunning the evaluation using only: (i) the immutable input ledger record, (ii) the manuscript equations referenced in the intermediate log fields, and (iii) the stored observational proxy values and uncertainties.

### 10.4 Criteria for real-time falsification or confirmation

**Explicit link to prior predictions.** Real-time confirmation or falsification is evaluated only against the explicit predictions and failure condition declared in Section 5 (P1–P$n$ and the failure list), using the same proxy definitions and the same uncertainty handling rule declared in Section 8. No alternative observable definitions are permitted in this protocol.

**Falsification rule (irreversible).** For any system in which: (i) the closed input set is present (Table 8), (ii) the evaluation yields a non-empty shearing window for at least one species (some $r_{\min}(Z) < r_{\max}(Z)$), and (iii) the observational data are sufficient to measure the relevant Section 5 proxies with recorded uncertainties or limits, then the absence of the predicted stratification and composition signatures constitutes a **FAIL** outcome for that system under the pre-committed protocol. No delay clause is permitted: falsification is immediate once (i) through (iii) are satisfied.

**Confirmation rule (operational).** A **PASS** outcome requires: (i) the closed input set is present, (ii) the evaluation yields a non-empty shearing window for at least one species, and (iii) the measured proxies match the predicted directional signatures from Section 5 within the uncertainty-handling rule declared in Section 8. Agreement must be stated in terms of measurable quantities (proxy values, gradients, ordering relations, or limits), not qualitative narrative.

**DATA-INSUFFICIENT handling (required specificity).** A DATA-INSUFFICIENT outcome is permitted only in the following cases:

- **Missing required inputs:** one or more symbols required by Eq. (211) are absent and no admissible bound exists. The missing symbols must be named explicitly.

- **Unmeasurable proxy under available data:** the required stratification proxies from Section 5 cannot be measured as detections or as robust non-detections with limits, at the resolution and sensitivity required to interpret the proxy. In this case, the specific missing observational artifact must be named (e.g., inadequate multi-frequency coverage for RM inference, insufficient angular resolution for transverse structure, or absent polarization calibration).

**Sufficiency of polarization and RM measurements (measurement-level, not physics-level).** When Section 5 requires RM structure or Faraday-depth inference, the dataset must provide polarization measurements over enough wavelength-squared coverage to support a stable inference method. One standard reconstruction approach is RM synthesis[7]. When depolarization behavior is invoked as a proxy distinction, the dataset must be capable of discriminating wavelength-dependent polarization behavior consistent with established depolarization formalisms[8]. This paragraph defines only measurement sufficiency; it does not introduce new interpretive latitude in outcome assignment.

**Commitment statement (anti-post-hoc enforcement).** The criteria above bind newly observed systems to the same irreversible outcomes as historical systems. If the evaluation yields shearing-dominant conditions and the Section 5 signatures are absent under sufficient data, the correct classification is FAIL for that system, without reinterpretation. If required inputs or required proxy measurements are missing, the correct classification is DATA-INSUFFICIENT, with the missing items named explicitly.

[7] M. A. Brentjens and A. G. de Bruyn (2005), *Astronomy & Astrophysics*, 441, 1217-1228, DOI: 10.1051/0004-6361:20052990.

[8] B. J. Burn (1966), *Monthly Notices of the Royal Astronomical Society*, 133, 67-83.

## 11 Limitations (Single, Disciplined Section)

**Automatic failure regimes (enforced co-motion).** The Coulomb Shearing construction yields $\chi_s = 0$ by construction under any of the following conditions:

- The plasma is dense and collisional such that $t_c \ll t_{\text{res}}$, enforcing rapid momentum relaxation and joint electron–ion motion.
- Differential forcing along resolved trajectories is insufficient to overcome Coulomb drag within the available residence time.
- Polar or near-polar trajectory access is geometrically inaccessible or transient such that extended residence does not occur.
- The admissibility inequality is not satisfied for any admissible velocity or trajectory subset, yielding empty species ranges.

These outcomes represent valid null predictions of the framework rather than inconsistencies or breakdowns.

**Modal language convention (interpretation rule).** Several statements in this manuscript use strong modal phrasing (e.g., "must occur," "required," "unavoidable") when describing the consequences of the Coulomb Shearing mapping. Unless explicitly stated otherwise, these modals are *conditional* and should be read as: (i) conditional on the declared force model and closure definitions, (ii) conditional on satisfying the admissibility criteria for a given system, and (iii) conditional on the availability of sufficient data to score the relevant observable tests. They are not intended as claims that the microphysical pathway is inevitable in all astrophysical plasmas, nor as claims that any system is confirmed in the absence of the required observables. Accordingly, whenever a system cannot be scored under the protocol, the correct disposition is **DATA-INSUFFICIENT**, and no stronger claim is implied by the modal phrasing.

**JWST LRDs and compact high-$z$ objects (data sufficiency for closure outputs).** JWST *Little Red Dots* (LRDs) are included as an explicit target class, but a full closure admissibility evaluation is often blocked by missing near-horizon environment inputs that are not routinely reported for compact high-$z$ sources (e.g. plasma density and temperature proxies, magnetic-field constraints, or a defensible $E_{\parallel}(r)$ bound). Under the binding protocol in Pre-Committed Evaluation Protocol (Anti-Post-Hoc), these missing quantities are not inferred from taxonomy, compactness, or broadband colors. Instead, LRD targets remain in-scope and are reported as **DATA-INSUFFICIENT** for the specific outputs that require those inputs, while still permitting mapping-only consequences that depend only on redshift and a traceable mass estimate.

1. **L1: Absence of sustained anisotropic forcing.**

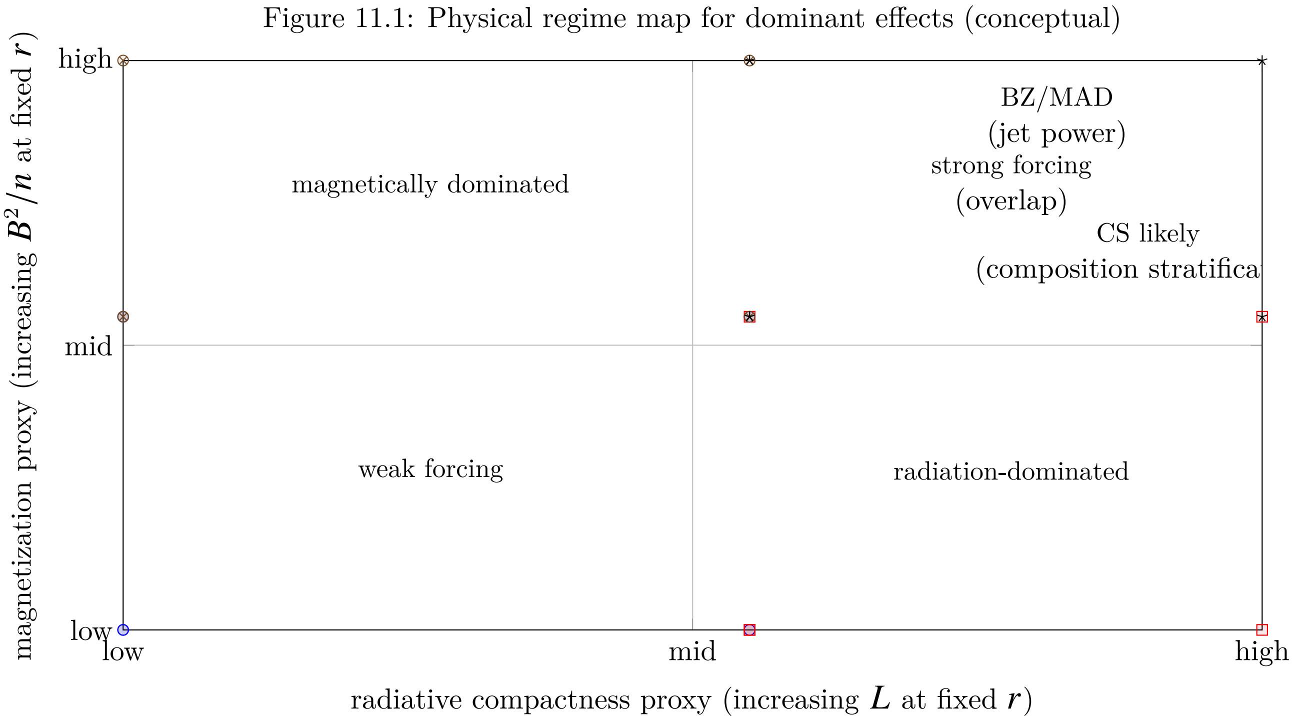


Figure 11: Parameter-space sketch indicating where different physical effects are expected to dominate. Coulomb Shearing (CS) is evaluated as a species-resolved composition stratification mechanism (closure failure and mapped windows), while BZ/MAD refer to jet-power extraction and high-flux magnetic states. The overlap region indicates that CS can operate as a stratification layer even when BZ/MAD set jet power and collimation. This figure is conceptual and is not used for validation.

*Physical limitation.* Coulomb Shearing requires a non-zero, sustained differential forcing component along polar or near-polar trajectories, as encoded in the evaluated $\Delta a(r)$ (Section Differential Forcing and the Onset of Coulomb Shearing). If the net external forcing is effectively symmetric (or negligibly small) on the evaluated trajectory family, then the electron and ion accelerations remain effectively co-directed and the computed displacement response does not reach the screening-length threshold. In that regime, the construction yields no admissible closure failure and no species-resolved shearing window.

*Regime boundary (explicit).* This limitation applies when, over the declared evaluation domain and the declared trajectory family, the executed equations return

$$\chi_s(r; Z) = 0 \quad \text{for all} \quad r \in \mathcal{R}_{\text{eval}} \text{ and all declared } Z,$$

because the computed forcing produces $\Delta x(t_{\text{res}}) < \kappa\lambda_D$ everywhere under the locked criterion (Section Differential Forcing and the Onset of Coulomb Shearing). Equivalently, the species windows are empty by definition: $r_{\min}(Z) \geq r_{\max}(Z)$ for all declared $Z$ under the range construction in Section Species-Dependent Shearing Ranges and Probabilities.

*Does not affect.* This limitation does not apply in any regime where the executed forcing model yields $\chi_s = 1$ in a non-empty radial interval for any declared species, and it may not be invoked to reinterpret a failed observational comparison. If anisotropic forcing is present and computed to exceed coupling thresholds anywhere in the evaluated domain, L1 is inapplicable by construction.

2. **L2: Enforced electron–ion coupling due to plasma state.**

   *Physical limitation.* In sufficiently dense and collisional plasma states, Coulomb momentum relaxation enforces rapid electron–ion co-motion. When relaxation is fast compared to the available residence time, relative drift is quenched and the displacement response remains below the screening threshold. In this regime, the mechanism does not generate sustained tracer partition and the evaluation returns $\chi_s = 0$.

   *Regime boundary (explicit).* This limitation applies when, for the executed coupling model and the declared inputs $(n_e, T_e, Z, \ln \Lambda)$, the computed coupling time satisfies the strong ordering

   $$t_c(r; Z) \ll t_{\rm res}(r) \quad \text{throughout the candidate shearing interval,}$$

   such that the full displacement solution yields $\Delta x(t_{\rm res}) \ll \kappa\lambda_D$ and hence $\chi_s(r; Z) = 0$ under the locked admissibility criterion (Section Differential Forcing and the Onset of Coulomb Shearing). The boundary is therefore a timescale ordering, not a narrative qualifier: the mechanism is suppressed because rapid relaxation enforces single-fluid behavior.

   *Does not affect.* This limitation does not excuse failure in systems where the executed evaluation yields $t_c(r; Z) \gtrsim t_{\rm res}(r)$ over any non-empty interval and returns $\chi_s = 1$ there. If finite coupling is demonstrated under the evaluated inputs, L2 is inapplicable and may not be invoked post hoc.

3. **L3: Geometric inaccessibility of polar trajectories.**

   *Physical limitation.* Coulomb Shearing is defined on polar or near-polar trajectory families. If those trajectories are physically absent in the system geometry, the mechanism cannot be instantiated. In that regime, the construction has no domain on which to evaluate sustained differential forcing and residence, and no tracer partition is physically available.

   *Regime boundary (explicit).* This limitation applies only under physical impossibility of polar access, meaning that the system admits no quasi-steady polar or near-polar transport channel on the scales where the forcing model is evaluated. In the notation already used elsewhere in the manuscript, this corresponds to a vanishing polar-access measure (for example, $f_{\rm polar}(r) = 0$ over the relevant radial range, if that function is part of the declared geometry model), so that a polar residence time is not physically definable as a trajectory property within the model's domain.

   *Does not affect.* This limitation applies only to physical absence of polar access, not to observational non-detection of polar structure. If polar access is demonstrated or is explicitly inferred under the same geometric model used in the evaluation, then L3 is inapplicable and may not be invoked to reclassify a FAIL outcome.

4. **L4: Kinetic ambipolar relocking and anomalously rapid momentum relaxation**

   *Physical limitation.* The construction evaluates closure failure using the coupling time model declared in Section Differential Forcing and the Onset of Coulomb Shearing. In collisionless funnels, additional kinetic processes (ambipolar electric-field response and wave–particle scattering) can, in principle, enforce single-fluid co-motion by providing *more rapid* momentum relaxation than binary Coulomb drag alone. When such processes enforce rapid relocking on timescales short compared to the residence time, the displacement criterion is not satisfied and tracer partition does not occur.

   *Regime boundary (explicit).* This limitation is triggered by an ordering of effective relaxation and residence times:

$$t_{c,\mathrm{true}} \ll t_{\mathrm{res}},$$

   where $t_{c,\mathrm{true}}$ denotes the actual momentum-relaxation time controlling relative electron–ion drift in the funnel plasma. In that ordering, the displacement response obeys $\Delta x(t_{\mathrm{res}}) \ll \kappa\lambda_D$ and the correct evaluation outcome is $\chi_s = 0$ under the executed admissibility criterion (Section Differential Forcing and the Onset of Coulomb Shearing), yielding empty shearing windows under the range construction (Section Species-Dependent Shearing Ranges and Probabilities).

   *Does not affect.* This limitation does not alter the forcing definitions, the Eddington derivation, or the PASS/FAIL logic. It cannot be used to neutralize an observational FAIL outcome. In particular:

   - If the executed evaluation (using the declared $t_c$ model) returns $\chi_s = 0$, the framework already yields a valid null prediction and no additional limitation argument is needed.
   - If the executed evaluation returns $\chi_s = 1$ and predicted signatures are absent under sufficient data, the outcome remains FAIL under Section Pre-Committed Evaluation Protocol (Anti-Post-Hoc). Post hoc appeals to unmodeled "extra relocking" are not an admissible reclassification pathway, because additional relaxation can only suppress shearing relative to the executed prediction and therefore cannot rescue a failed comparison.

**Regime map (auditable taxonomy over $(n_e, T_e, L/L_{\mathrm{Edd}}, M)$).** The framework's operating domain can be summarized as a regime map defined by two dimensionless diagnostics that are already computed in the executed evaluation:

$$\mathcal{C}(r;Z) \equiv \frac{t_{\mathrm{res}}(r)}{t_c(r;Z)}, \qquad \mathcal{F}(r;Z) \equiv \frac{\Delta x(t_{\mathrm{res}}(r))}{\kappa\,\lambda_D(r)}. \tag{286}$$

Here $\mathcal{C}$ measures whether coupling relaxes relative drift within the available residence time, while $\mathcal{F}$ is the locked admissibility ratio used by the displacement criterion. The executed outcome $\chi_s$ is equivalent to a threshold statement on $\mathcal{F}$:

$$\chi_s(r;Z) = 1 \iff \mathcal{F}(r;Z) \ge 1, \qquad \chi_s(r;Z) = 0 \iff \mathcal{F}(r;Z) < 1,$$

| Mechanism | Primary flux proxy | Key dependence | Dominant when |
|---|---|---|---|
| Coulomb Shearing | species-partition drive $\sim \Delta a_\parallel t_{\rm res}$ (with admissibility $\mathcal{F} \geq 1$) | $(L/L_{\rm Edd})$ for radiative forcing; $(B, v_A, \epsilon_{\rm rec})$ for reconnection $E_\parallel$; coupling via $(n_e, T_e, Z)$ | $\chi_s = 1$ over a non-empty interval, and additional gates (e.g. frozen-in check) are satisfied |
| BZ (spin extraction) | jet Poynting power $P_{\rm BZ} \sim B_H^2 r_H^2 a_*^2 c$ (order-of-magnitude) | horizon-threading $B_H$, spin $a_*$, field geometry | high $a_*$ and high $B_H$ with sustained open flux; governs jet power, not composition |
| MAD state | magnetic-pressure choking and interchange transport (global) | magnetization, $\beta \lesssim 1$, net flux accumulation | when magnetic pressure near the hole limits inflow and controls variability; sets global accretion and jet loading channels |
| Radiation-pressure winds | momentum flux $\dot{P}_{\rm rad} \sim L/c$ with optical-depth geometry | $L$, optical depth, line driving / opacity | when quasi-spherical or equatorial mass loss is efficient; governs wind mass loss and kinematics |
| Slim disk (advective) | advective energy transport enabling $\dot{M} > \dot{M}_{\rm Edd}$ | $\dot{M}$, trapping radius, advection fraction | when photon trapping and advection dominate cooling; governs accretion rate feasibility |

Table 9: Regime-aware comparison of Coulomb Shearing to competing mechanisms. The mechanisms control different observables: BZ/MAD primarily govern jet power and global transport, radiative winds govern outflow momentum and mass loss, and slim disks govern thermal viability of super-Eddington inflow. Coulomb Shearing, when admissible, governs species partition and stratification within the channels provided by the global transport state.

under the declared inputs and the declared forcing and coupling specializations.

A complete taxonomy is then obtained by classifying each evaluated system and species by the pair $(\mathcal{C}, \mathcal{F})$ over the declared evaluation domain $\mathcal{R}_{\rm eval}$, with polar-access as an external gate. A minimal regime table is:

The link to the requested parameter space $(n_e, T_e, L/L_{\rm Edd}, M)$ is explicit because the executed inputs determine $t_c$ and $\lambda_D$ through $(n_e, T_e, Z, \ln\Lambda)$, determine $t_{\rm res}$ through the declared trajectory and geometry (which scales with $M$ through $r_{\rm g}$ when radii are parameterized as multiples of $r_{\rm g}$), and determine $\Delta x(t_{\rm res})$ through the declared forcing model (whose radiative part is controlled by $L/L_{\rm Edd}$ under the Thomson specialization when used). A regime map for any target class is therefore produced by evaluating $\mathcal{C}$ and $\mathcal{F}$ on a grid of admissible inputs and reporting which of the above regions contains the computed $(\mathcal{C}, \mathcal{F})$ values over $\mathcal{R}_{\rm eval}$.

This taxonomy is not an additional escape clause: it is a reporting layer that makes the operating domains explicit in the same variables that the executed PASS/FAIL logic already uses.

## 11.1 Mechanism comparison and regime mapping (dominant vs complementary)

This manuscript is restricted to the *stratification and species-partition* consequences of differential electron–ion response on polar or near-polar trajectories. It is not a jet-launching theory and it does not replace electromagnetic jet power extraction models. Nevertheless, a regime-aware comparison to competing mechanisms is required to determine whether Coulomb Shearing is expected to be dominant, subdominant, or complementary for a given object class.

**What "dominant" means in this manuscript.** Because the compared mechanisms act on different observables (power extraction, accretion choking, wind mass loss, or composition stratification), "dominance" is defined operationally as follows:

- **Dominant for stratification:** Coulomb Shearing is dominant when the executed evaluation yields $\chi_s = 1$ with non-empty species windows and when competing mechanisms do not erase the implied ordering (e.g. via strong relocking, complete mixing, or frozen-in suppression of relative drift on the separation scale).
- **Complementary:** Coulomb Shearing is complementary when BZ/MAD/radiative-wind/slim-disk physics governs *power, collimation, and global mass flux*, while Coulomb Shearing governs *species partition within whatever outflow/inflow channels exist*.
- **Subdominant:** Coulomb Shearing is subdominant when the framework returns $\chi_s = 0$ (L1/L2/L3/L4) or when an additional gate (e.g. frozen-in suppression under Verification of magnetic decoupling (frozen-in check)) prevents electron displacement from being treated as an independent channel on the relevant scale.

**Comparison table (mechanism, scaling, and dominance conditions).** Table 9 summarizes the competing mechanisms referenced in the evaluation protocol, emphasizing which quantities are needed for an object-level comparison and which observable each mechanism primarily controls.

**Regime mapping with executed diagnostics.** The classification in Table 9 is tied to the executed diagnostics already defined in this manuscript:

- Coulomb Shearing admissibility is mapped by $(\mathcal{C}, \mathcal{F})$ in Eq. (286), and by the frozen-in gate $R_{m,\mathrm{local}}$ under Verification of magnetic decoupling (frozen-in check) when an electromagnetic channel is invoked.
- BZ/MAD dominance cannot be inferred from taxonomy alone; it requires declared or cited estimates of near-horizon magnetization and spin proxies (e.g. $a_*$ and an effective $B_H$ or equivalent flux measure).

- Radiative-wind dominance requires declared optical-depth and geometry assumptions; it cannot be inferred from luminosity alone.

- Slim-disk dominance requires declared $\dot{M}$ and trapping/advection characterization; it cannot be inferred solely from $L$ in radiatively inefficient flows.

Accordingly, an object-level statement of "dominant vs complementary" must report which of these inputs are available and which comparisons are **DATA-INSUFFICIENT** rather than guessed.

**Scope discipline.** This comparison section does not introduce a new reclassification pathway: if Coulomb Shearing is executed under declared inputs and returns a prediction that is not observed under sufficient data, the outcome remains FAIL under Pre-Committed Evaluation Protocol (Anti-Post-Hoc). The role of this section is to prevent category errors (power vs composition) and to state explicitly when Coulomb Shearing is expected to operate alongside, not instead of, other accretion/jet mechanisms.

# 12 Conclusions

## 12.1 What Coulomb Shearing explains in Eddington-limited regimes

Coulomb Shearing explains how sustained black hole growth can occur in radiation-dominated regimes where the emergent luminosity is bounded by electron Eddington saturation, without violating Eddington physics. The construction preserves the electron-level momentum balance that defines $L_{\mathrm{Edd}}$ (Eq. (137)) and targets only the *perfect-coupling inference* that transfers the electron force balance to the entire baryonic inflow. When the enforced co-motion closure fails on a resolved polar or near-polar trajectory subset, the electron-coupled channel remains radiatively regulated while the ion component follows a gravity-dominated inward branch (Eqs. (139)–(140), (143)), yielding an explicit two-channel growth law (Eq. (153) and the enhancement form Eq. (156)) rather than an altered Eddington limit.

The mechanism chain defined in this manuscript and bound for numerical execution is:

1. **Species-resolved differential forcing.** Along a declared trajectory family, species accelerations are computed from the force model in Section 3, and the differential acceleration $\Delta a$ is evaluated explicitly (e.g., Eq. (37) and the stated specializations such as Eq. (59)).

2. **Finite, time-dependent Coulomb coupling.** The electron-ion momentum relaxation time $t_c$ is computed from the coupling model declared in Section 3 (including the explicit dependence on $n_e$, $T_e$, $Z$, and $\ln \Lambda$), so that coupling is not treated as instantaneous by assumption.

3. **Admissibility, windows, and probabilities from executed inequalities.** The closure outcome $\chi_s$ is determined by direct evaluation of the displacement-based admissibility inequality (Section 3, Eqs. (72) and (78)). When evaluated over the declared phase-space measure, the

framework produces species-resolved shearing windows $[r_{\min}(Z), r_{\max}(Z)]$ and the corresponding shearing (decoupling) probabilities $P_s(Z)$ as defined and operationalized in Sections 4 and 9.

4. **Electron-dominated relativistic outflows and ion-dominated inflow as conditional consequences.** In the subset where $\chi_s = 1$ has nonzero measure, Section 5 maps the closure outcome to directional, measurable jet signatures (polarization and Faraday proxies), while Section 6 maps the same closure outcome to an ion-dominated inward channel and a computable growth enhancement through $f_{\text{ion}}$ (Eqs. (149) and (150)).

Under the pre-committed protocol (Section 8), the chain above is not treated as a conceptual explanation. It is a closed computational mapping: a traceable input set is mapped to $t_c$, $\Delta a$, $\chi_s$, $[r_{\min}, r_{\max}]$, and $P_s(Z)$, and the resulting predictions are compared to the measurable proxies defined in *Observable Consequences: Jets and Stratification* (Section 5) with outcomes constrained to PASS, FAIL, or DATA-INSUFFICIENT.

### 12.2 What would falsify Coulomb Shearing

Coulomb Shearing is falsified for an evaluated system if, under the locked input mapping and uncertainty handling of Section 8, the mechanism is *computed* to be dynamically admissible (nonempty shearing windows with nonzero $P_s(Z)$ for the declared species set), but the decisive observational signatures defined in Section 5 are absent or contradicted by the data used for that system. Such absence constitutes **FAIL** under the pre-committed protocol and may not be reclassified as inapplicability.

Falsifiability in this manuscript is therefore conditional on *data sufficiency* for the decisive observables. Where the required observational artifacts are not available at the necessary resolution, frequency coverage, or geometry to score the criteria in Section 5, the protocol outcome is **DATA-INSUFFICIENT** rather than PASS or FAIL, and no claim of observational confirmation or refutation is made for that system. Throughout this paper, modal verbs such as "must" and "required" are used in this protocol-conditional sense: if the computation yields admissibility and if the relevant observables are defined and measurable under the declared data conditions, then the listed inequalities are required outcomes of the mapping; they are not assertions that the underlying plasma microphysics is inevitable in all environments.

The decisive signatures already defined in this manuscript include:

- **Electron-loaded axial channel with reduced internal Faraday depth.** Where the calculation yields an open electron-loading zone, the on-axis emission region is predicted to exhibit systematically smaller $|\mathrm{RM}|$ and weaker depolarization than surrounding off-axis regions at matched beam and frequency coverage (Section 5, Eq. (113) and Tables 2–3).
- **Window-locked localization of transitions.** If $r_{\min}(Z)$ and $r_{\max}(Z)$ are computed, then the onset and termination of the electron-loading proxy behavior are required to co-locate with the computed window, rather than appearing at radii unrelated to the evaluated admissibility (Section 5, "Window-locked localization").

- **Species ordering implied by computed windows.** If multiple evaluated charge states are assigned open windows, their ordering is fixed by the computed $r_{\min}(Z)$ and $r_{\max}(Z)$ and must match the observed ordering of composition-linked proxies (Section 5, "Monotonic ordering rule from computed windows").
- **Consistency with the growth-channel closure.** In cases where the same computation yields $\chi_s = 1$ on a nonzero-measure subset, the framework simultaneously asserts that the ion component remains on an inward branch while the electron channel is radiatively regulated (Section 6, Eqs. (139)–(140)). If observations in that same system directly contradict the electron-loaded outflow channel together with an exterior, ion-bearing Faraday-active component, the outcome is FAIL rather than reinterpretation.

Conversely, if the computation yields $\chi_s = 0$ everywhere (or yields empty windows for all evaluated species), the framework makes a null prediction for the above tracer-partition signatures and does not claim a Coulomb Shearing-origin electron-loading channel for that system. This is a computational outcome determined by the executed inequality and the declared inputs, not a discretionary scope carve-out.

### 12.3 Relation to two-temperature accretion and collisionless coupling literature

The plasma literature on electron–ion equilibration, collisionless heating, and microinstability-mediated coupling is extensive and predates this work by decades. This manuscript does not claim novelty in the existence of two-temperature plasmas or in the general fact that collisionless processes can alter effective coupling rates. The novelty claim is narrower and structural: Coulomb Shearing treats enforced electron–ion co-motion as a *trajectory-local admissibility condition* under sustained, species-asymmetric forcing near polar and funnel-adjacent paths, and it maps closure failure into specific, window-locked transverse observables that can be scored under a pre-committed protocol.

Much of the prior two-temperature accretion-flow and related modeling literature is organized around (i) bulk, quasi-neutral flow kinematics with a shared velocity field and (ii) thermodynamic partitioning and emission modeling via electron-heating prescriptions, rather than (iii) spatial separation failure as a closure admissibility question with geometric and observational consequences. For this reason, the earlier literature does not typically produce the same species-ordering and matched-beam transverse inequalities (P1–P4) that are central to this manuscript. Where collisionless instabilities or quasilinear processes strengthen coupling beyond Coulomb expectations, the effect in this framework is not ignored; it moves the system toward explicit inadmissibility or toward reduced falsification leverage under the declared scoring rules, rather than being silently assumed away (see Differential Forcing and the Onset of Coulomb Shearing).

### 12.4 Direct comparison to recent alternative models

This subsection summarizes how the quantitative observables and falsification criteria defined in Sections 5 and 5.4.2 relate to recent alternative descriptions already cited in this manuscript. It does not introduce new claims; it records overlap versus divergence in the observable domain. Representative exemplars referenced in Table 10 include Yuan et al. (2025) [20], Mehlhaff et al. (2025) [21], and Sądowski & Narayan (2016) [11].

**Column definitions.** *Predicts electron jets?* = yields electron (or pair) dominated emission channels; does not imply jet launching by Coulomb Shearing. *Predicts super-Edd?* = sustained baryonic growth beyond the electron Eddington inference (two-channel construction; see Section 6). *Explains P1–P4?* = generates the same window-locked, matched-beam polarimetric inequalities and species-ordering consequences (see Sections 5 and 5.4.2).

Table 10: Direct comparison of observable scope versus alternative model classes.

| **Model class** | **Primary mechanism** | **Predicts electron jets?** | **Predicts super-Edd?** | **Explains P1–P4?** | **Distinguishing signature used here** |
|---|---|---|---|---|---|
| Coulomb Shearing (this work) | Species-resolved closure failure under differential forcing | Yes (electron-loaded axial channel) | Yes (ion-channel growth in Eddington-limited regimes) | Yes (by construction) | Computed admissibility windows + quantitative inequalities (P1–P4) + species-dependent ordering tied to computed windows |
| **Thick / slim disk inflow** | Photon trapping, advection, supercritical inflow with luminosity saturation | Not required | Yes (supercritical inflow possible) | No (does not predict window-locked RM/DP ordering and species-linked transverse signatures) | Accretion-regime signatures (optical depth, trapping radius, spectral/variability shifts) without the mapped-zone Faraday/polarization window locking |

*Continued on next page*

| Model class | Primary mechanism | Predicts electron jets? | Predicts super-Edd? | Explains P1–P4? | Distinguishing signature used here |
|---|---|---|---|---|---|
| **Hyper-Eddington inflow** | Radiation trapping and/or anisotropic escape enabling very large inflow rates | Not required | Yes (inflow rate can exceed Eddington) | No (does not imply species-resolved closure failure with P1–P4-style transverse tests) | Luminosity–accretion decoupling and trapping-driven observables, but no prediction of the specific spine–sheath Faraday and polarization inequalities defined here |
| **Heavy seeding scenarios** | High initial seed mass (direct-collapse, dense-cluster pathways, etc.) reduces required growth | Not required | Not a growth mechanism (changes initial condition) | No (does not address the tracer-partition signatures; can coexist with any jet model) | Population-level signatures (seed mass function, occupation fractions) but no intrinsic prediction of P1–P4 or computed species ordering |
| Yuan et al. (2025) | Pair-cascade / pair-loading gaps | Yes (pair/ electron-dominated emission) | No (does not create a baryonic ion-only growth channel) | No (pair-loading signatures are not equivalent to P1–P4 window locking) | High-energy pair-production cutoff / spectral pair-loading signatures (distinct from mapped-zone RM/DP inequalities) |

*Continued on next page*

| Model class | Primary mechanism | Predicts electron jets? | Predicts super-Edd? | Explains P1–P4? | Distinguishing signature used here |
|---|---|---|---|---|---|
| Mehlhaff et al. (2025) | Jet–corona coupling / kinetic reconnection channel | Yes (electron-dominated emission regions) | No (does not assert an ion-only inward channel under electron Eddington regulation) | No (does not yield the P1–P4 window locking as defined) | Corona temperature and reconnection-driven electron spectra (distinct from mapped-zone Faraday/polarization inequalities) |
| Sądowski & Narayan (2016) | Photon trapping / slim-disk supercritical inflow | Not required | Yes (supercritical inflow with luminosity saturation) | No (scope differs from P1–P4 window locking) | Optical-depth and trapping-profile signatures (distinct from closure-failure window predictions and species ordering) |

**Where overlap occurs and where it diverges.** The overlap among these descriptions is primarily qualitative (electron-dominated emission channels, strong-field environments, or supercritical inflow regimes). The divergence, as tested in this manuscript, is whether the predicted observables are *window-locked* to computed admissibility regions and satisfy the quantitative inequalities P1–P4 under matched-beam polarimetry, and whether a distinct ion-channel growth law is implied under electron Eddington regulation. These divergences are evaluated as PASS/FAIL/DATA-INSUFFICIENT under the locked protocol rather than by narrative preference.

**Comparison to thick-disk, hyper-Eddington, and heavy-seed classes (scope alignment).** The same distinction applies to three commonly invoked explanation classes that are sometimes treated as substitutes for one another in the growth-literature narrative: thick/slim-disk supercritical inflow, hyper-Eddington inflow enabled by trapping or anisotropic escape, and heavy-seed initial-condition scenarios. The first two can address *high inflow rates* and in some regimes can reconcile large accretion rates with modest emergent luminosity through photon trapping and/or geometric collimation, but they do not, by themselves, imply the specific *species-resolved closure-failure mapping* used here, and therefore do not generically predict the matched-beam, window-locked transverse polarimetry and Faraday inequalities P1–P4 nor the monotonic species-ordering tied to computed $[r_{\min}(Z), r_{\max}(Z)]$ windows. Heavy seeding is not a growth mechanism; it changes the required growth integral by altering the initial mass, and therefore does not compete on the same observable domain as the P1–P4 tests. Accordingly, the "quantitative superiority" asserted by this manuscript is not a claim of globally better fitting or broader explanatory power: it is the narrower statement that, where Coulomb Shearing is computed to be admissible under the locked protocol, it yields *numerically specified, falsifiable inequalities* that can be scored PASS/FAIL/DATA-INSUFFICIENT, whereas the alternative classes above typically do not produce the same window-locked, species-linked transverse tests in their base formulations.

### 12.5 Standing claim and consequence

Coulomb Shearing is proposed as a dominant growth mechanism in radiation-dominated, Eddington-limited regimes, and it stands or falls solely on whether, for systems where its admissibility and shearing windows are computed to be open under the locked protocol of Section 8, the directional jet and composition signatures in Section 5 are borne out by observation; contradiction in such a computed-dominance case is a FAIL outcome that defeats the mechanism for that system in the tested regime, while agreement within the pre-declared uncertainty handling rules yields PASS and retains the mechanism only in that evaluated regime.

# A Symbols and Definitions

### Notation conventions

This table defines notation conventions used throughout the manuscript to prevent symbol drift across sections.

| Symbol | Meaning |
|---|---|
| $G$ | Newtonian gravitational constant. Units: $\mathrm{m^3\,kg^{-1}\,s^{-2}}$. |
| $c$ | Speed of light in vacuum. Units: $\mathrm{m\,s^{-1}}$. |
| $k_B$ | Boltzmann constant. Units: $\mathrm{J\,K^{-1}}$. |
| $\varepsilon_0$ | Vacuum permittivity. Units: $\mathrm{F\,m^{-1}}$. |
| $\sigma_T$ | Thomson scattering cross section for electron scattering. Units: $\mathrm{m^2}$. |
| $e$ | Elementary charge magnitude. Units: C. Note: the manuscript also uses $e^{(\cdot)}$ to denote exponentiation by the base of natural logarithms; this is a notational overload. Wherever $e$ appears in Coulomb expressions (e.g., $e^2$ in $\lambda_D$ or $a_\mathrm{C}$), it denotes the elementary charge. Wherever $e^{(\cdot)}$ appears with a dimensionless exponent, it denotes the exponential function. |
| $\pi$ | Mathematical constant $\pi$. Dimensionless. |
| $M_\odot$ | Solar mass (used as a unit scale in worked examples). Units: kg. |

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| **Symbol** | **Meaning** |
|---|---|
| $ds^2$ | Spacetime line element in the Kerr metric. Units: $\mathrm{m}^2$ (or geometric units where stated). |
| $t$ | Coordinate time variable. Units: s (or geometric units where stated). |
| $r$ | Radial coordinate (Boyer–Lindquist radius in Kerr sections; ordinary radius elsewhere). Units: m. |
| $\theta$ | Polar angle coordinate. Units: radians. |
| $\phi$ | Azimuthal angle coordinate. Units: radians. |
| $M$ | Black hole mass parameter. Units: kg (or geometric units where stated). |
| $J$ | Black hole angular momentum. Units: $\mathrm{kg\,m^2\,s^{-1}}$ (or geometric units where stated). |
| $a$ | Kerr spin parameter, defined in the manuscript as $a \equiv J/M$ in geometric units ($G = c = 1$ in the Kerr subsection). Units: m in geometric units. |
| $a_*$ | Dimensionless Kerr spin, defined as $a_* \equiv a/M \in [0, 1)$. Dimensionless. |
| $\Sigma$ | Kerr metric function $\Sigma = r^2 + a^2 \cos^2\theta$. Units: $\mathrm{m}^2$ (geometric units in Kerr subsection). |
| $\Delta$ | Kerr metric function $\Delta = r^2 - 2Mr + a^2$. Units: $\mathrm{m}^2$ (geometric units in Kerr subsection). |
| $r_g$ | Gravitational radius defined as $r_g \equiv GM/c^2$. Units: m. |
| $t_g$ | Gravitational time defined as $t_g \equiv GM/c^3$. Units: s. |
| $r_{\mathrm{H}}$ | Outer Kerr event-horizon radius, defined as $r_{\mathrm{H}} \equiv r_+ = r_{\mathrm{g}}\left(1 + \sqrt{1 - a_*^2}\right)$ (Eq. (7)). Units: m. This symbol encodes the exterior-domain boundary for the evaluation protocol: all Coulomb Shearing conditions and bands are evaluated only for $r > r_{\mathrm{H}}$, with no evaluation or interpretation at $r \le r_{\mathrm{H}}$. |
| $a_*$ | Dimensionless Kerr spin parameter, $a_* \equiv Jc/(GM^2)$, with $0 \le a_* < 1$. Used to define the outer horizon radius $r_{\mathrm{H}} = r_+ = r_{\mathrm{g}}\left(1 + \sqrt{1 - a_*^2}\right)$ (Eq. (7)). Units: dimensionless. |
| $\Omega$ | Solid angle. Units: steradians. |
| $d\Omega$ | Differential solid angle element used in angular integrals. Units: steradians. |
| $\Omega_{\mathrm{pol}}$ | Polar-cap solid angle defined as $\Omega_{\mathrm{pol}} \equiv 4\pi(1 - \cos\theta_{\mathrm{pol}})$. Units: steradians. |
| $\theta_{\mathrm{pol}}$ | Polar-cap half-opening angle used to define the polar region for flux fraction diagnostics. Units: radians. |
| $30^\circ$ | Fixed numerical choice for the polar-cap half-opening angle used in evaluations ($\theta_{\mathrm{pol}} = 30^\circ$ in the manuscript). Units: radians. This encodes a modeling choice. |

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| Symbol | Meaning |
|---|---|
| $C$ | A trajectory or curve segment along which residence-time and along-path averages are evaluated. Physically represents the path of polar or near-polar transport. |
| $v(r)$ | Local flow speed along the evaluated transport segment. Units: $\mathrm{m\,s^{-1}}$. |
| $v_r$ | Radial velocity component. Units: $\mathrm{m\,s^{-1}}$. |
| $v_\theta$ | Polar-angle velocity component appearing in the illustrative MHD momentum balance. Units: $\mathrm{m\,s^{-1}}$. |
| $u^r$ | Contravariant radial component of the four-velocity used in the GRMHD mass-flux diagnostic. Dimensionless in relativistic units (or $\mathrm{m\,s^{-1}}$ if treated as a three-velocity proxy; the manuscript uses it as a GRMHD output quantity). |
| $v_{\mathrm{ff}}(r)$ | Newtonian free-fall speed magnitude used in residence-time bounds, $v_{\mathrm{ff}}(r) \equiv \sqrt{2GM/r}$. Units: $\mathrm{m\,s^{-1}}$. |
| $v_K$ | Keplerian orbital speed used in the heuristic inflow-speed scaling $\lvert v_r\rvert \sim \alpha(H/r)^2 v_K$. Units: $\mathrm{m\,s^{-1}}$. |
| $\eta$ | Transport slowdown factor defined in the manuscript as $\eta \equiv \langle \lvert v_r\rvert\rangle / v_{\mathrm{ff}}$, with $0 < \eta < 1$. Dimensionless. This encodes an externally constrained transport regime assumption. |
| $\alpha$ | Shakura–Sunyaev viscosity parameter appearing in the heuristic inflow scaling. Dimensionless. |
| $H$ | Disk scale height used in the aspect ratio $(H/r)$ in the inflow-speed heuristic. Units: m. Note: the manuscript also uses $H(\cdot)$ for the Heaviside step function in the probabilistic kernel; this is a notational overload. |
| $\Delta r$ | Local streamline-segment thickness used in the explicit example construction, parameterized as $\Delta r \equiv \zeta r_g$. Units: m. |
| $\zeta$ | Dimensionless segment-thickness parameter in $\Delta r \equiv \zeta r_g$. Dimensionless. Encodes a modeling choice for the local example geometry. |
| $x$ | Dimensionless radius defined as $x \equiv r/r_g$ in the explicit local example construction. Dimensionless. |
| $r_1, r_2$ | Lower and upper radial limits used in residence-time integrals (trajectory endpoints for the bound). Units: m. |
| $r_{\mathrm{in}}$ | Inner evaluation radius used in worked examples (lower limit in $t_{\mathrm{res}}$ integrals). Units: m. |
| $r_\star$ | Reference evaluation radius used in worked examples (the radius at which inputs are anchored and diagnostic quantities are explicitly computed). Units: m. |

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| Symbol | Meaning |
|---|---|
| $t_{\mathrm{res}}$ | Characteristic residence time that matter spends on a polar or near-polar trajectory channel before accretion, escape, or reintegration into the disk. Units: s. |
| $t_{\mathrm{dyn}}(r)$ | Local dynamical (orbital) timescale used in the non-homologous reservoir parameterization, $t_{\mathrm{dyn}}(r) \equiv 2\pi(r/r_g)^{3/2}t_g$. Units: s. |
| $t_{\mathrm{drive}}$ | Driving timescale for maintaining anisotropy, defined as $t_{\mathrm{drive}} \equiv M_{\mathrm{nh}}/\dot{M}_{\mathrm{pol}}$. Units: s. |
| $t_{\mathrm{relax}}$ | Effective relaxation timescale for damping anisotropy, defined either by a sum of rates $1/t_{\mathrm{relax}} \equiv 1/t_{\mathrm{qnm}} + 1/t_{\mathrm{mix}} + 1/t_{\mathrm{adv}} + \cdots$ or conservatively fixed to $t_g$. Units: s. Encodes a modeling choice when fixed. |
| $t_{\mathrm{qnm}}$ | Characteristic quasi-normal-mode relaxation timescale contributing to $t_{\mathrm{relax}}$ in the rate-sum model. Units: s. |
| $t_{\mathrm{mix}}$ | Characteristic mixing timescale contributing to $t_{\mathrm{relax}}$ in the rate-sum model. Units: s. |
| $t_{\mathrm{adv}}$ | Characteristic advective timescale contributing to $t_{\mathrm{relax}}$ in the rate-sum model. Units: s. |
| $\delta$ | Duty-cycle factor applied to the driving rate via $1/t_{\mathrm{drive}} \longrightarrow \delta/t_{\mathrm{drive}}$. Dimensionless. Encodes an intermittency modeling choice. |
| $\Delta t$ | Finite time interval used in angular and temporal averaging and in formation-time mappings. Units: s. |
| $t_0$ | Reference initial time used as an integration lower limit and as the start of averaging or growth intervals. Units: s. |
| $M(t)$ | Black hole mass as a function of time. Units: kg. |
| $M_0$ | Initial black hole mass at time $t_0$. Units: kg. |
| $M_f$ | Final black hole mass used in formation-time mappings. Units: kg. |
| $M_{\mathrm{classical}}(t)$ | Classical radiatively regulated mass-growth solution used as a baseline comparison, $M_{\mathrm{classical}}(t) = M_0 \exp((t - t_0)/t_S)$. Units: kg. |
| $\dot{M}$ | Mass inflow rate (supply entering the radiatively regulated channel). Units: $\mathrm{kg\,s^{-1}}$. |
| $\dot{M}_\bullet$ | Black hole mass growth rate after radiative losses, defined as $\dot{M}_\bullet = (1-\epsilon)\dot{M}$. Units: $\mathrm{kg\,s^{-1}}$. |
| $\dot{M}_{\mathrm{in}}$ | Total inflow rate reaching the inner system used as a normalization for polar fractions. Units: $\mathrm{kg\,s^{-1}}$. |
| $\dot{M}_{\mathrm{pol}}$ | Polar inflow component used in defining $f_{\mathrm{polar}}(r)$ and $t_{\mathrm{drive}}$. Units: $\mathrm{kg\,s^{-1}}$. |

Continued on next page

| Symbol | Meaning |
|---|---|
| $\dot{M}_{\rm sup}$ | Declared supplied inflow delivered into the shearing-evaluation region (boundary-condition supply for the Coulomb Shearing assessment). Units: $\mathrm{kg\,s^{-1}}$. Encodes an external boundary condition. |
| $\dot{M}_{\rm rad}$ | Radiatively regulated inflow term expressed as $\dot{M}_{\rm rad} = \lambda L_{\rm Edd}/(\epsilon c^2)$. Units: $\mathrm{kg\,s^{-1}}$. |
| $\dot{M}_{\rm ion}$ | Ion-dominated inflow component enabled by species decoupling, defined as $\dot{M}_{\rm ion} = f_{\rm ion}\dot{M}_{\rm sup}$. Units: $\mathrm{kg\,s^{-1}}$. |
| $\dot{M}_{\rm CS}$ | Total black hole growth rate in the Coulomb Shearing model, $\dot{M}_{\rm CS} = (1-\epsilon)\dot{M}_{\rm rad} + (1-\epsilon_{\rm ion})\dot{M}_{\rm ion}$. Units: $\mathrm{kg\,s^{-1}}$. |
| $\dot{M}_{\rm Edd}$ | Eddington accretion-rate scale referenced in the manuscript in the inequality $\dot{M} \gg \dot{M}_{\rm Edd}$. Units: $\mathrm{kg\,s^{-1}}$. |
| $L$ | Radiative luminosity used for radiative forcing and for defining the Eddington ratio. Units: W. |
| $L_{\rm Edd}$ | Eddington luminosity defined as $L_{\rm Edd} = 4\pi G M m_p c/\sigma_T$. Units: W. |
| $\epsilon$ | Radiative efficiency defined by $L = \epsilon \dot{M} c^2$. Dimensionless. Encodes a radiative closure choice. |
| $\epsilon_{\rm ion}$ | Effective radiative efficiency assigned to the ion-dominated inflow channel in $\dot{M}_{\rm CS}$. Dimensionless. Encodes a modeling choice for ion-channel radiative losses. |
| $\lambda$ | Symbol used in two distinct contexts in the manuscript: (i) observing wavelength in the Faraday rotation relation $\psi(\lambda) = \psi_0 + \mathrm{RM}\lambda^2$, and (ii) Eddington ratio in $\dot{M}_{\rm rad} = \lambda L_{\rm Edd}/(\epsilon c^2)$ and in $t_S \propto 1/\lambda$. This is a notational overload. In the growth equations, $\lambda$ represents the dimensionless Eddington ratio $L/L_{\rm Edd}$. In the polarization equations, $\lambda$ represents observing wavelength. Units: dimensionless (Eddington ratio) or m (wavelength), depending on context as stated in each section. |
| $f_{\rm Edd}$ | Eddington fraction used in the explicit local example for $\Delta a$ scaling, representing a dimensionless luminosity fraction relative to Eddington. Dimensionless. |
| $\Phi$ | Supply ratio defined as $\Phi \equiv \dot{M}_{\rm sup}/\dot{M}_{\rm rad}$. Dimensionless. Encodes a declared boundary-condition ratio. |
| $\Phi_{\rm req}$ | Required supply ratio derived from a target growth multiplier, $\Phi_{\rm req} = (\mathcal{G}-1)\left[(1-\epsilon)/(1-\epsilon_{\rm ion})\right]/f_{\rm ion}$. Dimensionless. |
| $t_{\rm Sal}$ | Salpeter timescale for Eddington-limited growth at $\lambda = 1$, $t_{\rm Sal} = \frac{\epsilon}{1-\epsilon}\frac{c\sigma_T}{4\pi G m_p}$. Units: s. |

Continued on next page

| Symbol | Meaning |
|---|---|
| $t_S$ | Generalized classical e-folding timescale at Eddington ratio $\lambda$, $t_S = \frac{\epsilon}{1-\epsilon}\frac{\sigma_T c}{4\pi G m_p}\frac{1}{\lambda}$. Units: s. |
| $t_{\text{form}}$ | Formation-time interval (available assembly time) used in inverse mapping relations, defined as $t_{\text{form}} = \Delta t$ in the mapping equation. Units: s. |
| $\mathbf{F}$ | Generic force vector. Units: N. |
| $\mathbf{F}_e$, $\mathbf{F}_i$ | Total force vectors acting on electrons and ions, respectively. Units: N. |
| $\mathbf{F}_{\text{rad},e}$ | Radiative force on an electron under the adopted scattering model (Thomson in the baseline expression). Units: N. |
| $\mathbf{F}_{\text{rad},i}$ | Radiative force on an ion under the adopted model (taken as negligible in the Thomson-limit baseline expression). Units: N. |
| $\mathbf{F}_{g,e}$, $\mathbf{F}_{g,i}$ | Gravitational force vectors acting on an electron and an ion, respectively. Units: N. |
| $\mathbf{g}$ | Gravitational acceleration vector (used as a generic symbol in the force-balance discussion). Units: $\mathrm{m\,s^{-2}}$. |
| $\hat{\mathbf{r}}$ | Radial unit vector. Dimensionless. |
| $\hat{\mathbf{s}}$ | Unit vector tangent to the evaluated trajectory or separation direction used in along-path projections. Dimensionless. |
| $\mu_s$ | Geometric projection factor defined as $\mu_s \equiv \hat{\mathbf{r}} \cdot \hat{\mathbf{s}} = \cos\theta_s$. Dimensionless. |
| $\theta_s$ | Angle between $\hat{\mathbf{r}}$ and $\hat{\mathbf{s}}$ used in defining $\mu_s$. Units: radians. |
| $a_e$, $a_i$ | Species accelerations along the chosen projection direction, defined as $a_e \equiv (1/m_e)\mathbf{F}_e \cdot \hat{\mathbf{s}}$ and $a_i \equiv (1/m_i)\mathbf{F}_i \cdot \hat{\mathbf{s}}$. Units: $\mathrm{m\,s^{-2}}$. |
| $\Delta a$ | Differential acceleration between electron and ion components under the adopted species-resolved forcing model, evaluated along the same trajectory. The sign convention is fixed as $\Delta a \equiv a_e - a_i$. Units: $\mathrm{m\,s^{-2}}$. |
| $\Delta a_\parallel$ | Magnitude or component of the differential acceleration projected along the relevant separation or trajectory direction in the shearing condition (denoted by the subscript $\parallel$ in the manuscript). Units: $\mathrm{m\,s^{-2}}$. |
| $F_{e,\parallel}$, $F_{i,\parallel}$ | Projected force components along the $\parallel$ direction used in $\Delta a_\parallel = \left\lvert \frac{F_{e,\parallel}}{m_e} - \frac{F_{i,\parallel}}{m_i} \right\rvert$. Units: N. |
| $m_e$ | Electron mass. Units: kg. |
| $m_p$ | Proton mass. Units: kg. |
| $m_i$ | Ion mass for the species being evaluated. Units: kg. |
| $m_Z$ | Mass of an ion or nucleus associated with atomic number $Z$ (used in thermal-speed definitions and species scaling). Units: kg. |
| $Z$ | Atomic number of the species. Dimensionless integer. |

Continued on next page

| Symbol | Meaning |
|---|---|
| $q$ | Ion charge state (integer describing ionization level, used as an argument in $a_{\mathrm{C}}(Z,q)$ and $\lambda_D(Z,q)$). Dimensionless integer. |
| $Z_{\mathrm{eff}}$ | Effective charge used in the Coulomb binding-acceleration threshold, representing the net charge coupling relevant for electron binding to the ion core at charge state $q$. Dimensionless. |
| $t_c$ | Characteristic Coulomb coupling or relaxation timescale governing momentum exchange between electrons and ions under the specified plasma interaction model. Units: s. |
| $\tau_{ei}$ | Electron–ion collision (relaxation) time used as an equivalent notation for $t_c$ in the manuscript's explicit Spitzer form. Units: s. |
| $\ln\Lambda$ | Coulomb logarithm entering the Spitzer coupling time. Dimensionless. |
| $\ln\Lambda_0$ | Reference Coulomb logarithm used in the sensitivity scaling $t_c(\ln\Lambda) = t_c(\ln\Lambda_0)\ \ln\Lambda_0/\ln\Lambda$, with $\ln\Lambda_0 \equiv 20$ in the manuscript. Dimensionless. Encodes a fixed numerical choice. |
| $\lambda_D$ | Debye length, $\lambda_D \equiv (\varepsilon_0 k_B T_e/(n_e e^2))^{1/2}$ (or the equivalent form used in worked examples). Units: m. |
| $\ell(Z,q)$ | Characteristic electron–ion separation scale entering the Coulomb binding threshold, defined piecewise as $\ell = \ell_{\mathrm{bound}}(Z,q)$ (partially ionized regime) or $\ell = \kappa\lambda_D$ (fully ionized separation regime). Units: m. Encodes a regime choice. |
| $\ell_{\mathrm{bound}}(Z,q)$ | Bound-state dominated separation scale used in the partially ionized regime. Physically represents a characteristic bound-electron orbital scale for the charge state. Units: m. |
| $\kappa$ | Dimensionless displacement threshold multiplier in the shearing condition $\Delta x(t_{\mathrm{res}}) \geq \kappa\lambda_D$. Dimensionless. Encodes a closure choice for when charge separation is considered dynamically significant. |
| $a_{\mathrm{C}}(Z,q)$ | Coulomb binding-acceleration threshold defined as $a_{\mathrm{C}} = \frac{Z_{\mathrm{eff}} e^2}{4\pi\varepsilon_0 m_e \ell^2}$. Physically represents the characteristic acceleration needed to separate an electron from an ion over scale $\ell$. Units: $\mathrm{m\,s^{-2}}$. |
| $\Delta v(t)$ | Relative drift velocity between electrons and ions induced by differential forcing under finite coupling, appearing in the coupled relaxation solution. Units: $\mathrm{m\,s^{-1}}$. |
| $\Delta x(t)$ | Relative displacement between electron and ion components along the evaluated separation direction after time $t$ under the adopted forcing and coupling model. Units: m. |

Continued on next page

| Symbol | Meaning |
|---|---|
| $\Delta x_Z(t_{\text{res}}; v, \theta)$ | Species-resolved displacement used inside the probabilistic kernel, explicitly indicating dependence on residence time and kinematic parameters. Units: m. |
| $\mathcal{S}(r; \theta, v, Z, q)$ | Dimensionless shearing functional defined as $\mathcal{S} \equiv \Delta x(t_{\text{res}})/(\kappa\lambda_D)$. Physically represents how strongly a given state exceeds (or fails) the charge-separation threshold. Dimensionless. |
| $\mathcal{A}_Z^{\text{seg}}(\theta, v, q)$ | Set (region) of radii that satisfy the shearing admissibility condition for species $Z$ under specified $(\theta, v, q)$. Physically represents the shearing-admissible radial segment(s) in the constructed model. |
| $r_{\text{min}}, r_{\text{max}}$ | Lower and upper bounds of the species-dependent region in which the Coulomb Shearing criterion is satisfied, defined as $r_{\text{min}} \equiv \inf \mathcal{A}_Z^{\text{seg}}$ and $r_{\text{max}} \equiv \sup \mathcal{A}_Z^{\text{seg}}$. Units: m. |
| $\chi_s$ | Binary indicator representing the physical outcome of the shearing criterion for a given species and trajectory. $\chi_s = 1$ denotes coupling enforcement failure (shearing succeeds), and $\chi_s = 0$ denotes enforced coupling. Dimensionless. |
| $\Pr(\cdot)$ | Probability operator used in defining decoupling and shearing probabilities. Dimensionless. |
| $P_{\text{dec}}(Z)$ | Decoupling probability for a species with atomic number $Z$, defined as $P_{\text{dec}}(Z) \equiv \Pr(\chi_s = 1 \mid Z)$. Dimensionless. |
| $P_s(Z)$ | Probability that a species with atomic number $Z$ undergoes Coulomb Shearing within the available residence time, with $P_s(Z) \equiv P_{\text{dec}}(Z)$ in the manuscript. Dimensionless. |
| $w_Z$ | Species weighting coefficients used in the definition $f_{\text{ion}} \equiv \sum_Z w_Z P_{\text{dec}}(Z)$. Dimensionless; typically normalized such that $\sum_Z w_Z = 1$ within the modeled composition. Encodes a composition assumption. |
| $f_{\text{ion}}$ | Ion-channel mass fraction defined as $f_{\text{ion}} = \sum_Z w_Z P_{\text{dec}}(Z)$, used to map species probabilities into a net ion-dominated inflow fraction. Dimensionless. |
| $\mathcal{K}_{\text{shear}}(Z, v, \theta)$ | Shearing kernel used in the probability integral, defined as a Heaviside decision functional of whether the displacement exceeds the threshold. Dimensionless. |
| $H(\cdot)$ | Heaviside step function used in $\mathcal{K}_{\text{shear}} = H(\Delta x_Z - \kappa\lambda_D)$ to encode a binary pass/fail of the displacement criterion. Dimensionless. Note: symbol conflicts with disk scale height $H$ used elsewhere; this is a notational overload. |

Continued on next page

| Symbol | Meaning |
|---|---|
| $f(v)$ | Speed probability density function used in $P_s(Z) = \int \mathcal{K}_{\rm shear} f(v)\,dv$. Units: $(\mathrm{m\,s^{-1}})^{-1}$. |
| $F_v(v_{\rm crit})$ | Cumulative distribution function associated with $f(v)$, defined by $P_s(Z) = \int_0^{v_{\rm crit}} f(v)\,dv \equiv F_v(v_{\rm crit})$. Dimensionless. |
| $v_{\rm crit}$ | Critical speed threshold such that states with $v \le v_{\rm crit}$ satisfy the shearing condition in the adopted mapping. Units: $\mathrm{m\,s^{-1}}$. |
| $f_{\rm MB}(v)$ | Maxwell–Boltzmann speed distribution used as an explicit example for $f(v)$. Units: $(\mathrm{m\,s^{-1}})^{-1}$. |
| $v_{\rm th}$ | Thermal speed scale in the Maxwell–Boltzmann distribution, defined as $v_{\rm th} \equiv \sqrt{2k_BT_i/m_i}$. Units: $\mathrm{m\,s^{-1}}$. |
| $T_i$ | Ion temperature used in the thermal-speed definition. Units: K. |
| $\mathrm{erf}(x)$ | Error function appearing in the closed-form Maxwell–Boltzmann evaluation of $P_s(Z)$. Dimensionless. |
| $x$ (in $P_s$ closed form) | Dimensionless ratio $x \equiv v_{\rm crit}/v_{\rm th}$ used as the argument of erf. Dimensionless. |
| $W_{\rm ion}$ | Work done by differential acceleration in driving relative motion, defined as $W_{\rm ion} = \int_0^{t_{\rm res}} m_e \Delta a_\parallel(t)\, v_{\rm rel}(t)\,dt$. Units: J. |
| $v_{\rm rel}(t)$ | Relative speed entering the ionization-work integral, representing electron–ion relative motion during the forcing interval. Units: $\mathrm{m\,s^{-1}}$. |
| $I_j(Z,q)$ | Ionization energy for the $j$-th electron removed in the sequence for species $(Z,q)$, used in the admissibility inequality $\sum_{j=1}^{k} I_j \le W_{\rm ion}$. Units: J (or eV if consistently used in inputs). |
| $k$ | Number of electrons removed (upper limit of the ionization-energy sum) in the ionization admissibility condition. Dimensionless integer. |
| $n_e$ | Electron number density. Units: $\mathrm{m^{-3}}$. |
| $n_i$ | Ion number density. Units: $\mathrm{m^{-3}}$. |
| $T_e$ | Electron temperature. Units: K. |
| $\theta_e$ | Dimensionless electron temperature parameter used in worked examples, defined as $\theta_e \equiv k_BT_e/(m_ec^2)$. Dimensionless. Encodes an input parameterization choice. |
| $\rho$ | Mass density appearing in the GRMHD mass-flux diagnostic. Units: $\mathrm{kg\,m^{-3}}$. |
| $\rho_q$ | Charge density defined as $\rho_q \equiv e(Zn_i - n_e) \simeq 0$ under quasi-neutrality. Units: $\mathrm{C\,m^{-3}}$. |
| $j_\parallel$ | Parallel current density component defined as $j_\parallel \equiv e(Zn_iv_{i,\parallel} - n_ev_{e,\parallel})$. Units: $\mathrm{A\,m^{-2}}$. |

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| Symbol | Meaning |
|---|---|
| $\mathbf{j}$ | Current density vector entering the charge-continuity equation. Units: $\mathrm{A\,m^{-2}}$. |
| $v_{e,\parallel}$, $v_{i,\parallel}$ | Electron and ion velocity components along the $\parallel$ direction used in the current definition. Units: $\mathrm{m\,s^{-1}}$. |
| $\omega_{pe}$ | Electron plasma frequency, $\omega_{pe} \equiv \sqrt{n_e e^2/(\varepsilon_0 m_e)}$. Units: $\mathrm{s^{-1}}$. |
| $t_E$ | Characteristic electrostatic response timescale, taken as $t_E \sim \omega_{pe}^{-1}$ in the manuscript. Units: s. |
| $\nabla$ | Spatial gradient operator (and $\nabla\cdot$ divergence) used in the charge-continuity equation. Units: $\mathrm{m^{-1}}$ when acting on fields. |
| $\partial/\partial t$ | Time partial-derivative operator used in the charge-continuity equation. Units: $\mathrm{s^{-1}}$ when acting on fields. |
| $F(r,\theta,\phi,t)$ | Radiative flux field used in the angular and temporal averaging definition. Units: $\mathrm{W\,m^{-2}}$. |
| $\langle F\rangle_{\Omega,t}(r)$ | Angular and time averaged flux at radius $r$, defined by $\langle F\rangle_{\Omega,t}(r) \equiv \frac{1}{4\pi\Delta t}\int_{t_0}^{t_0+\Delta t}\int F\,d\Omega\,dt$. Units: $\mathrm{W\,m^{-2}}$. |
| $\psi(\lambda)$ | Observed electric-vector position angle (EVPA) as a function of observing wavelength in the Faraday rotation relation. Units: radians. |
| $\psi_0$ | Intrinsic EVPA at $\lambda = 0$ in the Faraday rotation relation. Units: radians. |
| RM | Rotation measure appearing in $\psi(\lambda) = \psi_0 + \mathrm{RM}\lambda^2$. Units: $\mathrm{rad\,m^{-2}}$. |
| $I_{\mathrm{pol}}$ | Polarized intensity used as an observable input in the evaluation protocol. Units: depends on observing convention (e.g., $\mathrm{Jy\ beam^{-1}}$); treated as an observational input quantity. |
| $\mathcal{F}_M(r,\theta)$ | Mass-flux diagnostic per solid angle defined as $\mathcal{F}_M \equiv -r^2\rho u^r$. Units: $\mathrm{kg\,s^{-1}\,sr^{-1}}$ (interpreting the diagnostic as a flux per solid angle). |
| $f_{\mathrm{polar}}$ | Polar fraction of supplied inflow. Used both as $f_{\mathrm{polar}} \equiv \dot{M}_{\mathrm{sup}}/\dot{M}_{\mathrm{in}}$ and as a radius-dependent diagnostic $f_{\mathrm{polar}}(r) \equiv \dot{M}_{\mathrm{pol}}(r)/\dot{M}_{\mathrm{in}}(r)$. Dimensionless. |
| $f_{\mathrm{iso}}$ | Isotropic expectation for the polar fraction given the polar-cap solid angle, $f_{\mathrm{iso}} \equiv \Omega_{\mathrm{pol}}/(4\pi) = 1 - \cos\theta_{\mathrm{pol}}$. Dimensionless. |
| $a_2(r)$ | Quadrupole anisotropy coefficient of the mass-flux distribution, defined by angular projection onto $P_2(\cos\theta)$. Dimensionless. |
| $P_2(\mu)$ | Second Legendre polynomial, $P_2(\mu) = \frac{1}{2}(3\mu^2 - 1)$. Dimensionless. |
| $\mu$ | Cosine of polar angle used as the argument of $P_2$, $\mu \equiv \cos\theta$ in the quadrupole definition. Dimensionless. |
| 0.30 | Fixed numerical threshold used in the criterion $a_2(r_{\mathrm{nh}}) \geq 0.30$. Dimensionless. Encodes a modeling choice. |

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| Symbol | Meaning |
|---|---|
| $r_{\mathrm{nh}}$ | Characteristic radius defining the non-homologous reservoir region used in the maintenance model. Units: m. Encodes a modeling choice. |
| $\mathcal{V}_{\mathrm{nh}}$ | Non-homologous reservoir volume used in the definition $M_{\mathrm{nh}} = \int_{\mathcal{V}_{\mathrm{nh}}} \rho\, dV$. Units: m$^3$. |
| $dV$ | Differential volume element used in the reservoir mass definition. Units: m$^3$. |
| $M_{\mathrm{nh}}$ | Mass contained in the non-homologous reservoir region, defined by $M_{\mathrm{nh}} = \int_{\mathcal{V}_{\mathrm{nh}}} \rho\, dV$ and parameterized as $M_{\mathrm{nh}} = \dot{M}_{\mathrm{in}} t_{\mathrm{dyn}}(r_{\mathrm{nh}})$. Units: kg. |
| $\mathcal{A}$ | Dimensionless anisotropy amplitude defined as $\mathcal{A} \equiv (f_{\mathrm{polar}} - f_{\mathrm{iso}})/(1 - f_{\mathrm{iso}})$. Dimensionless. |
| $\mathcal{A}_{\mathrm{drive}}$ | Driving amplitude entering the forced-damped anisotropy model $d\mathcal{A}/dt = \mathcal{A}_{\mathrm{drive}}/t_{\mathrm{drive}} - \mathcal{A}/t_{\mathrm{relax}}$. Dimensionless. Encodes a modeling choice. |
| $\mathcal{A}_{\mathrm{ss}}$ | Steady-state anisotropy amplitude predicted by the forced-damped model, $\mathcal{A}_{\mathrm{ss}} = \mathcal{A}_{\mathrm{drive}}(t_{\mathrm{relax}}/t_{\mathrm{drive}})$. Dimensionless. |
| $\mathcal{A}_{\mathrm{crit}}$ | Critical anisotropy threshold defined as $\mathcal{A}_{\mathrm{crit}} \equiv 0.30$. Dimensionless. Encodes a fixed threshold choice. |
| 0.30 | Fixed numerical constant used to set $\mathcal{A}_{\mathrm{crit}}$ and to map the polar-fraction maintenance requirement. Dimensionless. Encodes a modeling choice. |
| $\mathcal{M}_{\mathrm{maint}}$ | Maintenance margin defined as $\mathcal{M}_{\mathrm{maint}} \equiv (\mathcal{A}_{\mathrm{drive}}/\mathcal{A}_{\mathrm{crit}})(t_{\mathrm{relax}}/t_{\mathrm{drive}})$; values $\geq 1$ indicate maintenance under the adopted forced-damped model. Dimensionless. |
| $P$ | Fluid pressure appearing in the illustrative polar-momentum balance equation. Units: Pa. |
| $B$ | Magnetic field magnitude appearing in the illustrative polar-momentum balance equation via $B^2$. Units: T (or G if cgs is used consistently in an input set). |
| $D$ | Luminosity distance used as an input in the evaluation protocol. Units: m (or Mpc as an input unit). |
| $z$ | Cosmological redshift used as an alternative to $D$ when a cosmology is declared. Dimensionless. |
| LRD | Little Red Dot: JWST-identified class label for extremely compact high-redshift sources used here as an explicit target class label (descriptive only; not an assumed physical model). |
| quasi-star / black hole star | Conditional hypothesis class in which a black hole accretes inside a massive envelope; treated as an interpretation to be tested only when required external inputs are traceable and declared. |

Continued on next page

| Symbol | Meaning |
|---|---|
| $L_{X,\mathrm{obs}}$ | Observed X-ray luminosity used as a conservative radiative forcing proxy in worked examples. Units: W (or $\mathrm{erg\,s^{-1}}$ in cgs inputs). |
| $L_{\mathrm{jet}}$ | Observed or inferred jet extent used as a falsification diagnostic in the evaluation protocol (treated as a length scale). Units: m. |
| $L_{\mathrm{jet,ref}}$ | Reference jet-length threshold used for pass/fail comparisons in the evaluation protocol. Units: m. Encodes a chosen falsification threshold. |
| $L_{\mathrm{jet,pred}}$ | Predicted jet-length comparator used in P5, defined by Eq. (210) as a product of a Coulomb Shearing maintenance margin, a spin-dependent hybrid multiplier, and the reference threshold $L_{\mathrm{jet,ref}}$. Units: m. |
| $L^-_{\mathrm{jet,pred}}$, $L^+_{\mathrm{jet,pred}}$ | Lower and upper bounds on the predicted comparator when the spin is bounded as an interval $a_* \in [a_{\min}, a_{\max}]$, defined in the P5 protocol block in Effective Inflow Geometry and Axially-Concentrated Mass-Loading. Units: m. |
| $a_{\min}$, $a_{\max}$ | Lower and upper bounds on the dimensionless Kerr spin parameter $a_*$ when spin is constrained as an interval for protocol evaluation. Dimensionless. |
| $\mathcal{M}_{\mathrm{hyb}}(a_*)$ | Hybrid Kerr multiplier used in the jet-length comparator, defined by Eq. (206) as $\mathcal{M}_{\mathrm{hyb}} \equiv M_{\mathrm{spin}}\,\kappa_{\mathrm{align}}$. Dimensionless. Encodes a spin-dependent scaling (not a universal constant). |
| $M_{\mathrm{spin}}(a_*)$ | Spin-dependent factor entering the hybrid multiplier, defined by Eq. (207) using the horizon angular frequency $\Omega_H$ and the spin-suppression function $S(a_*)$, normalized to a fixed reference spin $a_0$. Dimensionless. |
| $\kappa_{\mathrm{align}}(a_*)$ | Alignment gain factor entering the hybrid multiplier, defined by Eq. (208) and capped ($\kappa_{\mathrm{align}} \leq 1.2$). Dimensionless. Encodes a bounded modeling choice for spin-assisted alignment. |
| $\Omega_H(a_*)$ | Kerr horizon angular frequency used in the hybrid multiplier, defined by Eq. (209) as $\Omega_H \equiv a_* c/(2r_H)$. Units: $\mathrm{s^{-1}}$. |
| $S(a_*)$ | Spin-suppression function used in the hybrid multiplier, defined in Eq. (207) as $S(a_*) \equiv 1/(1+2a_*^2)$. Dimensionless. |
| $a_0$ | Fixed reference spin used only for normalization in the definition of $M_{\mathrm{spin}}$ (Eq. (207)), set in-text as $a_0 \equiv 0.5$. Dimensionless. Encodes a normalization convention (not a fitted parameter). |
| $\mathcal{G}(t)$ | Growth multiplier defined as $\mathcal{G}(t) \equiv M(t)/M_{\mathrm{classical}}(t)$. Dimensionless. |
| $\mathcal{E}$ | Instantaneous accretion-rate enhancement factor defined as $\mathcal{E} \equiv \dot{M}_{\mathrm{CS}}/\dot{M}_{\mathrm{classical}}$. Dimensionless. |

Table 11: Notation conventions.

| Quantity | Use this | Meaning / rule |
|---|---|---|
| Differential acceleration (general) | $\Delta a$ | Species differential acceleration (electron minus ion) when direction does not matter or is implicitly projected. |
| Differential acceleration (parallel component) | $\Delta a_{\parallel}$ | Along-trajectory (field-aligned / tangent) component used in the closure and displacement calculations. Use $\Delta a_{\parallel}$ whenever the parallel component is intended. |
| Reference evaluation radius | $r_{\star}$ | Single canonical symbol for the reference evaluation radius (formerly written in mixed forms). Use $r_{\star}$ everywhere for the reference radius used in worked examples and control runs. |
| Accretion rate (generic) | $\dot{M}$ | Generic mass inflow rate when no comparison class is intended. |
| Eddington reference accretion rate | $\dot{M}_{\mathrm{Edd}}$ | Eddington reference accretion rate used only as a normalization. |
| Ion-only accretion channel rate | $\dot{M}_{\mathrm{ion}}$ | Ion-only (or ion-dominated) delivery rate through the sheared channel, as defined by the framework. |
| Observed total accretion proxy | $\dot{M}_{\mathrm{obs}}$ | Observed or inferred total accretion rate proxy (bolometric, spectral, or model-derived). |

# B Derivation Details

## B.1 Eddington baseline and coupling assumption

**Radiative force on an electron (Thomson channel).** Consider an isotropic luminosity $L$ at radius $r$. The radiative energy flux is

$$F(r) = \frac{L}{4\pi r^2}.$$

The associated photon momentum flux is $F(r)/c$. For Thomson scattering, the momentum transferred per unit time to a single electron is the momentum flux multiplied by the Thomson cross section $\sigma_T$. Therefore the radiative force magnitude per electron is

$$f_{\mathrm{rad},e}(r) = \sigma_T \frac{F(r)}{c} = \sigma_T \frac{1}{c} \frac{L}{4\pi r^2} = \frac{\sigma_T}{c} \frac{L}{4\pi r^2},$$

which is Eq. (135) in the main text. In vector form (radially outward) this is the Thomson specialization

$$\mathbf{F}_{\text{rad},e} \simeq \frac{\sigma_T L}{4\pi r^2 c}\,\hat{\mathbf{r}}, \qquad \mathbf{F}_{\text{rad},i} \simeq \mathbf{0},$$

consistent with Eq. (38).

**Gravitational force on a particle.** For a central mass $M$, the gravitational force magnitude on a particle of mass $m$ at radius $r$ is

$$f_g(m;r) = \frac{GMm}{r^2}.$$

Applied to electrons and ions, this is Eq. (136) in the per-particle form, and Eq. (39) in the vector form.

**The coupling assumption enters at the bulk (electron plus ion) step.** The physically enforced statement is the electron force balance: radiative momentum transfer couples directly to electrons (Thomson channel). The classical Eddington construction then makes one additional assumption:

*Classical coupling assumption:* an electrostatic restoring field maintains near-co-motion of electrons and ions on the macroscopic inflow scale, so that the radiative force deposited into the electron component is communicated to the ion component, enabling one to treat the electron and ion as an effectively locked pair for the purpose of bulk force balance.

This is the exact step at which electron balance is extended to bulk inflow.

**Coupled-pair force balance and the Eddington luminosity.** In the locked-pair limit, the electron and proton (or representative ion) share a common bulk acceleration. The classical Eddington limit sets the outward radiative force on the electron equal to the inward gravitational force on the bulk mass that must be supported. In the standard hydrogenic baseline, that bulk mass is taken as $m_p$ (proton mass), so the balance condition is

$$f_{\text{rad},e}(r) = f_g(m_p;r).$$

Substituting the explicit expressions gives

$$\frac{\sigma_T}{c}\,\frac{L}{4\pi r^2} = \frac{GMm_p}{r^2}.$$

Multiply both sides by $4\pi r^2 c/\sigma_T$:

$$L = \frac{4\pi r^2 c}{\sigma_T}\,\frac{GMm_p}{r^2} = \frac{4\pi GMm_p c}{\sigma_T}.$$

Thus the Eddington luminosity is

$$L_{\text{Edd}} = \frac{4\pi GMm_p c}{\sigma_T},$$

which matches Eq. (137) exactly.

**Separation of enforced vs assumed statements (explicit).**

- **Enforced:** radiative momentum couples to electrons according to Thomson scattering, giving Eq. (135) (or Eq. (38) in vector form).
- **Assumed (classical Eddington step):** electron-ion locking is sufficiently strong that the electron force balance can be extended to the bulk mass inflow by supporting $m_p$ (or a representative ion mass) against gravity.

Later sections modify only this coupling extension, not the Thomson electron channel itself.

## B.2 Force model closures used for electrons and ions

**Projected, species-resolved accelerations.** Along a shared polar or near-polar trajectory with tangent unit vector $\hat{\mathbf{s}}$, the species-projected accelerations are defined as in Eq. (34):

$$a_e(t) \equiv \frac{1}{m_e}\,\mathbf{F}_e(t)\cdot\hat{\mathbf{s}}, \qquad a_i(t) \equiv \frac{1}{m_i}\,\mathbf{F}_i(t)\cdot\hat{\mathbf{s}}.$$

**Force decomposition (no hidden closures).** The net force on each species is decomposed into physically distinct contributions exactly as in Eqs. (35)–(36):

$$\mathbf{F}_e(t) = \mathbf{F}_{\mathrm{rad},e}(t) + \mathbf{F}_{\mathrm{EM},e}(t) + \mathbf{F}_{g,e}(t),$$
$$\mathbf{F}_i(t) = \mathbf{F}_{\mathrm{rad},i}(t) + \mathbf{F}_{\mathrm{EM},i}(t) + \mathbf{F}_{g,i}(t).$$

**Gravity (species-independent acceleration, species-dependent force).** The gravitational forces (vector form) are Eq. (39):

$$\mathbf{F}_{g,e} = -\frac{GMm_e}{r^2}\,\hat{\mathbf{r}}, \qquad \mathbf{F}_{g,i} = -\frac{GMm_i}{r^2}\,\hat{\mathbf{r}}.$$

Projecting along $\hat{\mathbf{s}}$ introduces the geometric factor

$$\mu_s \equiv \hat{\mathbf{r}}\cdot\hat{\mathbf{s}} = \cos\theta_s,$$

so that the projected gravitational accelerations satisfy

$$a_{g,e} = \frac{1}{m_e}\,\mathbf{F}_{g,e}\cdot\hat{\mathbf{s}} = -\mu_s\,\frac{GM}{r^2},$$
$$a_{g,i} = \frac{1}{m_i}\,\mathbf{F}_{g,i}\cdot\hat{\mathbf{s}} = -\mu_s\,\frac{GM}{r^2}.$$

Thus gravity cancels identically in the differential acceleration $\Delta a$ (it contributes equally to both accelerations).

**Radiation pressure (Thomson specialization).** The Thomson-channel closure used in the manuscript is Eq. (38):

$$\mathbf{F}_{\mathrm{rad},e} \simeq \frac{\sigma_T L}{4\pi r^2 c}\,\hat{\mathbf{r}}, \qquad \mathbf{F}_{\mathrm{rad},i} \simeq \mathbf{0}.$$

Projecting and dividing by mass yields the projected radiative accelerations

$$\begin{aligned} a_{\mathrm{rad},e} &= \frac{1}{m_e}\,\mathbf{F}_{\mathrm{rad},e}\cdot\hat{\mathbf{s}} = \mu_s\,\frac{\sigma_T L}{4\pi r^2 c\, m_e}, \\ a_{\mathrm{rad},i} &= \frac{1}{m_i}\,\mathbf{F}_{\mathrm{rad},i}\cdot\hat{\mathbf{s}} \simeq 0. \end{aligned}$$

**Electromagnetic forcing (explicitly retained, optionally specialized).** Electromagnetic forcing is retained at the force level via $\mathbf{F}_{\mathrm{EM},e}$ and $\mathbf{F}_{\mathrm{EM},i}$ in Eqs. (35)–(36). Its contribution to the projected accelerations is

$$a_{\mathrm{EM},e}(t) \equiv \frac{1}{m_e}\,\mathbf{F}_{\mathrm{EM},e}(t)\cdot\hat{\mathbf{s}}, \qquad a_{\mathrm{EM},i}(t) \equiv \frac{1}{m_i}\,\mathbf{F}_{\mathrm{EM},i}(t)\cdot\hat{\mathbf{s}}.$$

**Downstream specializations used by the worked-example machinery (Worked Examples and Source-Grounded Outputs):** the electromagnetic term is retained in the general force decomposition, and downstream runs may choose either (i) the ideal-MHD baseline $E_\parallel = 0$ (control specialization) or (ii) a declared non-ideal $E_\parallel(r)$ model (benchmark specialization). For the control specialization, the electromagnetic contribution is suppressed in the executed $\Delta a$ evaluation by setting

$$\mathbf{F}_{\mathrm{EM},e} = \mathbf{0}, \qquad \mathbf{F}_{\mathrm{EM},i} = \mathbf{0}.$$

Benchmark runs may instead evaluate the projected electromagnetic contribution through a declared non-ideal $E_\parallel(r)$ (Eq. (43)), which enters the full projected law (Eq. (58)). Control runs adopting $E_\parallel = 0$ are archived in Appendix E.

**Differential acceleration.** With the sign convention fixed, the differential acceleration along the shared trajectory is Eq. (37) in Section 03:

$$\Delta a \equiv a_e - a_i.$$

Substituting the decompositions and using the exact cancellation of gravity gives

$$\begin{aligned} \Delta a &= \left(a_{\mathrm{rad},e} - a_{\mathrm{rad},i}\right) + \left(a_{\mathrm{EM},e} - a_{\mathrm{EM},i}\right) + \left(a_{g,e} - a_{g,i}\right) \\ &= \left(a_{\mathrm{rad},e} - a_{\mathrm{rad},i}\right) + \left(a_{\mathrm{EM},e} - a_{\mathrm{EM},i}\right), \end{aligned}$$

since $a_{g,e} - a_{g,i} = 0$ identically.

**Thomson-only differential acceleration (control specialization, $E_\parallel = 0$).** Under the control specialization $E_\parallel = 0$ (ideal-MHD baseline), the electromagnetic contribution to the projected differential acceleration is suppressed, so the above reduces to

$$\Delta a \simeq a_{\mathrm{rad},e} = \mu_s \, \frac{\sigma_T L}{4\pi r^2 c \, m_e},$$

which is Eq. (59).

**Evaluation at the Eddington luminosity (scale expression).** Insert $L = L_{\mathrm{Edd}}$ from Eq. (137) into Eq. (59):

$$\begin{aligned}\Delta a(L_{\mathrm{Edd}}) &\simeq \mu_s \, \frac{\sigma_T}{4\pi r^2 c \, m_e} \left( \frac{4\pi G M m_p c}{\sigma_T} \right) \\ &= \mu_s \, \frac{G M m_p}{r^2 m_e} = \mu_s \, \frac{m_p}{m_e} \, \frac{G M}{r^2},\end{aligned}$$

which is Eq. (60).

**Connection to the Section 04 projected-force form.** Section 04 uses the trajectory-parallel magnitude

$$\Delta a_{\parallel} = \left| \frac{F_{e,\parallel}}{m_e} - \frac{F_{i,\parallel}}{m_i} \right|,$$

as stated in Eq. (80), where $F_{e,\parallel} \equiv \mathbf{F}_e \cdot \hat{\mathbf{s}}$ and $F_{i,\parallel} \equiv \mathbf{F}_i \cdot \hat{\mathbf{s}}$. This is algebraically identical to the definition in Section 03 after (i) projection onto $\hat{\mathbf{s}}$ and (ii) taking an absolute value when only magnitudes are needed to test the inequality criteria.

### B.3 From $\Delta a(t)$ to $\Delta x(t_{\mathrm{res}})$

**Relative-velocity dynamics with continuous Coulomb drag.** Define the species-relative velocity along the shared trajectory as

$$\Delta v(t) \equiv v_e(t) - v_i(t).$$

The adopted closure is that collisional momentum exchange acts as a linear relaxation of the relative drift on the timescale $t_c$, while external forcing contributes the differential acceleration $\Delta a(t)$. This gives the first-order ODE (Eq. (66)):

$$\begin{aligned} \frac{d\,\Delta v}{dt} &= \Delta a(t) - \frac{1}{t_c}\,\Delta v, \\ \Longleftrightarrow \qquad \frac{d\,\Delta v}{dt} + \frac{1}{t_c}\,\Delta v &= \Delta a(t). \end{aligned}$$

**Kernel solution for arbitrary forcing history (no skipped steps).** Multiply the ODE by the integrating factor $e^{t/t_c}$:

$$e^{t/t_c}\frac{d\,\Delta v}{dt} + \frac{1}{t_c}e^{t/t_c}\Delta v = e^{t/t_c}\Delta a(t).$$

Recognize the left-hand side as a total derivative:

$$\frac{d}{dt}\left(e^{t/t_c}\Delta v(t)\right) = e^{t/t_c}\Delta a(t).$$

Integrate from $t' = 0$ to $t' = t$:

$$e^{t/t_c}\Delta v(t) - e^{0}\Delta v(0) = \int_0^t e^{t'/t_c}\Delta a(t')\,dt'.$$

With the initial condition used for the displacement build-up (zero initial drift),

$$\Delta v(0) = 0,$$

we obtain

$$\begin{aligned}\Delta v(t) &= e^{-t/t_c}\int_0^t e^{t'/t_c}\Delta a(t')\,dt' \\ &= \int_0^t e^{-(t-t')/t_c}\Delta a(t')\,dt',\end{aligned}$$

which matches Eq. (67).

**Relative displacement as the time integral of relative velocity.** Define the species-relative displacement along the trajectory as

$$\Delta x(t) \equiv \int_0^t \Delta v(\tau)\,d\tau.$$

Insert the kernel form for $\Delta v(\tau)$:

$$\Delta x(t) = \int_0^t \left[\int_0^\tau e^{-(\tau-t')/t_c}\Delta a(t')\,dt'\right]d\tau.$$

Swap the order of integration over the triangular domain $0 \le t' \le \tau \le t$:

$$\Delta x(t) = \int_0^t \left[\int_{t'}^t e^{-(\tau-t')/t_c}\,d\tau\right]\Delta a(t')\,dt'.$$

Evaluate the inner integral using $u \equiv \tau - t'$:

$$\int_{t'}^{t} e^{-(\tau-t')/t_c}\, d\tau = \int_{0}^{t-t'} e^{-u/t_c}\, du = \left[-t_c e^{-u/t_c}\right]_0^{t-t'}$$
$$= t_c\left(1 - e^{-(t-t')/t_c}\right).$$

Therefore

$$\Delta x(t) = \int_0^t \left[t_c\left(1 - e^{-(t-t')/t_c}\right)\right] \Delta a(t')\, dt',$$

which is Eq. (68).

**Constant-$\Delta a$ specialization (explicit integration).** If $\Delta a(t) = \Delta a$ is approximately constant over $[0, t]$, then Eq. (66) becomes

$$\frac{d\,\Delta v}{dt} + \frac{1}{t_c}\Delta v = \Delta a, \qquad \Delta v(0) = 0.$$

Using the already-derived kernel form,

$$\Delta v(t) = \int_0^t e^{-(t-t')/t_c} \Delta a\, dt' = \Delta a \int_0^t e^{-(t-t')/t_c}\, dt'.$$

Let $u = t - t'$ so that $dt' = -du$ and the limits map $t' = 0 \to u = t$ and $t' = t \to u = 0$:

$$\Delta v(t) = \Delta a \int_{u=t}^{u=0} e^{-u/t_c}(-du) = \Delta a \int_0^t e^{-u/t_c}\, du$$
$$= \Delta a \left[-t_c e^{-u/t_c}\right]_0^t = \Delta a\, t_c\left(1 - e^{-t/t_c}\right).$$

Integrate once more for the displacement:

$$\begin{aligned}
\Delta x(t) &= \int_0^t \Delta v(\tau)\, d\tau = \Delta a\, t_c \int_0^t \left(1 - e^{-\tau/t_c}\right) d\tau \\
&= \Delta a\, t_c \left[\tau + t_c e^{-\tau/t_c}\right]_0^t \quad \text{since} \quad \int -e^{-\tau/t_c} d\tau = t_c e^{-\tau/t_c} \\
&= \Delta a\, t_c \left(t + t_c e^{-t/t_c} - 0 - t_c\right) \\
&= \Delta a \left[t_c\, t - t_c^2\left(1 - e^{-t/t_c}\right)\right],
\end{aligned}$$

which matches Eq. (69). Replacing $\Delta a$ by the trajectory-parallel $\Delta a_{\parallel}$ and setting $t = t_{\mathrm{res}}$ gives the Section 04 local bound form (Eq. (84)).

**Residence-time evaluation.** The paper evaluates the physical separation accumulated over the available residence time as $\Delta x(t_{\rm res})$, and compares it directly to the adopted screening threshold in the Coulomb-closure criterion (Eq. (72) and Eq. (83)).

### B.4 Species-dependent threshold modeling

**Coulomb restoring acceleration scale.** The threshold $a_{\rm C}(Z, q)$ used in the shearing admissibility conditions is defined in Section 04 as Eq. (81). It follows directly from the Coulomb force magnitude at a separation scale $\ell$.

Take an electron of charge magnitude $e$ interacting with an effective ion core characterized by an effective charge $Z_{\rm eff}e$ at separation $\ell$. The Coulomb force magnitude is

$$F_{\rm C}(\ell) = \frac{Z_{\rm eff}\, e^2}{4\pi\varepsilon_0\, \ell^2}.$$

The corresponding electron acceleration scale is the force divided by the electron mass:

$$a_{\rm C}(Z, q) \equiv \frac{F_{\rm C}(\ell)}{m_e} = \frac{1}{m_e}\,\frac{Z_{\rm eff}\, e^2}{4\pi\varepsilon_0\, \ell^2} = \frac{Z_{\rm eff}\, e^2}{4\pi\varepsilon_0\, m_e\, \ell^2},$$

which is exactly Eq. (81).

**Operational separation length $\ell(Z, q)$ (explicit regime encoding).** The manuscript encodes two operational regimes through the choice of $\ell(Z, q)$ (Eq. (82)):

$$\ell(Z, q) \equiv \begin{cases} \ell_{\rm bound}(Z, q), & \text{partially ionized / bound-state dominated,} \\ \kappa\, \lambda_D, & \text{fully ionized / free-charge separation.} \end{cases}$$

This is a modeling disclosure, not a fit parameter:

- In the bound-state dominated regime, $\ell_{\rm bound}(Z, q)$ is the characteristic bound-electron scale used to test partial or shell-level stripping.
- In the fully ionized regime, the operational separation scale is the screened length $\kappa\, \lambda_D$, with $\kappa \sim O(1)$ retained explicitly as stated in Section 03 surrounding Eq. (72).

**Debye length (explicit plasma-parameter dependence).** The Debye length used in the fully ionized regime is defined in Section 03 (Eq. (65)):

$$\lambda_D \equiv \left( \frac{\varepsilon_0\, k_B T_e}{n_e\, e^2} \right)^{1/2}.$$

Thus the fully ionized separation scale depends on plasma temperature and density as $\lambda_D \propto (T_e/n_e)^{1/2}$, consistent with Eq. (106).

**From restoring acceleration to the base shearing condition.** The Section 04 acceleration-form admissibility condition (Eq. (79)) is obtained by comparing the trajectory-parallel differential acceleration to the restoring acceleration scale:

$$\Delta a_{\parallel}(r, \theta, v) \geq a_{\mathrm{C}}(Z, q).$$

Substituting Eq. (81) makes explicit that this condition is a statement about whether the external differential forcing exceeds the Coulomb restoring acceleration at the operational separation scale $\ell(Z, q)$.

**Displacement-form threshold used for the executable criterion.** The manuscript's executable closure-failure test is posed as a displacement comparison at the residence time:

$$\Delta x(t_{\mathrm{res}}) \geq \kappa\, \lambda_D,$$

which is Eq. (72) (Section 03) and appears in Section 04 as Eq. (83). The displacement $\Delta x(t_{\mathrm{res}})$ is computed from $\Delta a(t)$ under continuous Coulomb drag using the kernel solution (Eq. (68)) or its constant-$\Delta a$ specialization (Eq. (69) and Eq. (84)).

**Coupling time closure used in the displacement build-up.** The coupling time $t_c$ is an explicit input controlling the suppression of relative drift through Eq. (66). The adopted normalization recorded in the manuscript is the Spitzer-form scaling (Eq. (63)):

$$t_c \approx 3 \times 10^5\ \mathrm{s} \left(\frac{T_e}{10^8\ \mathrm{K}}\right)^{3/2} \left(\frac{10^8\ \mathrm{cm}^{-3}}{n_e}\right) \left(\frac{20}{\ln \Lambda}\right) \frac{1}{Z^2}.$$

This expression is used as a declared closure imported from standard plasma transport theory [1]. Its explicit scaling with $\ln \Lambda$ is separated as Eq. (64), and the evaluation protocol requires the chosen $\ln \Lambda$ (value or bound) to be declared for each worked example or regime map.

### B.5 Limiting-case consistency checks (if included)

**Limit $\Delta a \to 0$.** From the kernel expression (Eq. (68)),

$$\Delta x(t) = \int_0^t \left[t_c\left(1 - e^{-(t-t')/t_c}\right)\right] \Delta a(t')\, dt',$$

if $\Delta a(t') \equiv 0$ for all $t'$, then the integrand vanishes and therefore

$$\Delta x(t) \equiv 0 \quad \forall\, t.$$

Thus no separation accumulates if there is no differential forcing.

**Limit $t_{\rm res} \to 0$.** Using the constant-$\Delta a$ form (Eq. (69)) and expanding $e^{-t/t_c}$ for small $t$:

$$\begin{aligned} e^{-t/t_c} &= 1 - \frac{t}{t_c} + \frac{1}{2}\left(\frac{t}{t_c}\right)^2 + O\left(\left(\frac{t}{t_c}\right)^3\right), \\ 1 - e^{-t/t_c} &= \frac{t}{t_c} - \frac{1}{2}\left(\frac{t}{t_c}\right)^2 + O\left(\left(\frac{t}{t_c}\right)^3\right). \end{aligned}$$

Insert into Eq. (69):

$$\begin{aligned} \Delta x(t) &= \Delta a\left[t_c t - t_c^2\left(1 - e^{-t/t_c}\right)\right] \\ &= \Delta a\left[t_c t - t_c^2\left(\frac{t}{t_c} - \frac{1}{2}\left(\frac{t}{t_c}\right)^2 + O\left(\left(\frac{t}{t_c}\right)^3\right)\right)\right] \\ &= \Delta a\left[t_c t - \left(t_c t - \frac{1}{2}t^2 + O\left(\frac{t^3}{t_c}\right)\right)\right] \\ &= \frac{1}{2}\Delta a\, t^2 + O\left(\frac{\Delta a\, t^3}{t_c}\right). \end{aligned}$$

Therefore $\Delta x(t_{\rm res}) \to 0$ as $t_{\rm res} \to 0$, and the leading behavior recovers the free-drift short-time limit in Eq. (70).

**Strong coupling limit $t_c \ll t_{\rm res}$.** In the constant-$\Delta a$ case, for $t \gg t_c$ we have $e^{-t/t_c} \to 0$, so Eq. (69) reduces to

$$\Delta x(t) \simeq \Delta a\left[t_c t - t_c^2\right] = \Delta a\, t_c\, t - \Delta a\, t_c^2.$$

When $t \gg t_c$, the $t_c^2$ term is negligible compared to $t_c t$, giving

$$\Delta x(t) \simeq \Delta a\, t_c\, t,$$

which is the drag-limited accumulation in Eq. (71). In the enforced-coupling limit $t_c \to 0$ at fixed $t$, this implies

$$\Delta x(t) \to 0,$$

so no macroscopic separation accumulates when collisional coupling becomes arbitrarily fast.

**Transition criterion used to demarcate suppression vs growth.** Section 04 introduces the regime-separation inequality (Eq. (104)). It is obtained by requiring that, in the short-time free-drift limit, the separation reaches the operational scale $\ell(Z, q)$ within the available time.

Start with the short-time limit (Eq. (70)):

$$\Delta x(t) \simeq \frac{1}{2}\Delta a_{\parallel}\, t^2 \qquad (t \ll t_c).$$

Impose the requirement $\Delta x(t_{\rm res}) \geq \ell(Z,q)$:

$$\begin{aligned}
\frac{1}{2}\Delta a_{\parallel}\, t_{\rm res}^2 &\geq \ell(Z,q),\\
t_{\rm res}^2 &\geq \frac{2\,\ell(Z,q)}{\Delta a_{\parallel}},\\
\left(\frac{t_{\rm res}}{t_c}\right)^2 &\geq \frac{2\,\ell(Z,q)}{\Delta a_{\parallel}\, t_c^2},\\
\frac{t_{\rm res}}{t_c} &\geq \left(\frac{2\,\ell(Z,q)}{\Delta a_{\parallel}\, t_c^2}\right)^{1/2}.
\end{aligned}$$

Equation (104) is the corresponding order-unity regime-demarcation form written in the manuscript (with the numerical factor absorbed into the explicitly retained order-unity cutoff bookkeeping already present in $\ell(Z,q)$ through $\kappa$):

$$\frac{t_{\rm res}}{t_c} \;\geq\; \left(\frac{\ell(Z,q)}{\Delta a_{\parallel}\, t_c^2}\right)^{1/2}.$$

This criterion is used only to separate coupling-suppressed from separation-allowing behavior under the declared approximations; it does not redefine the executable displacement test itself, which remains Eq. (83) with $\Delta x(t_{\rm res})$ computed from the drag-limited dynamics.

# C Data Sources and Provenance Rules

## C.1 Accepted repositories and publication types

Numerical inputs are admissible only if they originate from one of the accepted source classes below and satisfy the exact provenance and unit rules in this appendix. These rules apply uniformly to every evaluated object and every symbol in the required set (Eq. (211) in Pre-Committed Evaluation Protocol (Anti-Post-Hoc)).

**Accepted source classes (numerical values).** The following sources are admissible for primary numerical inputs (values, bounds, uncertainties, and object-level measurements):

1. **Peer-reviewed primary observational papers with object-level data tables.** Acceptable venues include (non-exhaustive) ApJ/ApJL/ApJS, MNRAS, A&A, Nature, and Science. The key

admissibility condition is that the paper reports object-level numerical values in tables (main text or appendix) or in a machine-readable supplement with stable identifiers.

2. **Mission archives and official observatory repositories.** Mission archive products are admissible when retrieved from an official archive with a documented pipeline and release identifier, including (non-exhaustive): MAST (HST/JWST), Chandra Data Archive, XMM-Newton Science Archive, ALMA Science Archive, NRAO archive products (VLA/VLBA where applicable), and official EHT data releases.

3. **Established astronomical catalogs with catalog identifiers and machine-readable columns.** VizieR-hosted catalogs are admissible when the catalog identifier is provided and the queried columns and row identifiers are recorded exactly. Catalog-derived values must be object-level, not population-level summary statistics.

4. **Official survey data products with documented pipelines and object-level outputs.** Survey releases are admissible only when the release is public, versioned, and provides object-level measurements in table form (for example: a released catalog, a mission data product table, or a formally archived machine-readable table).

**JWST compact high-$z$ objects (LRDs): mandatory provenance fields.** JWST compact high-$z$ objects, including *Little Red Dots* (LRDs), are an explicit target class in this manuscript and therefore use the same governance rules as all other systems. In addition, because many LRD results originate from mission-archive products and pipeline-processed catalogs, any JWST-derived numerical value used anywhere in the manuscript must record the following fields verbatim in its provenance string or in an adjacent locked retrieval note:

1. **Program and observation identity:** JWST Program ID, target/field name, observation date range if provided, and the stable archive identifier for the product used (MAST product identifier or an equivalent stable product key).

2. **Instrument and configuration:** instrument (NIRCam or NIRSpec), observing mode, and configuration needed to interpret the value:

   - Imaging: filter name(s), pixel scale if explicitly stated, and the method used to report sizes (for example, PSF-convolved profile fitting versus aperture-based upper bounds).
   - Spectroscopy: grating and filter (or prism), stated spectral resolution if provided, and the extraction convention if the paper defines one that affects reported line widths.

3. **Pipeline or catalog release identity:** the published pipeline or catalog release identifier if the value is taken from a release table, plus any version string explicitly given in the source.

For LRD exemplars where a black hole mass estimate is reported, the admissibility condition is stricter than name-only citation. A mass value may be used only when the cited source reports an explicit estimator and ties it to stated measurements (for example, broad-line width and the accompanying luminosity proxy). If the estimator inputs are not provided in traceable form, the mass is treated as a bound or as unavailable, and the object remains eligible but may become **DATA-INSUFFICIENT** for any result requiring that mass.

The LRD exemplar packet in Appendix D is constructed to satisfy this rule set and uses the same admissible-source classes as all other objects (peer-reviewed object-level papers and mission-archive products). The initial exemplar uses object-level values and observing identifiers from [7] and uses sample-level context only for selection discipline [5, 6, 8].

**Accepted source classes (identifiers only).** The following sources may be used only for *object identification and disambiguation* (names, aliases, and coordinates), not for primary numerical modeling inputs:

- SIMBAD: identifiers and coordinates only.

**Preprints and non-refereed artifacts (narrow conditional admissibility).** Non-refereed artifacts are inadmissible for primary numerical values *except* under the following locked condition:

- A mission archive product is non-refereed by construction. It is admissible if it is retrieved from an official archive and satisfies the mission provenance format in this appendix.

For journal articles that also exist as arXiv eprints, the journal version is the authoritative source class. The arXiv PDF may be used as an access copy only if the provenance string still cites the journal publication metadata and provides a verifiable location (page, section, table, row, and column) that uniquely identifies the value.

**Explicitly prohibited sources (numerical values).** The following are inadmissible for primary numerical inputs regardless of plausibility:

- Review articles, perspective pieces, and whitepapers as the *origin* of a numerical value.
- Secondary compilations that do not provide object-level traceability to the primary source locations.
- Meta-analyses that only report aggregate statistics without object-level values.
- Private datasets, personal communications, unpublished notes, conference slides, and blog posts.
- Preprints or manuscripts lacking explicit data tables (main text, appendix, or machine-readable supplements).
- Values obtained by digitizing figures when the underlying numerical dataset is not provided in a table or machine-readable supplement.

**Figure-backed datasets (allowed only when the numbers exist).** Rule 5 does not prohibit use of results that are *shown* in a figure. It prohibits values obtained by visual inference alone. A figure is admissible as a pointer only if the source also provides the corresponding numerical dataset in one of: (i) a table in the paper, (ii) a machine-readable table in the journal supplement, or (iii) an archived dataset with stable identifiers. In such cases, the provenance must cite the table or machine-readable dataset, not the figure.

### C.2 Exact provenance format (mandatory)

Every numerical input row in Section 9 and every input packet in Appendix D must include a provenance string in one of the locked formats below, recorded verbatim. A BibTeX citation may appear alongside the provenance string, but it is not a substitute for it.

**Format 1: journal article value (locked).** Use the following exact string structure:

```
Author(s) Year, Journal, Volume, Page, Section X, Table Y, Row Z, Column C
```

**Mandatory fields:** Author(s), Year, Journal, Volume, Page, Section identifier (or “Section n/a” if not labeled), Table identifier, Row identifier, and Column name. **Location rule:** If the table spans multiple pages, include the specific page containing the row used.

**Example (format only):**

Event Horizon Telescope Collaboration 2019, ApJL, 875, L1, Sec. 3.1, Table 3, Row "M87*", Col. "$L_{X,\mathrm{obs}}$"

**Format 2: catalog value (locked).** Use the following exact string structure:

```
Catalog name, Catalog ID, Column name, Row ID, Query URL, Access date
YYYY-MM-DD
```

**Mandatory fields:** Catalog name, catalog identifier, column name, row identifier used to select the object, full query URL, and access date. **Query URL rule:** The URL must reproduce the same row selection and columns (or be a stable permalink to the exact query result).

**Example (format only):**

```
VizieR, J/ApJ/875/L5, Col.  "ne0", Row "M87*", Query https://vizier.cds.
unistra.fr/, Access 2026-01-10
```

**Format 3: mission archive product (locked).** Use the following exact string structure:

```
Mission, Program ID, Observation ID, Data product name, Product version,
Access date YYYY-MM-DD
```

**Mandatory fields:** Mission, Program ID, Observation ID, data product name, product version (or pipeline/release identifier when provided), and access date.

**Example (format only):**

```
JWST (MAST), Program 1234, ObsID jw01234-o001, Product "jw01234-o001_t001
_nirspec_clear-prism_x1d.fits",
Pipeline/Release "CRDS 11.17.0", Access 2026-01-10
```

**Deterministic derivations from admissible values (allowed only with full audit trail).** A quantity may be computed from admissible published values only if the mapping is deterministic and fully recorded. In that case, the provenance string must follow:

```
Computed: [target quantity] from [source provenance...] and constants [constant
set], with full unit conversion shown
```

This clause permits unit conversions and definition-based transforms (for example, converting a published dimensionless temperature parameter into $T_e$ using the explicit definition). It does not permit introduction of new empirical scalings, tuning, or model-dependent substitutions not declared in Pre-Committed Evaluation Protocol (Anti-Post-Hoc).

**Prohibited provenance shortcuts.** The following are invalid and must be treated as missing inputs:

- BibTeX citation alone (no location fields).
- “See [paper]” without page, table, row, and column.
- “From figure X” without a corresponding published numeric table or machine-readable dataset pointer.
- Any provenance lacking an access date for catalogs or mission archives.

### C.3 Unit normalization rules

**Canonical unit system (locked).** All *computational inputs* are normalized to SI units before evaluation. Source values may be recorded in their published units in the input table, but the SI-converted value used in computation must be shown explicitly in the same row (or in an immediately adjacent row) with a conversion chain.

**Astrophysical display exceptions (allowed, not computational).** For readability only, the manuscript may display the following conventional astrophysical units alongside SI:

$$M_\odot, \quad \mathrm{pc},\ \mathrm{kpc},\ \mathrm{Mpc}, \quad \mathrm{G}, \quad \mathrm{erg\,s^{-1}}, \quad \mathrm{cm^{-3}}.$$

These are display exceptions. They do not alter the requirement that the computation uses SI-normalized quantities.

**Locked conversion constants and rules.** The following conversions are used verbatim in worked examples and in Appendix D input packets:

- **Mass:**
$$1\,M_\odot = 1.98847 \times 10^{30}\ \text{kg}.$$
- **Distance:**
$$1\,\text{pc} = 3.085677581 \times 10^{16}\ \text{m}, \qquad 1\,\text{kpc} = 10^3\ \text{pc}, \qquad 1\,\text{Mpc} = 10^6\ \text{pc}.$$
- **Energy and luminosity:**
$$1\,\text{eV} = 1.602176634 \times 10^{-19}\ \text{J}, \qquad 1\,\text{keV} = 10^3\ \text{eV}, \qquad 1\,\text{erg} = 10^{-7}\ \text{J}, \qquad 1\,\text{erg}\,\text{s}^{-1} = 10^{-7}\ \text{W}.$$
- **Magnetic field:**
$$1\,\text{G} = 10^{-4}\ \text{T}.$$
- **Number density:**
$$1\,\text{cm}^{-3} = 10^6\ \text{m}^{-3}.$$
- **Length and time (when needed for source-unit conversion):**
$$1\,\text{cm} = 10^{-2}\ \text{m}, \qquad 1\,\text{day} = 86400\ \text{s}, \qquad 1\,\text{yr} = 365.25\ \text{day}.$$

**Cosmology-dependent conversions (locked rule).** If a source reports redshift $z$ and uses a specific cosmology to infer distances or luminosities, the conversion from $z$ to distance used in this manuscript must adopt the same cosmology as the cited source for that object. Substituting a different cosmology is prohibited unless the source explicitly provides an alternative conversion and uncertainty, in which case both must be recorded and propagated under the same rules.

**Mandatory conversion-chain disclosure.** Each worked example in Section 09 must show at least one full conversion chain explicitly in-line (not as an implied step), including the source units, the conversion factors used, and the final SI value used in computation. A worked example lacking at least one explicit conversion chain is invalid under this appendix.

**Unit-system consistency requirement.** A calculation is invalid if any intermediate step mixes unit systems without explicit conversion. Where a source provides values in cgs-like units (for example, erg $\text{s}^{-1}$, $\text{cm}^{-3}$, G), those values must be converted to SI before being substituted into equations written in SI form.

### C.4 Uncertainty propagation rules

**Locked method choice (this manuscript).** This manuscript uses **Monte Carlo propagation with fixed distributions** uniformly across all evaluated systems, matching the locked propagation statement in Pre-Committed Evaluation Protocol (Anti-Post-Hoc). The sampling size and seed are fixed globally:

$$N = 10{,}000, \qquad s_0 = 314159.$$

Changing $N$ or $s_0$ after any results exist is prohibited and requires a protocol revision and a full re-run for all objects.

**Locked distribution assignment rules (no discretion).** Input uncertainties are mapped to sampling distributions using the following deterministic rules:

1. **Symmetric uncertainty:** if a source reports $x \pm \sigma_x$, sample $x$ from a normal distribution $\mathcal{N}(x, \sigma_x)$ truncated to the physical domain of $x$.
2. **Asymmetric uncertainty:** if a source reports $x^{+\sigma_+}_{-\sigma_-}$, sample from a split-normal distribution with left width $\sigma_-$ and right width $\sigma_+$, truncated to the physical domain.
3. **Bound-only reporting:** if a source reports $x \in [x_{\min}, x_{\max}]$ with no distributional form, sample uniformly on the interval. For strictly positive scale parameters that span more than one decade (for example $n_e$, $T_e$, $B$, and $L$), sample uniformly in $\log_{10} x$ over $[\log_{10} x_{\min}, \log_{10} x_{\max}]$. This rule is fixed and applies uniformly.
4. **Hard bounds:** if a source reports a one-sided bound (upper or lower), the bound is treated as a hard truncation limit. If no additional uncertainty model is reported, the input is treated as data-insufficient for Monte Carlo sampling unless the protocol in Pre-Committed Evaluation Protocol (Anti-Post-Hoc) explicitly permits bound-only substitution for that symbol in that computation.

**Locked multi-source rule (this manuscript).** When multiple eligible literature values exist for a symbol, this manuscript adopts the **envelope rule** as the uniform mapping rule:

- All eligible values and their provenance strings must be recorded in a source ledger for that symbol.
- The computational sampling domain for that symbol is the union envelope across eligible values after unit normalization and after applying any pre-declared scale constraints in Pre-Committed Evaluation Protocol (Anti-Post-Hoc).
- The envelope rule is applied identically to every object and every symbol. It may not be replaced by a mixture model, weighting, or selective exclusion after evaluation.

**Locked reporting interval (uniform).** All propagated outputs that feed outcome classification are reported as:

$$\text{median}, \quad \text{and the central 90\% interval (5th to 95th percentile)}.$$

The 90% level is fixed and is used uniformly for both predicted and observed intervals in agreement tests.

**Locked decision rules (classification gate).** Outcome assignment follows the pre-committed three-outcome rule in Pre-Committed Evaluation Protocol (Anti-Post-Hoc). The uncertainty-handling component of that rule is locked as:

- **PASS:** predicted interval(s) and the corresponding observed interval(s) overlap under the locked 90% representation and the required comparator dataset is present in admissible form.
- **FAIL:** predicted signatures are contradicted by admissible comparator data under the same locked interval representation, or the predicted signature is absent where the comparator is decisive under the protocol.
- **DATA-INSUFFICIENT:** missing required inputs, missing required comparators, or uncertainty so large that the protocol cannot discriminate under the locked tests.

**Prohibited uncertainty behavior.** The following invalidate the associated result and require reclassification as DATA-INSUFFICIENT:

- Per-object changes to the distribution assignment rules above.
- Post-hoc tightening or relaxation of bounds.
- Changing the credibility level, sampling size, seed, truncation rule, or multi-source rule after any outcomes exist.

## C.5 Cross-locking to worked examples

**Binding scope.** This appendix binds *all* numerical content in the manuscript, including (but not limited to): (i) the Section 09 worked examples, (ii) any tables of adopted constants or inputs, (iii) any parameter values used to generate regime maps, and (iv) all input packets in Appendix D.

**Worked example enforcement.** Every numerical input appearing in Section 09 must satisfy:

- admissible source class (this appendix),
- exact provenance string format (this appendix),
- unit normalization and explicit conversion disclosure (this appendix),
- locked uncertainty propagation method and distribution rules (this appendix and Pre-Committed Evaluation Protocol (Anti-Post-Hoc)).

Any Section 09 input row lacking a compliant provenance string is treated as absent. If the missing value is required for the executed computation, the worked example cannot be assigned PASS or FAIL and must be DATA-INSUFFICIENT.

**No dual standards clause.** It is prohibited to apply stricter provenance rules in the theoretical sections and weaker rules in worked examples, or vice versa. The same admissibility, provenance, unit, and uncertainty rules apply everywhere.

**Violation handling (no correction after evaluation).** If a provenance violation is discovered after a worked example has been evaluated (for example: the source location is not reproducible, the value is figure-only without a dataset, or the unit conversion was implicit), the response is not to "fix" the input. The response is:

- reclassify the affected example as DATA-INSUFFICIENT (or FAIL if the protocol specifies contradiction based on remaining admissible data), and
- archive the violation in the computation log.

Substitution of a new source or a revised value to preserve an outcome is prohibited by Rule 1 (Source Finality).

**Cross-lock to Appendix D.** Appendix D contains the worked-example input packets used in Section 09. Every value in Appendix D must carry an Appendix C compliant provenance string and must match the corresponding Section 09 input table entry exactly (value-for-value and unit-for-unit). Any discrepancy invalidates the worked example.

# D Worked Example Input Packets

## D.1 Object 1: M87*

### D.1.1 Identifiers

- Primary common name: M87*
- Alternative names / aliases: Messier 87; M87; NGC 4486; Virgo A; 3C 274
- Catalog / database identifiers (non-exhaustive): NGC 4486; 3C 274
- Coordinates: not required for disambiguation in this packet (unique under the above identifiers)

### D.1.2 Input table (final)

**Scope note:** This table lists the verbatim numerical inputs used in the Section 09 worked example for M87*. Only direct inputs are included; intermediate quantities and derived values are omitted.

| Quantity | Symbol | Value | Units | Exact provenance string |
|---|---|---|---|---|
| Black hole mass | $M_\bullet$ | $6.5 \times 10^9$ | $M_\odot$ | `Event Horizon Telescope Collaboration 2019, ApJL, 875, L5, Appendix A (object parameter listing), row:` $M_\bullet$. |
| One-zone region radius (example geometry) | $r_h$ | $5\, r_\mathrm{g}$ | dimensionless | `Event Horizon Telescope Collaboration 2019, ApJL, 875, L5, arXiv:1906.11242, PDF p. 2, one-zone model parameter block, entry:` $r_h = 5\, r_g$. |
| Electron temperature parameter (emission peak) | $\theta_{e,\mathrm{bpk}}$ | 10 | dimensionless | `Event Horizon Telescope Collaboration 2019, ApJL, 875, L5, arXiv:1906.11242, PDF p. 2, one-zone model parameter block, entry:` $\theta_{e,\mathrm{bpk}} = 10$. |
| Electron density normalization | $n_{e,0}$ | $2.9 \times 10^4$ | $\mathrm{cm}^{-3}$ | `Event Horizon Telescope Collaboration 2019, ApJL, 875, L5, arXiv:1906.11242, PDF p. 2, one-zone model parameter block, entry:` $n_{e,0} = 2.9 \times 10^4\ \mathrm{cm}^{-3}$. |

Continued on next page

| Quantity | Symbol | Value | Units | Exact provenance string |
|---|---|---|---|---|
| Density slope | $\alpha$ | 1.1 | dimensionless | `Event Horizon Telescope Collaboration 2019, ApJL, 875, L5, arXiv:1906.11242, PDF p. 2, one-zone model parameter block, entry:` $\alpha = 1.1$. |
| Magnetic field normalization | $B_0$ | 4.9 | G | `Event Horizon Telescope Collaboration 2019, ApJL, 875, L5, arXiv:1906.11242, PDF p. 2, one-zone model parameter block, entry:` $B_0 = 4.9\,\mathrm{G}$. |
| Observed X-ray luminosity (quiescent constraint) | $L_{X,\mathrm{obs}}$ | $7 \times 10^{40}$ | $\mathrm{erg\,s^{-1}}$ | `Event Horizon Telescope Collaboration 2019, ApJL, 875, L5, Appendix A, Table 3, row:` $L_{X,\mathrm{obs}} = 7 \times 10^{40}\,\mathrm{erg\,s^{-1}}$. |
| Adopted polar inflow speed (evaluation input) | $v$ | $0.1\,c$ | $c$ | `This manuscript (Coulomb Shearing V7), Section 09, Worked Example 1 (M87*), Inputs table, row:` $v = 0.1\,c$. |
| Adopted residence time (evaluation input) | $t_{\mathrm{res}}$ | $2 \times 10^5$ | s | `This manuscript (Coulomb Shearing V7), Section 09, Worked Example 1 (M87*), Inputs table, row:` $t_{\mathrm{res}} = 2 \times 10^5\,\mathrm{s}$. |

Continued on next page

| Quantity | Symbol | Value | Units | Exact provenance string |
|---|---|---|---|---|
| Coulomb logarithm (evaluation input) | $\ln\Lambda$ | 20 | dimensionless | `This manuscript (Coulomb Shearing V7), Section 09, Worked Example 1 (M87*), Inputs table, row:` $\ln\Lambda = 20$. |
| Magnetic multiplier (evaluation input) | $\eta_B$ | 1.0 | dimensionless | `This manuscript (Coulomb Shearing V7), Section 09, Worked Example 1 (M87*), Inputs table, row:` $\eta_B = 1.0$. |
| Lorentz factor (evaluation input) | $\eta_H$ | 1.0 | dimensionless | `This manuscript (Coulomb Shearing V7), Section 09, Worked Example 1 (M87*), Inputs table, row:` $\eta_H = 1.0$. |
| Threshold factor (evaluation input) | $\kappa$ | 3 | dimensionless | `This manuscript (Coulomb Shearing V7), Section 09, Worked Example 1 (M87*), Inputs table, row:` $\kappa = 3$. |

### D.1.3 Source links and extraction anchors

- **EHT M87* Paper V (one-zone anchor and basic parameters).** DOI: `https://doi.org/10.3847/2041-8213/ab0f43`. arXiv: `https://arxiv.org/abs/1906.11242`. PDF: `https://arxiv.org/pdf/1906.11242.pdf`. Accessed: `2026-01-10`. Extraction anchors: PDF p. 2 (one-zone block), Appendix A Table 3 (quiescent X-ray constraint), and Appendix A object parameters, as recorded in the provenance strings in the input table above.
- **MOJAVE (M87 = 1228+126; multifrequency and 15 GHz epoch products).** Source page: https://www.cv.nrao.edu/MOJAVE/sourcepages/1228%2B126.shtml. Accessed: `2026-01-16`. Multifrequency VLBA products for epoch `2006-06-15`: 8.1 GHz: https://www.cv.nrao.edu/2cmVLBA/data/multifreq/1228%2B126.x.2006_06_15.uvf, https://www.cv.nrao.edu/2cmVLBA/data/multifreq/1228%2B126.x.2006_06_15.i_0.1.fits.gz,

https://www.cv.nrao.edu/2cmVLBA/data/multifreq/1228%2B126.x.2006_06_15_color.png, https://www.cv.nrao.edu/2cmVLBA/data/multifreq/1228%2B126.x.2006_06_15.pdf. 8.4 GHz: https://www.cv.nrao.edu/2cmVLBA/data/multifreq/1228%2B126.y.2006_06_15.uvf, https://www.cv.nrao.edu/2cmVLBA/data/multifreq/1228%2B126.y.2006_06_15.i_0.1.fits.gz, https://www.cv.nrao.edu/2cmVLBA/data/multifreq/1228%2B126.y.2006_06_15_color.png, https://www.cv.nrao.edu/2cmVLBA/data/multifreq/1228%2B126.y.2006_06_15.pdf. 12.1 GHz: https://www.cv.nrao.edu/2cmVLBA/data/multifreq/1228%2B126.j.2006_06_15.uvf, https://www.cv.nrao.edu/2cmVLBA/data/multifreq/1228%2B126.j.2006_06_15.i_0.1.fits.gz, https://www.cv.nrao.edu/2cmVLBA/data/multifreq/1228%2B126.j.2006_06_15_color.png, https://www.cv.nrao.edu/2cmVLBA/data/multifreq/1228%2B126.j.2006_06_15.pdf. 15 GHz epoch `2006-06-15` (VLBA code `BL137F`) table entry: $I(\text{mJy}) = 2395$, $P(\text{mJy}) < 1.5$, $P(\%) < 0.6$. 15 GHz products: https://www.cv.nrao.edu/2cmVLBA/data/1228%2B126/2006_06_15/1228%2B126.u.2006_06_15.icn.fits.gz, https://www.cv.nrao.edu/2cmVLBA/data/1228%2B126/2006_06_15/1228%2B126.u.2006_06_15.uvf, https://www.cv.nrao.edu/2cmVLBA/data/1228%2B126/2006_06_15/1228%2B126.u.2006_06_15.pcn.png, https://www.cv.nrao.edu/2cmVLBA/data/1228%2B126/2006_06_15/1228%2B126.u.2006_06_15.pcn.pdf, https://www.cv.nrao.edu/2cmVLBA/data/1228%2B126/2006_06_15/1228%2B126.u.2006_06_15.montage.png.

- **Rotation measure comparator (MOJAVE RM not available for this source).** Nikonov et al. (2023): https://arxiv.org/abs/2307.11660, PDF: https://arxiv.org/pdf/2307.11660.pdf. Accessed: `2026-01-17`. RM summary values used: RM changes from approximately $-5000$ to $-2000$ rad m$^{-2}$, $\langle \text{RM} \rangle = -4070 \pm 1030$ rad m$^{-2}$, and $\sigma_{\text{pix}} \sim 300$ rad m$^{-2}$, quoted from Sec. 5.4 (paragraph beginning "The Rotation Measure map (Figure 9) ...").

## D.2 Object 2: Sgr A*

### D.2.1 Identifiers

- Primary common name: Sgr A*
- Alternative names / aliases: Sagittarius A*; Galactic Center SMBH
- Catalog / database identifiers (non-exhaustive): radio source at Galactic Center; commonly indexed as Sgr A* in major archives
- Coordinates: not required for disambiguation in this packet (unique under the above identifiers)

### D.2.2 Input table (final)

**Scope note:** This table lists the verbatim numerical inputs used in the Section 09 worked example for Sgr A*. Only direct inputs are included; intermediate quantities and derived values are omitted.

| Quantity | Symbol | Value | Units | Exact provenance string |
|---|---|---|---|---|
| Black hole mass | $M_\bullet$ | $4.1 \times 10^6$ | $M_\odot$ | This manuscript (Coulomb Shearing V7), Section 09, Worked Example 2 (Sgr A*), Inputs table, row: $M_\bullet = 4.1 \times 10^6 M_\odot$; cited there as Genzel et al. 2010 (mass review). |
| One-zone region radius (example geometry) | $r_h$ | $5\,r_{\rm g}$ | dimensionless | Event Horizon Telescope Collaboration 2022, ApJL, 930, L16, Sec. 2.1 (one-zone model parameter block), entry: $r_h = 5\,r_g$. |
| Electron temperature parameter (emission peak) | $\theta_{e,\rm bpk}$ | 10 | dimensionless | Event Horizon Telescope Collaboration 2022, ApJL, 930, L16, Section 2.1 (one-zone model parameter block), entry: $\theta_{e,\rm bpk} = 10$. |
| Electron density normalization | $n_{e,0}$ | $1.0 \times 10^6$ | $\rm cm^{-3}$ | Event Horizon Telescope Collaboration 2022, ApJL, 930, L16, Section 2.1 (one-zone model parameter block), entry: $n_{e,0} = 1.0 \times 10^6\ \rm cm^{-3}$. |

Continued on next page

| **Quantity** | **Symbol** | **Value** | **Units** | **Exact provenance string** |
|---|---|---|---|---|
| Density slope | $\alpha$ | 1.1 | dimensionless | `Event Horizon Telescope Collaboration 2022, ApJL, 930, L16, Section 2.1 (one-zone model parameter block), entry:` $\alpha = 1.1$. |
| Magnetic field normalization | $B_0$ | 29 | G | `Event Horizon Telescope Collaboration 2022, ApJL, 930, L16, Section 2.1 (one-zone model parameter block), entry:` $B_0 = 29\,\mathrm{G}$. |
| Observed X-ray luminosity (quiescent constraint) | $L_{X,\mathrm{obs}}$ | $2 \times 10^{33}$ | $\mathrm{erg\,s^{-1}}$ | `Baganoff et al. 2003, ApJ, 591, 891, PDF p. 22 (printed page label "912" within PDF), Section 10 (Conclusions), item 1: "The 2-10 keV luminosity is` $\sim 2\times10^{33}$ `ergs` $\mathrm{s}^{-1}$`".` |
| Adopted polar inflow speed (evaluation input) | $v$ | $0.1\,c$ | $c$ | `This manuscript (Coulomb Shearing V7), Section 09, Worked Example 2 (Sgr A*), Inputs table, row:` $v = 0.1\,c$. |

Continued on next page

| Quantity | Symbol | Value | Units | Exact provenance string |
|---|---|---|---|---|
| Adopted residence time (evaluation input) | $t_{\mathrm{res}}$ | $2 \times 10^5$ | s | `This manuscript (Coulomb Shearing V7), Section 09, Worked Example 2 (Sgr A*), Inputs table, row:` $t_{\mathrm{res}} = 2 \times 10^5\,\mathrm{s}$. |
| Coulomb logarithm (evaluation input) | $\ln \Lambda$ | 20 | dimension-less | `This manuscript (Coulomb Shearing V7), Section 09, Worked Example 2 (Sgr A*), Inputs table, row:` $\ln \Lambda = 20$. |
| Magnetic multiplier (evaluation input) | $\eta_B$ | 1.0 | dimension-less | `This manuscript (Coulomb Shearing V7), Section 09, Worked Example 2 (Sgr A*), Inputs table, row:` $\eta_B = 1.0$. |
| Lorentz factor (evaluation input) | $\eta_H$ | 1.0 | dimension-less | `This manuscript (Coulomb Shearing V7), Section 09, Worked Example 2 (Sgr A*), Inputs table, row:` $\eta_H = 1.0$. |
| Threshold factor (evaluation input) | $\kappa$ | 3 | dimension-less | `This manuscript (Coulomb Shearing V7), Section 09, Worked Example 2 (Sgr A*), Inputs table, row:` $\kappa = 3$. |

#### D.2.3 Source links and extraction anchors

- **EHT Sgr A* Paper V (one-zone anchor and basic parameters).** DOI: `https://doi.org/10.3847/2041-8213/ac6672`. Accessed: `2026-01-10`. Extraction anchors: Section 2.1 (one-zone model parameter block), as recorded in the provenance strings in the input table above.

- **Quiescent 2–10 keV luminosity anchor.** ADS abstract: `https://ui.adsabs.harvard.edu/abs/2003ApJ...591..891B/abstract`. Accessed: `2026-01-10`. Extraction anchors: Conclusions section, item 1 (PDF p. 22; printed page label "912" within the PDF), as recorded in the provenance string for $L_{X,\mathrm{obs}}$.

## D.3 Object 3: JWST LRD J0647_1045

### D.3.1 Identifiers

- Primary common name: JWST Little Red Dot (LRD) J0647_1045
- Alternative names / aliases: MACS J0647 field LRD; NIRSpec catalog ID 1045 (DAWN JWST Archive)
- Catalog / database identifiers (non-exhaustive): arXiv:2312.03065; A&A 691, A52 (2024); DOI: 10.1051/0004-6361/202348857
- Coordinates: RA = 101.933406 deg; Dec = 70.198268 deg (as printed in the cited source)

### D.3.2 Input table (final)

**Scope note:** This table lists the verbatim numerical inputs used for the Section 09 JWST LRD exemplar anchor. Only direct inputs are included; intermediate quantities and derived values are omitted.

| Quantity | Symbol | Value | Units | Exact provenance string |
|---|---|---|---|---|
| Object designation | — | J0647_1045 | — | `Killi et al. 2024, arXiv:2312.03065, Abstract: object name "J0647_1045".` |
| Spectroscopic redshift | $z$ | $4.5321 \pm 0.0001$ | dimensionless | `Killi et al. 2024, arXiv:2312.03065, Abstract: "J0647_1045 at` $z = 4.5321 \pm 0.0001$`".` |
| Right ascension | RA | 101.933406 | deg | `Killi et al. 2024, arXiv:2312.03065, Sec. 2.1 (Observations and Data): "RA=101.933406".` |

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| Quantity | Symbol | Value | Units | Exact provenance string |
|---|---|---|---|---|
| Declination | Dec | 70.198268 | deg | `Killi et al. 2024, arXiv:2312.03065, Sec. 2.1 (Observations and Data): "Dec=70.198268".` |
| JWST program identifier | — | GO-1433 | — | `Killi et al. 2024, arXiv:2312.03065, Sec. 2.1 (Observations and Data): "GO program (ID: GO-1433; PI: Dan Coe)".` |
| NIRSpec catalog identifier | — | 1045 | — | `Killi et al. 2024, arXiv:2312.03065, Sec. 2.1 (Observations and Data): "NIRSpec catalog ID 1045".` |
| Effective radius (long-wavelength filters; upper bound) | $r_{e,\mathrm{LW}}$ | $< 0.17$ | kpc | `Killi et al. 2024, arXiv:2312.03065, Abstract: "unresolved (`$r_e < 0.17\,\mathrm{kpc}$`) in the three NIRCam long-wavelength filters".` |
| Effective radius (short-wavelength filters) | $r_{e,\mathrm{SW}}$ | $0.45 \pm 0.06$ | kpc | `Killi et al. 2024, arXiv:2312.03065, Abstract: "extended (`$r_e = 0.45 \pm 0.06\,\mathrm{kpc}$`) in the three short-wavelength filters".` |
| Continuum attenuation (UV component) | $A_{V,\mathrm{UV}}$ | $0.54 \pm 0.01$ | mag | `Killi et al. 2024, arXiv:2312.03065, Abstract: "`$A_V = 0.54 \pm 0.01$` (UV)".` |

Continued on next page

| Quantity | Symbol | Value | Units | Exact provenance string |
|---|---|---|---|---|
| Continuum attenuation (optical/NIR component) | $A_{V,\mathrm{opt}}$ | $5.7 \pm 0.2$ | mag | `Killi et al. 2024, arXiv:2312.03065, Abstract: "`$A_V = 5.7 \pm 0.2$` (optical/NIR)".` |
| Broad H$\alpha$ width (FWHM) | $\mathrm{FWHM}_{\mathrm{H}\alpha,}$ broad | $\sim (4300 \pm 300)$ | $\mathrm{km\,s^{-1}}$ | `Killi et al. 2024, arXiv:2312.03065, Abstract: "broad (full width at half-maximum `$\sim 4300 \pm 300\,\mathrm{km\,s^{-1}}$`)".` |
| Balmer decrement attenuation (narrow lines) | $A_{V,\mathrm{BD,nar}}$ | $1.1 \pm 0.2$ | mag | `Killi et al. 2024, arXiv:2312.03065, Abstract: "`$A_V = 1.1 \pm 0.2$` from the Balmer decrement using narrow H`$\alpha$` and H`$\beta$`".` |
| Balmer decrement attenuation (broad-line bound) | $A_{V,\mathrm{BD,broad}}$ | $> 4.1 \pm 0.2$ | mag | `Killi et al. 2024, arXiv:2312.03065, Abstract: "`$A_V > 4.1 \pm 0.2$` from broad H`$\alpha$` and upper limit on broad H`$\beta$`".` |
| Black hole mass estimate (single-epoch broad H$\alpha$) | $M_\bullet$ | $(8 \pm 1) \times 10^8$ | $M_\odot$ | `Killi et al. 2024, arXiv:2312.03065, Abstract: "mass of the central black hole is `$8 \pm 1 \times 10^8\, M_\odot$`".` |

#### D.3.3 Source links and extraction anchors

- **Killi et al. (2024) LRD exemplar J0647_1045.** DOI: `https://doi.org/10.1051/0004-6361/202348857`. arXiv: `https://arxiv.org/abs/2312.03065`. PDF: `https://arxiv.org/pdf/2312.03065.pdf`. Accessed: `2026-01-12`. Extraction anchors: Abstract (redshift, size, attenuation, line-width summary values) and Section 2.1 (Observations and Data) for observing-program metadata, as recorded in the provenance strings in the input table above.

# E Control Runs and Forcing-Term Sensitivity

This appendix collects control runs that are not used as the primary evidentiary basis for any claim in the main text, but are retained for traceability and sensitivity auditing. A control run is defined here as a computation that either (i) enforces a strict specialization of the declared force model (for example $E_{\parallel} = 0$), or (ii) varies a single declared modeling choice while holding all extracted inputs fixed, solely to quantify how outputs change under that constrained variation.

The role of this appendix is strictly procedural:

- Preserve conservative baselines (lower-bound forcing specializations) for audit purposes.
- Prevent dilution of the main worked-example narrative by separating baseline checks from the benchmark mechanism demonstration.
- Ensure that any baseline case remains fully reproducible using the same input packets and symbol declarations as the benchmark run.

Unless explicitly stated otherwise within a given control run, all object inputs are identical to the corresponding benchmark worked example in Section 09 and are sourced from the same input packet set in Appendix D.

## E.1 Control Run E.1: M87* Thomson-only baseline ($E_{\parallel} = 0$)

This control run reproduces the M87* admissibility evaluation with the electromagnetic term disabled, i.e. $E_{\parallel} = 0$ in Eq. (58), so that $\Delta a_{\parallel}$ reduces to the Thomson radiative specialization (Eq. (59)). It exists solely to document the limiting case previously executed in the main text and to preserve a strict baseline.

At the reference radius $r_\star = 5\, r_g$, using the same declared inputs as the benchmark run, the Thomson-only differential acceleration is

$$\Delta a_{\parallel}(r_\star) = \mu_s\, a_{\mathrm{rad},e}(r_\star) \approx 4.07\ \mathrm{cm\,s^{-2}} \quad (\mu_s = 1), \tag{287}$$

which implies (free-drift limit, $t_{\mathrm{res}} \ll t_c$)

$$\Delta x(r_\star; Z) \simeq \tfrac{1}{2}\Delta a_{\parallel}(r_\star)\, t_{\mathrm{res}}(r_\star)^2 \approx 2.94 \times 10^{10}\ \mathrm{cm}, \tag{288}$$

and therefore

$$\frac{\Delta x(r_\star; Z)}{\kappa\, \lambda_D(r_\star)} \approx 2.98 \times 10^6 \gg 1. \tag{289}$$

Thus, for M87*, the Thomson-only limit already fails the closure criterion strongly. The benchmark run in Section 9.1 prevents marginalization of magnetic effects by executing a non-ideal $E_{\parallel}$ prescription; the control run here documents the $E_{\parallel} = 0$ limit previously used.

### E.2 Control Run E.2: M87* relativistic-corrected robustness check (Thomson-only)

This control run applies the minimal relativistic correction factors defined in section 3.5 to the M87* Thomson-only baseline in Control Run E.1, without changing any extracted inputs or any closure definition. It exists solely to evaluate the conservative revision gate on the dimensionless displacement ratio $\Delta x/(\kappa\lambda_D)$ at the near-horizon reference radius $r_\star = 5\,r_g$.

**Applied corrections.** For the Thomson-only track ($E_\parallel = 0$), the only required correction is the radiative forcing factor in Eq. (74) and the proper-time mapping in Eq. (76). Writing the baseline (Control Run E.1) quantities with the superscript "(base)" and the corrected quantities with "(corr)", we have

$$\Delta a_\parallel^{(\mathrm{corr})}(r_\star) = \Delta a_\parallel^{(\mathrm{base})}(r_\star)\, g^{1/2}\left(1 + \frac{v_r}{c}\right), \tag{290}$$

and

$$t_{\mathrm{res}}^{(\mathrm{proper})}(r_\star) = t_{\mathrm{res}}^{(\mathrm{coord})}(r_\star)\, g^{1/2}. \tag{291}$$

In the free-drift limit used in Control Run E.1, $\Delta x \propto \Delta a\, t_{\mathrm{res}}^2$, so the corrected displacement obeys

$$\frac{\Delta x^{(\mathrm{corr})}(r_\star; Z)}{\Delta x^{(\mathrm{base})}(r_\star; Z)} = g^{3/2}\left(1 + \frac{v_r}{c}\right). \tag{292}$$

If $\lambda_D$ is evaluated from the same extracted plasma state at the same radius (Appendix D inputs unchanged), the relativistic corrections act primarily through the multiplier in Eq. (292).

**Numerical robustness bound at $r_\star = 5\,r_g$.** Using the adopted near-horizon redshift factor $g \approx 0.71$ at $r_\star = 5\,r_g$,

$$g^{3/2} \approx 0.71^{3/2} \approx 0.60.$$

Therefore,

$$\frac{\Delta x^{(\mathrm{corr})}}{\Delta x^{(\mathrm{base})}} \approx 0.60\left(1 + \frac{v_r}{c}\right),$$

which is order unity for any sub-relativistic radial component and remains $\lesssim 1.2$ even in the limiting outward case $v_r \to c$. The conservative revision gate in section 3.5.4 is a factor-of-3 change in $\Delta x/(\kappa\lambda_D)$; the multiplier above is far smaller than that threshold in magnitude.

**Outcome of the gate check.** Under the minimal relativistic correction factors declared in section 3.5, the M87* Thomson-only control run does not trigger the factor-of-3 revision gate. The closure failure conclusion for the Thomson-only track is therefore robust to this minimal near-horizon relativistic correction.

### E.3 Control Run E.3: M87* polar delivery fraction bottleneck check

This control run instantiates the continuity-based polar delivery estimator defined in section 2.2.4 for M87*. The purpose is to determine whether polar inflow constitutes a potentially growth-relevant fraction of the total accretion budget under conservative, externally motivated bounds. No Coulomb Shearing criteria are modified, and no worked-example inputs are altered.

**Evaluation surface and geometry.** The control surface is taken at the same near-horizon reference radius used elsewhere for M87*,

$$r_\star = 5\, r_g,$$

with a symmetric two-cap polar region of half-opening angle $\theta_{\rm cap} = 10^\circ$. The corresponding polar solid-angle fraction is

$$\frac{\Omega_{\rm polar}}{4\pi} = 1 - \cos\theta_{\rm cap} \approx 1 - \cos(10^\circ) \approx 1.5 \times 10^{-2}.$$

**Density contrast bound.** GRMHD simulations of low-luminosity AGN disks with magnetically dominated funnels typically show that the polar-adjacent inflow sheath is not evacuated to arbitrarily low density. A conservative bound consistent with such simulations is

$$\frac{\rho_{\rm polar}}{\rho_{\rm disk}} \sim 0.3,$$

which intentionally disfavors polar mass loading relative to the disk body.

**Radial velocity contrast bound.** Near the spin axis, the centrifugal barrier is reduced and the inward radial velocity can exceed that of the disk body. A conservative, order-of-magnitude bound is adopted:

$$\frac{|v_{r,\rm polar}|}{|v_{r,\rm disk}|} \sim 5,$$

which lies in the middle of the commonly cited range $3-10$.

**Resulting polar delivery fraction.** Substituting these bounds into Eq. (16) yields

$$f_{\rm polar}(r_\star) \;\approx\; (1.5 \times 10^{-2}) \times 0.3 \times 5 \;\approx\; 2.3 \times 10^{-2}.$$

**Bottleneck assessment.** The resulting estimate satisfies

$$f_{\rm polar}(r_\star) \gtrsim 10^{-2},$$

so the polar delivery fraction does not fall into the strongly suppressive bottleneck regime defined in section 2.2.4. Under these conservative bounds, polar inflow is therefore capable of contributing at the several-percent level to the total mass supply, which is sufficient for the growth enhancement evaluated elsewhere in this manuscript to remain non-negligible for M87*.

This control run does not claim universality. It documents that, for M87* under conservative and auditable assumptions, disk-to-polar transport is not the dominant limiting factor for the applicability of Coulomb Shearing.

# F Change Log

**Version 1.00 — January 20, 2026**

**Change Type: Initial Release**

**Scientific Status:** Original publication.

**Version 1.01 — April 20, 2026**

**Change Type: Formatting and Clarification (No Scientific Model Changes)**

**Scientific Status:** No changes to equations, derivations, or model predictions.

- **Change Log Added:** Introduced Appendix F to provide version traceability.
- **Force Model Clarification (Section 3):** Explicitly stated that electron and ion force expressions are presented in a minimal Newtonian form for auditable scaling. Clarified that GR-consistent treatments are handled through local radiative flux and redshift relations to luminosity at infinity.
- **Eddington Framework Clarification (Section 6):** Distinguished Newtonian normalization from GR-local force balance by introducing the redshifted Schwarzschild Eddington condition. Removed ambiguity between local electron forcing and global luminosity constraints.
- **Formatting Correction (Section 11 – Limitations):** Replaced long optional enumerate labels with in-text bold headers to eliminate margin overflow at page breaks.

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